Escaping Unemployment Traps*

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Abstract

Sluggish economic activity since the Great Recession has raised the question whether temporary downturns can inflict permanent scars on the economy. In view of this, we ask how monetary policy should be conducted. In our model, unemployed workers lose skill and are expensive to re-train, generating multiple steady state unemployment rates. Large shocks cause the zero lower bound on nominal interest rates to bind, generating deflation. With nominal wage rigidities, this reduces hiring, sending the economy towards the high-unemployment steady state. Neutral monetary policies that seek to replicate allocations as in the flexible wage economy are insufficiently accommodative, generating at best, a slow recovery, and at worst, a complete failure to return the economy back to full employment. More expansionary monetary policy which sets prices higher than warranted in the flexible wage benchmark, prevents a recession and can even engineer quick escapes from an unemployment trap.

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1 Introduction

In the aftermath of the global financial crisis, global economic activity has remained subdued, suggesting that the world economy may be on a lower growth trajectory compared to the pre-2007 period. This raises the threat that temporary downturns can inflict permanent damage to the productive capacity of an economy. How should monetary policy be conducted differently in light of this possibility?

We build a stylized model to answer this question. The central ingredient is an externality in the labor market: unskilled workers are costly to retrain and workers lose human capital whilst unemployed. The possibility of skill loss combined with the cost of retraining workers gives rise to multiple steady state unemployment rates. One desirable steady state is a high-pressure economy: job finding rates are high, unemployment is low and new hires require little training. While wages are high because tight labor markets improve workers' outside option, firms still find it attractive to create jobs as these higher wages are tempered by lower training costs stemming from the higher proportion of skilled job-seekers. However, the economy can also be trapped in an undesirable steady state . Such steady states are characterized by a low pressure economy where job finding rates are low, unemployment is high, and many of the unemployed have lost human capital. Poor outside options for workers drive down wages, but hiring is still limited as firms find it costly to retrain these workers. Crucially, even in the absence of nominal rigidities, an economy near the low pressure steady state can never transition back to a high pressure economy.

To study monetary policy, we introduce nominal wage rigidities to this economy and ask whether monetary policy can prevent temporary demand and supply shocks from permanently setting the economy on a trajectory away from the high pressure steady state. We then ask whether monetary policy can resuscitate a low pressure economy and usher it back towards full employment. The presence of nominal wage rigidities potentially allows monetary policy to boost hiring by increasing prices and effectively lowering real wages. However, the effectiveness of monetary policy may be constrained by the zero lower bound (ZLB) on nominal interest rates.

We consider three classes of monetary policy rule. The first is a price level target that keeps prices at a level consistent with a high pressure steady state. The second is a *neutral* monetary policy which seeks to replicate allocations in the flexible wage economy. In response to small enough transitory shocks, these two policy regimes are able to engineer a quick recovery to the full employment steady state. However, large temporary shocks cause the lower bound on interest rates to bind. In this scenario, these regimes at best generate a slow recovery and at worst, completely fail to return the economy to the high pressure steady state. Thus, these two apparently reasonable monetary regimes can fail to curb the advent of hysteresis, allowing for the possibility of permanent scarring.

Under price level targeting, a large enough transitory demand shock causes a permanent decline in output. Adverse demand shocks cause prices to fall, causing real wages to rise and thus, reducing hiring. This decline in hiring lengthens the average duration of unemployment and increases the incidence of skill loss, leading to a worsening of the skill composition of the unemployed. This deterioration of the skill composition implies that even after the shock has abated, the economy still requires lower real wages to sustain hiring. A commitment to return prices to their steady state level neglects to address the higher retraining costs that firms face and has the perverse effect of keeping real wages too high. This causes hiring and employment to further decline. In fact in our stylized model, the economy transitions towards zero employment.

Neutral monetary policy, that instead tries to replicate the real wage as in the flexible wage economy, fares only slightly better. This regime recognizes that large adverse demand shocks reduce *potential* output, and require lower real wages in order to sustain hiring. In trying to replicate the trajectory of the real economy, such rules only lower wages enough to successfully keep the economy from collapsing to zero employment. As the flexible wage economy may itself feature slow transitions back to full employment or even stagnation, a monetary policy which tries to replicate this can do no better. Thus, such a monetary regime can leave the economy in a persistently sluggish trajectory.

Next, to study if monetary policy can prevent permanent scars, we consider rules which deviate from replicating real allocations in order to ensure the economy returns to full employment after adverse shocks. We show that these alternative monetary policies can prevent a recession from ever occurring. These policies work through a commitment to keep prices temporarily *higher* than warranted by conditions in the real economy. When the demand shock hits, the monetary authority is constrained by the zero bound, but promises to keep prices higher in the future, even if the economy remains at full employment. Higher future prices raise prices today, mitigating the rise in real wages; they also promise firms higher profits in the future, encouraging hiring even if current wages remain high. Thus, a monetary policy stance that is more accommodative than neutral monetary policy is necessary to avoid permanent scarring.

Furthermore, if the economy is near or at the low pressure steady state, this alternative monetary policy may still be able to pull the economy out of the high unemployment trap. By committing to keep prices elevated for an extended period of time, this policy can boost hiring today and over the near future until the economy returns to full employment. However, the feasibility of such a policy depends critically on the monetary authority's ability to create a boom, and not just avert a recession.

For example, if nominal wages are downwardly rigid but perfectly flexible upwards, an attempt by monetary policy to raise prices is nullified by the nominal wage rising by the same extent.[1] Since monetary policy cannot engineer booms in hiring, expansionary monetary policy is powerless to make the economy escape an unemployment trap. In such a setting, it become all the more important to prevent the economy from entering such a trap in the first place. Thus, the results in our model echo the arguments made by DeLong (2015) that the risks associated with monetary

[1] In contrast, monetary policy can always lower prices, raise real wages and reduce hiring. Thus, monetary policy is capable of creating recessions but not booms.

easing are asymmetric: excessively easy policy can be reversed but excessively tight policy causes irreversible damage.

Importantly, we make the case that monetary rules that seek to track the natural rates of unemployment, interest rate and etc, are insufficient to steer the economy back to full employment once large adverse shocks shift the economy to or near the low pressure steady state. Because different steady states are associated with different natural rates of unemployment and so forth, monetary policy should actually select the natural rate and consequently, the steady state it would like the economy to achieve. In this sense, we argue that accommodative monetary policy has the ability to stem the advent of hysteresis and prevent the economy from observing long-lasting declines in output and employment.

Our model is deliberately highly stylized, allowing us to analytically characterize equilibria in a monetary economy with multiple steady states.[2] In comparison, standard New Keynesian (NK) models typically study stationary fluctuations around a unique steady state. By construction, this precludes the possibility that short run disturbances can cause permanent damage, ruling out out any possibility for monetary policy to influence long run outcomes.[3] Furthermore, in standard NK models, the failure of monetary policy to equate the real interest rate to the *Wicksellian rate* r^* due to the ZLB typically precipitates recessions, implying that any policy which can help equate these two rates should be sufficient to prevent a recession. We abstract from this problem altogether by studying an environment in which the real rate is always equal to the Wicksellian rate, and show that this is still not enough to prevent the economy from entering a recession.

The remainder of the paper is structured as follows. The rest of this section discusses the related literature. In section 2, we present the model economy . In section 3, we characterize steady states and equilibria in the flexible wage benchmark. In section 4, we introduce nominal rigidities, and study the economy's response to both demand and supply shocks under different monetary policy regimes. We conclude in Section 5.

Related Literature The empirical literature generally finds evidence consistent with the hysteresis hypothesis - the productive capacity of the economy falls after a recession. Dickens (1982) finds that recessions can temporarily lower productivity while Haltmaier (2012) finds that trend output falls by 3 percentage points on average in developed economies four years after a pre-recession peak. Ball (2009) finds that in a sample of 20 developed economies, large increases in the natural rate of unemployment are associated with disinflations, and large decreases with inflation. Ball (2014) shows that countries with a larger fall in output during the Great Recession experienced a larger decline in potential output. However, this evidence also admits the alternative interpretation that the same forces that caused potential output to fall were also the cause of the recession. Focusing on employment, Song and von Wachter (2014) find that the persistent decline in employment

[2]It is important to distinguish our approach from that of Farmer (2012) who considers economies with a continuum of steady states but focuses on how beliefs cause an economy to transition between these.

[3] Summers (2015) expounds on this criticism of New Keynesian models at length.

following job displacement is more pronounced during recessions, suggesting that a severe spike in job-destruction rates can have a lasting impact on unemployment rates.

Blanchard and Summers (1986) proposed an explanation of hysteresis based on insider-outsider labor markets while Fatas (2000) and Bianchi and Kung (2014) find that declines in R&D expenditures during recessions can account for persistent effects of cyclical shocks on growth. Our model instead draws on Pissarides (1992), who presented a model in which skill depreciation gives rise to multiple steady states. Sterk (2016) also uses a similar model and argues that such a model can empirically explain observed job finding rates remarkably well. Drazen (1985) also argues that the loss of human capital in recessions can lead to delayed recoveries. Relative to these papers, our contribution is to introduce nominal rigidities and study monetary policy. Similar to recent work by Schaal and Taschereau-Dumouchel (2016) who consider how the level of aggregate demand affects firms' hiring decisions, the unemployment rate is a key state variable in our model. While knowledge of the unemployment rate helps firms forecast future demand in Schaal and Taschereau-Dumouchel (2016), the unemployment rate in our model reflects the extent of skill deterioration in the economy and affects firms' incentive to create jobs via the cost channel.

A few papers study hysteresis and monetary policy within the New Keynesian framework. Reifschneider et al. (2013) augment the Federal Reserve Board's FRB/US model to include reduced form equations relating the NAIRU and labor force participation rate to unemployment, and study optimal policy using a quadratic loss function. Kapadia (2005) performs a similar exercise in the 3 equation model. Galí (2016) studies optimal policy in a version of the canonical model in which hysteresis arises due to insider-outside labor markets. These papers use first order methods to study economies with a unique steady state, in which hysteresis adds persistence, but does not create unemployment traps. We consider a model with multiple steady states, and study how monetary policy affects long run outcomes.

Our paper also relates to the recent literature on secular stagnation. Eggertsson and Mehrotra (2014) and Caballero and Farhi (2016) present models in which the market clearing interest rate is negative for an extended period of time, or even permanently, leading to persistently low output, as nominal interest rates are prevented from falling to clear markets by the ZLB. In these models, typically a permanent change in fiscal or monetary policy (such as an increase in the inflation target) is required to prevent stagnation. We share this literature's concern with long run outcomes, but consider a different mechanism: *temporary* falls in market clearing interest rates can have permanent effects on potential output, and a *temporary* burst of accommodative monetary policy can prevent this. Recent work by Benigno and Fornaro (2015) also studies an economy in which pessimistic expectations can drive the economy to the ZLB and lead to very persistent or even permanent slowdowns. They argue that subsidizing innovation activity can help economies exit such *stagnation traps*. This resonates with our finding that if an economy is stuck in a high-unemployment steady state, monetary policy may be incapable of repairing the damage without the help of other tools such as fiscal policy.

2 The Model Economy

We use a standard Diamond-Mortensen-Pissarides model of labor-market frictions. The model is formulated in discrete time and for simplicity, there is no uncertainty. The only addition to the standard model is that we assume that workers can lose skill following an unemployment spell. Next, we describe the economic agents and environment in detail.

Workers The economy consists of a unit mass of workers who are ex-ante identical. These workers are risk neutral and discount the future at a rate β. Workers can either be employed or unemployed. We denote the mass of employed workers as n and the mass of unemployed as $u = 1 - n$. All unemployed workers produce $b > 0$ as home-production. Households can borrow and save in a nominal bond which pays a certain nominal return of $1 + i_t$, yielding the Euler equation:

$$\beta(1 + i_t)\frac{P_t}{P_{t+1}} = 1 \tag{1}$$

where P_t is the price of consumption at date t. The number of employed workers at any date t is given by:

$$n_t = [1 - \delta(1 - q_t)]\, n_{t-1} + q_t u_{t-1} \tag{2}$$

where δ is the exogenous rate at which workers get separated from their current jobs and q_t is the job-finding rate. Note that equation (2) implies that a worker that gets separated at the beginning of period t can find another job in the same period. Next, let $\mathbb{W}_t$ denote the value of an employed worker and $\mathbb{U}_t$ denote the value of an unemployed worker at time t. These can be expressed as follows:

$$\mathbb{W}_t = w_t + \beta\Big\{ [1 - \delta(1 - q_{t+1})]\, \mathbb{W}_{t+1} + \delta(1 - q_{t+1})\mathbb{U}_{t+1} \Big\} \tag{3}$$

$$\mathbb{U}_t = b + \beta\Big\{ q_{t+1}\mathbb{W}_{t+1} + (1 - q_{t+1})\mathbb{U}_{t+1} \Big\} \tag{4}$$

where w_t denotes the real wage at date t and b is the value of home production. Notice that the net value of being employed for a worker can be written as:

$$\mathbb{W}_t - \mathbb{U}_t = w_t - b + \beta(1 - \delta)\,(1 - q_{t+1})(\mathbb{W}_{t+1} - \mathbb{U}_{t+1})$$

All else being equal, the worker's gain from matching is smaller (her outside option is higher) if q_{t+1} is higher.

Labor Market As in Pissarides (1992), we assume that a worker who gets separated from her job and is unable to transition back to employment immediately, loses skill that she acquired while employed. In other words, we label any worker who has been unemployed for at least 1 period as *unskilled*. Importantly, we assume that if a firm employs an unskilled worker, then it must incur a training cost $\chi > 0$. Once the firm incurs the training cost, the worker acquires skill and remains *skilled* until the next unemployment spell of at least 1 period. At any time t, μ_t denotes the fraction of unskilled workers in the pool of job-seekers (l_t) and is defined as:

$$\mu_t = \frac{u_{t-1}}{l_t} \equiv \frac{u_{t-1}}{1-(1-\delta)(1-u_{t-1})} \tag{5}$$

As can be seen from the equation above, a higher level of unemployment in the past corresponds to a lower fraction of skilled job-seekers. As such, there is a one-to-one mapping between u_{t-1} and μ_t.

Matching Technology Search is random. The number of successful matches m_t between searchers l_t and vacancies v_t is given by a CRS matching technology $m(v_t, l_t)$. We define market tightness θ_t as the ratio of vacancies to job seekers. Then we can express the job-finding rate q_t as:

$$q(\theta_t) = \frac{m(v_t, l_t)}{l_t} \tag{6}$$

In a similar fashion, we can define the job-filling rate as:

$$f(\theta_t) = \frac{m(v_t, l_t)}{v_t} = \frac{q(\theta_t)}{\theta_t} \tag{7}$$

We consider a particular form of the matching function in the economy:

$$m_t = \min\{l_t, v_t\}$$

which implies $q(\theta_t) = \min\{\theta_t, 1\}$, $f(\theta_t) = \min\{1/\theta_t, 1\}$. This matching function simplifies the analysis without losing any generality. In particular, it implies that the short side of the marker matches with probability 1. We refer to the case with $\theta_t < 1$ the *slack labor market regime* and $\theta_t \geq 1$ the *tight labor market regime.*

Firms The production side of the economy is summarized by a representative competitive CRS firm which only uses labor as an input to produce the final good. The production function is given by:

$$y_t = An_t \tag{8}$$

where $A > b$ is the level of aggregate productivity and n_t is the number of employed workers in period t. A firm must incur a vacancy posting cost of $\kappa > 0$ and an additional training cost of χ for each unskilled worker. The problem of the firm that has n_{t-1} workers at the beginning of period

t can be expressed as choosing vacancies (taking wages as given) in order to maximize lifetime discounted profit:

$$\mathcal{J}_t = \max_{v_t \geq 0} (A - w_t) n_t - (\kappa + \chi \mu_t f_t) v_t + \beta \mathcal{J}_{t+1}$$

subject to

$$n_t \quad = \quad (1-\delta) n_{t-1} + f_t v_t \tag{9}$$

where w_t is the wage that the firm pays all its workers.[4] Notice that this is the standard problem of a firm in search models with one difference. The total cost of job creation effectively depends on the skill composition of job-seekers. Since the firm must pay a cost χ to train each unskilled job-seeker it hires, the effective average cost of creating a job increases in the the fraction of unskilled job-seekers. Recall from equation (5) that this fraction depends on past unemployment rates, making the cost of job creation *increasing* in the unemployment rate. The job-creation condition can then be written as:

$$\frac{\kappa}{f_t} + \chi \mu_t + \lambda_t = A - w_t + \beta(1-\delta) \left\{ \frac{\kappa}{f_{t+1}} + \chi \mu_{t+1} + \lambda_{t+1} \right\} \tag{10}$$

where $\lambda_t f_t \geq 0$ is the multiplier on the non-negativity constraint on vacancies. Using the Envelope Theorem, the value of a filled vacancy, $J_t = \partial \mathcal{J}_t / \partial n_t$, for the firm can be written as:[5]

$$J_t = A - w_t + \beta(1-\delta) J_{t+1} \tag{12}$$

Resource Constraint The resource constraint in the real economy can be written as:

$$c_t = A n_t + b(1 - n_t) - \kappa v_t - \chi \mu_t f_t v_t$$

To close the model, we now need to specify how wages and prices are determined.

Wage and Price Determination Our ultimate goal in this paper is to analyze the conduct of monetary policy in an environment with sticky nominal wages. However, as a benchmark it is useful to define an economy with no nominal rigidities i.e, with flexible wages. To this end, we assume that in the flexible wage economy, wages are determined by Nash bargaining every period. Because Nash Bargaining occurs after all hiring and training costs have been paid, this ensures that

[4] We have assumed that firms pays the same wage to both initially skilled and unskilled hires as well as existing skilled workers. We discuss this in more detail in the section on wage determination.

[5] Using the notation J_t, the job creation condition can also be written as:

$$f_t [J_t - \chi \mu_t] \leq \kappa \ , \ \theta_t \geq 0 \tag{11}$$

all workers are paid the same wage.[6] Formally the Nash-Bargaining problem can be written as:

$$\max_{w_t} J_t^{1-\eta} \left(\mathbb{W}_t - \mathbb{U}_t\right)^{\eta}$$

where $\eta \in [0, 1)$ denotes the bargaining power of the workers. The Nash-Bargained wage can be written as:

$$w_t^* = \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1} J_{t+1} \tag{13}$$

Plugging in the Nash bargained wage into the expression for J_t yields:

$$J_t = a + \beta(1-\delta)(1-\eta q_{t+1}) J_{t+1} \tag{14}$$

where we define $a = (1-\eta)(A-b)$. Again, at every future date, next period's job finding rate puts upward pressure on the Nash wage because it increases the worker's outside option. The result is a smaller profit to the firm or a lower J_t. Iterating this forward and using equation (10), the job creation condition can be rewritten as :

$$J_t = a \sum_{s=0}^{\infty} \beta^s (1-\delta)^s \prod_{\tau=0}^{s} (1-\eta q_{t+\tau}) \leq \kappa + \chi \mu_t \ , \ \theta_t \geq 0 \ , \text{ with at least one strict equality} \tag{15}$$

In this benchmark, the classical dichotomy holds and the price level is irrelevant in characterizing real allocations. Thus, it is not necessary to describe the conduct of monetary policy which determines prices. In Section 4, we describe the economy with sticky nominal wages and specify how monetary policy is conducted in that economy. Next, we analyze the behavior of the flexible wage benchmark.

3 Flexible Wage Benchmark

We now characterize equilibria in the flexible wage benchmark economy. From the previous section, it follows that equilibria are completely described by the following system:

$$\mu_{t+1} = \frac{1-q_t}{1+\gamma[1-q_t-\mu_t]} \tag{16}$$

$$J_t = a + \beta\gamma(1-\eta q_t) J_{t+1} \tag{17}$$

$$J_t \leq \frac{\kappa}{f_t} + \chi\mu_t, \theta_t \geq 0, \text{ at least one strict equality} \tag{18}$$

$$q_t = \min\{\theta_t, 1\} \tag{19}$$

$$f_t = \min\left\{\frac{1}{\theta_t}, 1\right\} \tag{20}$$

[6]The assumption of random search and the fact that the training cost is sunk at the time of bargaining implies that all workers have the same probability of finding a job and hence they share the same outside options.

where we define $\gamma = 1 - \delta$ and (16) is derived by combining equations (5) and (9). Before proceeding further, it is useful to note the following:

Lemma 1. *The value of a filled vacancy to the firm is contained in the closed interval:*

$$J_{min} \le J_t \le J_{max} \tag{21}$$

where $J_{min} := \dfrac{a}{1-\beta(1-\delta)(1-\eta)}$ *and* $J_{max} := \dfrac{a}{1-\beta(1-\delta)}$. *Moreover, if* $J_t = J_{min}$*, then* $q_{t+1} = 1$*, i.e.* $\theta_{t+1} \ge 1$ *and* $J_{t+1} = J_{min}$

Proof. The proof of the first part follows from the fact that $q_t \in [0, 1]$. For the second claim, we know that

$$J_t = a + \beta(1-\delta)(1-\eta q_{t+1})J_{t+1}$$

If $J_t = J_{min}$, one way to attain this value is $q_{t+1} = 1$ and $J_{t+1} = J_{min}$. This is in fact the only way because $q_{t+1} < 1$ and $J_{t+1} \ge J_{min}$ and the expression is decreasing in q_{t+1} and increasing in J_{t+1}. □

3.1 Steady States

Our ultimate goal is to describe an economy in which transitory increases in unemployment can permanently scar the economy and to ask whether monetary policy can do anything about it. We choose to interpret these permanent changes through the lens of a model economy in which there are multiple steady state unemployment rates and temporary shocks can move the economy between these steady states. The different steady-state unemployment rates can be interpreted as different possible values of the NAIRU. To this end, we now describe conditions under which multiple steady states can exist in this economy.

In our economy, multiplicity of steady state unemployment rates is possible because workers' skills depreciate during spells of unemployment and firms must pay a cost to train unskilled workers. Consider an economy plagued with high unemployment. Since the average duration of unemployment is high, the average skill composition of the workforce is low. Consequently, firms need to spend more on training workers which makes them less willing to post vacancies. Thus, a high rate of unemployment can be self-sustaining. Conversely, if unemployment is low, average unemployment duration is low and the average skill of the workforce is high. Since the required outlay on training workers is lower, firms are more willing to post vacancies, thus sustaining a low level of unemployment.

3.1.1 Full employment steady state

Regardless of multiplicity of steady states in the economy, a full employment steady state always exists under conditions that we describe next. In the full employment steady state, $n = 1$ and so it must be the case from equation (2) that $q = 1$ (and $f = 1/\theta \leq 1$). Job seekers are on the short side of the market, and always find a job within one period. Since separated workers immediately find jobs in this steady state, skill depreciation never occurs, and equation (5) implies that $\mu = 0$. The complementary slackness condition (15) becomes

$$\kappa \theta^{FE} = J_{min}$$

This has a solution with $\theta^{FE} \geq 1$ provided that $J_{min} \geq \kappa$. In what follows, we will assume that this is always true and that a full employment steady state (FESS) exists.

Assumption 1 (Full Employment Steady State)**.**

$$J_{min} > \kappa.$$

Note that this assumption requires that the vacancy creation cost is relatively low.

For some parameter values, there may also exist a steady state with no hiring and no employment. Since this case is uninteresting, we rule out this possibility with the following assumptions on parameters. Qualitatively, none of our results would change if we allowed for a zero employment steady state.

Assumption 2 (No Zero Employment Steady State)**.**

$$J_{max} > \kappa + \chi$$

This assumption broadly requires that training costs are not too large.

3.1.2 Existence of Multiple Steady States

Finally, we look for steady states in which firms are on the short side of the labor market, and there is some skill depreciation. To establish the existence of multiple steady states, it is convenient to work with the fraction of unskilled workers μ rather than the unemployment rate u. In the same vein, it convenient to work with a quasi-value function of the firm defined in terms of μ as opposed to J_t. In this section and in what follows, we will use μ as the state variable of interest.[7]

Definition 1 (Quasi-Value Function)**.** *Define the quasi-value function $\mathcal{Q}(\mu)$ as:*

$$\mathcal{Q}(\mu) = \frac{a}{1 - \beta\gamma \left[1 - \eta(1 - \mu)\right]}$$

By construction, $\mathcal{Q}(\mu)$ is the value of the firm as long as the job-finding rate is $1 - \mu$ forever. Note that $\mathcal{Q}'(\mu) > 0$ and $\mathcal{Q}''(\mu) > 0$.

[7]Notice that equation (5) defines a one-to-one map between μ_t and u_{t-1}.

Any interior steady state must satisfy $\mathcal{Q}(\mu) = \kappa + \chi\mu$. Notice that this relation describes a quadratic equation in μ which has at most two solutions. In general, these solutions may not be economically meaningful and may lie outside the closed interval $[0, 1]$. The following assumptions are sufficient to guarantee that economically meaningful solutions exist.

Assumption 3 (High η and χ)**.**

$$\eta > \max\left\{\frac{1-\beta\gamma}{\beta\gamma}, \frac{\gamma}{1+\gamma}\right\}$$

and

$$\chi \in [\underline{\chi}, J_{\max} - \kappa]$$

where $\underline{\chi} := e[2 - k + 2\sqrt{1-k}]J_{min}$ *and* $e = \dfrac{\beta\gamma\eta}{1-\beta\gamma(1-\eta)}$, $k = \dfrac{\kappa}{J_{min}}$.

The first part of this assumption states that workers' bargaining power η must be sufficiently high; the second part states that training cost χ must be sufficiently high to allow for multiplicity. High bargaining power increases workers' share of the surplus; it also increases the sensitivity of wages and profits to labor market conditions. When unemployment is low, wages are high because workers' outside option is relatively favorable; however, firms are willing to pay high wages because training costs are low. When unemployment is high, workers are relatively unskilled and expensive to train; however, firms are willing to spend a lot on training because wages are relatively low, owing to the weak labor market. The following Lemma summarizes the result.

Lemma 2 (Existence of Multiple Steady States)**.** *Under Assumptions 1, 2, and 3 there exist two interior steady states* $0 < \widetilde{\mu} < \overline{\mu} < 1$.

Proof. See Appendix A.2. □

Figure 2 graphically depicts the arguments above. The red curve plots the quasi-value function $\mathcal{Q}(\mu)$, the blue line plots $\kappa + \chi\mu$ for different values of χ. When χ is too low, the two curves do not intersect and there are no interior steady states. When χ is too high, the blue line lies above the red curve at $\mu = 1$ and there exists a zero-employment steady state, violating Assumption 2. When χ is in the appropriate range, then there are two steady states, $\widetilde{\mu}$ and $\overline{\mu}$. Finally, recall that there is always a full employment steady state at $\mu = 0$.

3.2 Dynamics

Having described steady states in the flexible wage economy, we now study its transitional dynamics. We can split initial conditions on μ into 3 regions: (i) healthy (ii) convalescent and (iii) stagnant.

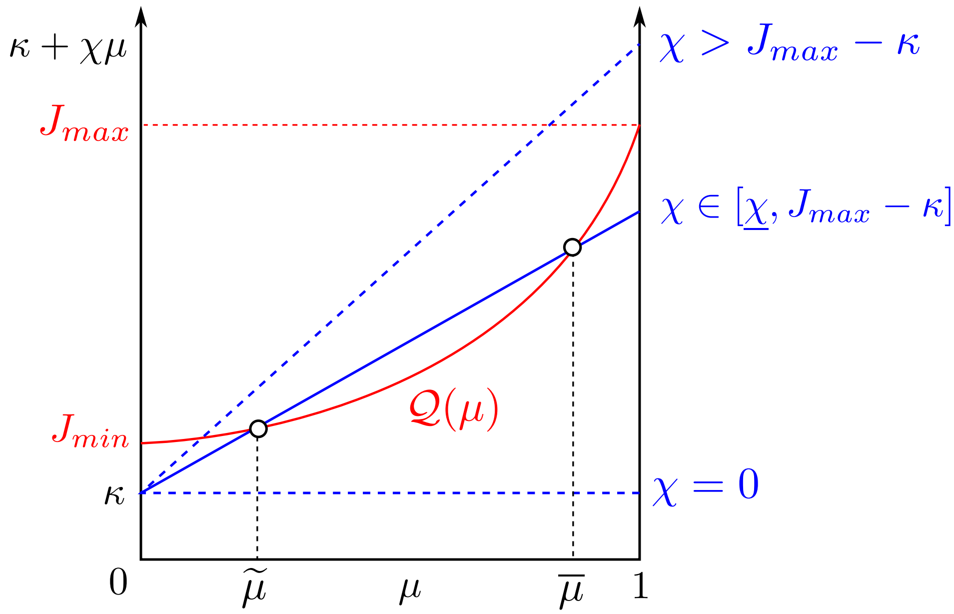


Figure 1. Multiple Steady States

3.2.1 Healthy Region (High Pressure Economy)

The healthy region is defined as the set $\mu \in [0, \underline{\mu})$ where $\underline{\mu}$ is defined as:

$$\underline{\mu} = \frac{J_{min} - \kappa}{\chi} \tag{22}$$

In other words, $\underline{\mu}$ is defined as the highest value of μ for J_{min} is still attainable. For any $\mu < \underline{\mu}$, $q = 1$. The following Lemma says that if the economy starts in the healthy region, then labor markets are tight, $\theta > 1$ and the economy immediately converges to the full employment steady state.

Lemma 3. *Suppose $\mu_0 < \underline{\mu}$. Then the equilibrium is unique, with $n_t = 1$, $\mu_t = 0$ for all $t \geq 1$, and*

$$\theta_t = \begin{cases} \dfrac{J_{min} - \chi\mu_0}{\kappa} & \text{for } t = 0 \\ \dfrac{J_{min}}{\kappa} & \text{for } t \geq 1 \end{cases}$$

Consequently, the value of a filled vacancy to a firm is constant for all $\mu \in [0, \underline{\mu})$, and is given by J_{min}.

Proof. See Appendix A.3. □

Intuitively, when the skill composition of job-seekers is very high, firms are willing to post a large number of vacancies. This creates a tight labor market, and job-seekers who are on the short side of the market find jobs very easily. This creates an environment in which unemployment duration is short (and equal to zero after the first period), and hence the skill quality of the work-force is high.

While we have not introduced any shocks to this economy, one way to interpret the result above is that the full-employment steady state is stable with respect to shocks which only cause a small deterioration in the average skill composition of job-seekers. In particular, if the fraction of unskilled job-seekers rises to a level in the interval $(0, \underline{\mu})$, this shock will be immediately reversed as job-seekers are still largely skilled, and firms are willing to post enough vacancies to hire and retrain them all.

If there is zero hiring for one period, starting from full employment, then the fraction of unskilled job seekers is $\mu_R = 1/(1+\gamma)$. The following result, which will be useful later, states that $\mu_R > \underline{\mu}$. That is, a hiring freeze takes the economy out of the healthy region.

Lemma 4. *Under Assumption 3, if multiple steady states exist, then* $\mu_R = \dfrac{1}{1+\gamma} > \underline{\mu}$.

Proof. See Appendix A.4. □

3.2.2 Convalescent Region

We now describe dynamics in the convalescent region, defined as the set $[\underline{\mu}, \widetilde{\mu})$. In this region, the economy eventually returns to full employment, and thus we can solve the model by backward induction. First we characterize equilibrium at $\underline{\mu}$, the point at which the labor market just becomes tight.

Lemma 5 (On the verge of a full recovery)**.** *If* $\mu_0 = \underline{\mu}$ *then in any equilibrium* θ_0 *lies in the compact set* $[1-\underline{\mu}, 1]$*. Furthermore,* $\mu_1 = \dfrac{1-\theta_0}{1+\gamma[1-\theta_0-\mu_0]} \leq \mu_0$ *and* $\theta_1 = \dfrac{J_{min} - \chi\mu_1}{\kappa} \geq 1$*. For all* $t > 1$*,* $\theta_t = \theta^{FE}$ *and* $\mu_t = 0$*.*

Proof. See Appendix A.5. □

The Lemma states that the economy at $\underline{\mu}$ today will reach the healthy region in the next period. This in turn implies that θ today must be greater than $1-\underline{\mu}$. In fact, any θ between $1-\underline{\mu}$ and 1 is consistent with equilibrium. Next, we describe how the economy evolves starting in the interior of the convalescent region. We proceed by backward induction. First, we characterize the set of μ's in the convalescent region from which the economy can reach $\underline{\mu}$ in one period. We refer to this set as I^1. We then proceed to construct the sets $I^2, I^3, \ldots$ from which the economy can reach $\underline{\mu}$ in $2, 3, \cdots$ periods respectively. Crucially, we show that the union of all these sets is the entire convalescent region. In other words, starting from any point in the convalescent region, there is an equilibrium in which the economy converges to $\underline{\mu}$ in finite time and conversely, any equilibrium that converges to full employment must start from some point in the convalescent region. Furthermore, starting from any μ_0 in the convalexcent region, there is a unique path that the economy takes back to $\underline{\mu}$ and subsequently to steady state. This is formalized in the Proposition below.

Proposition 1 (Dynamics in the Convalescent Region)**.** *For β sufficiently close to 1, there exists a strictly unique increasing sequence $\{\mu^n\}_{n=-1}^{\infty}$ with $\mu^{-1} = \mu^0 \equiv \underline{\mu}$, $\lim_{n\to\infty} \mu^n = \widetilde{\mu}$, such that if $\mu_0 \in I^n \equiv (\mu^{n-1}, \mu^n]$, the economy escapes the convalescent region in $n+1$ periods and reaches the full-employment steady state in $n+2$ periods, i.e., $\mu_n = \underline{\mu}$, $\mu_{n+1} \in (0, \underline{\mu})$ and $\mu_{n+2} = 0$.*

Proof. Appendix A.6 proves the existence of such a path starting from any μ_0 in the convalescent region and shows that it is unique. The appendix also provides an algorithm to construct such a path. □

Figure 2 illustrates this proposition by depicting the equilibrium starting from a point μ_0 in the convalescent region which lies between $\underline{\mu}$ and $\widetilde{\mu}$. In particular, μ_0 lies in the interval $(\mu^{n-1}, \mu^n]$ and consequently, it takes $n+2$ periods for the economy to reach the full employment steady state. The red circles depict the trajectory at dates $t = 0, 1, \cdots$ while the blue line depicts the nullcline associated with equation (16). When $\theta_t > 1 - \mu_t$, $\mu_{t+1} < \mu_t$, i.e. employment is growing over time and the proportion of unskilled individuals in the pool of job-seekers is shrinking. The light-gray line describe the equilibrium mapping from μ_t to θ_t. Also note that the horizontal dashed line at $\theta = 1$ separates the slack and tight labor market regimes. For $t \leq n$ μ_t is in the convalescent region and $\theta_t < 1$ which implies a slack labor market. After n periods, μ_t reaches the healthy region and $\theta_t > 1$ which implies a tight labor market regime.

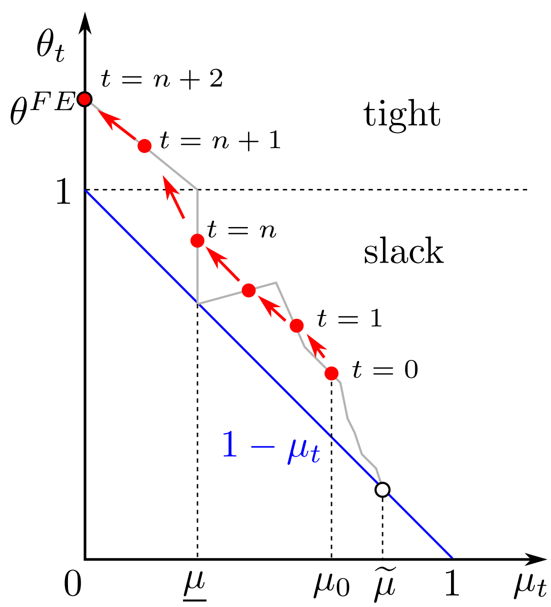


Figure 2. Equilibrium in Convalescent Region

A feature of the equilibrium described above is that the economy can spend an arbitrarily long time in the the convalescent region before transitioning to the healthy region. In particular, as μ_0 gets closer to $\widetilde{\mu}$, the economy takes longer recover and is particularly slow in the early stages. Comparing this with the result in Lemma 3, we see that while the economy quickly recovers to the full-employment steady state following small shocks, it takes a much longer time to recover from any shock that precipitates a large deterioration in the proportion of skilled job-seekers.

Lemma 6 (Slow Recoveries). *Take any $T \in \mathbb{N}$. Then there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > 0$ for all $t < T$. That is, recoveries can be arbitrarily long. More generally, take any $\delta > 0$, $T \in \mathbb{N}$. Then there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > \mu_0 - \delta$ for all $t < T$. That is, recoveries can be arbitrarily slow.*

Proof. First we prove the second part. Fix $\delta > 0, T \in \mathbb{N}$ and let n be the smallest integer such that $\mu^n \geq \widetilde{\mu} - \delta$ (this exists, since $\mu^n \to \widetilde{\mu}$ and $\delta > 0$. Set $\varepsilon = \widetilde{\mu} - \mu^{n+T}$. Take any $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu}) = (\mu^{n+T}, \widetilde{\mu})$. Then $\mu_0 \in (\mu^{m-1}, \mu^m]$ for some $m > n+T+1$. We know from 1 that $\mu_T \in (\mu^{m-T-1}, \mu^{m-T}]$. In particular,

$$\mu_T > \mu^{m-T-1} > \mu^n \geq \widetilde{\mu} - \delta > \mu_0 - \delta$$

Finally, since $\{\mu_t\}$ is monotonically decreasing, we have $\mu_t > \mu_0 - \delta$ for all $t < T$, as claimed. Next, note that the first part of the lemma is a special case of the second part with $\delta = \widetilde{\mu}$. □

3.2.3 Stagnant Region (Low-pressure Economy)

An important corollary of the above results is that any equilibrium converging to full employment must start either in the healthy region or the convalescent region.To see this note that any trajectory which starts to the right of $\underline{\mu}$ and reached full employment at some date T has to be at $\underline{\mu}$ at date $T - 2$. But, in the proposition we considered all trajectories that reached $\underline{\mu}$ and showed that each of these lies entirely within the convalescent region. It follows that if the economy starts in the stagnant region, it can never converge to full employment. In this sense, the stagnant region is an *unemployment trap.*

Corollary 1 (Unemployment Traps). *If $\mu_0 \geq \widetilde{\mu}$, the economy never reaches the full employment steady state.*

To get some intuition for this result, consider the unstable steady state $\widetilde{\mu}$. Starting from this unfavorable position, can this economy ever reach full-employment rather than remaining at this steady state? Recall that the job creation condition ensures firms are just breaking even in equilibrium. Furthermore, since this is a steady state, firms are just posting enough vacancies to keep the skill composition constant at $\tilde{\mu}$. For this economy to move towards full employment, it would require firms to create more vacancies. However, if the economy were to eventually reach full employment, labor markets would be tight and wages high. Thus, firms would expect lower profits along this trajectory compared to the unstable steady state. Since firms were just breaking-even in the steady state, they would certainly be unwilling to tolerate lower profits along this alternative trajectory and would post fewer vacancies. Thus, such a recovery is not feasible. Clearly, this holds a fortiori for $\mu > \widetilde{\mu}$.

For our purposes, it is not important to characterize dynamics within the stagnant region; all that matters it that if the economy reaches this region, it never returns to full-employment. Having said that, the dynamics in the stagnant region are completely described by the following equations:

$$\theta_{t+1} = \frac{1}{\eta}\left[1 - \frac{(\kappa + \chi\mu_t - a)(1+\gamma(1-\theta_t-\mu_t))}{\beta\gamma[\kappa(1+\gamma(1-\theta_t-\mu_t))+\chi(1-\theta_t)]}\right] \tag{23}$$

$$\mu_{t+1} = \frac{1-\theta_t}{1+\gamma[1-\theta_t-\mu_t]} \tag{24}$$

In the slack labor regime, there is a decreasing relation between current market tightness (equivalently, job finding rate) θ_t and next period's tightness (or job finding rate) θ_{t+1}. The intuition is as follows. The Euler equation implies that the firm should be indifferent between posting a vacancy today or posting one tomorrow adjusting for the opportunity cost of money. Higher future job finding rates increase the worker's outside option and cause the worker to bargain for higher wages, reducing the firm's benefit from creating a match today. Thus, in order for the firm to be indifferent between creating a match today and tomorrow, the benefit from creating a match tomorrow must also be lower. This means that it must be more costly to create a match tomorrow, i.e. the expected cost of retraining new hires tomorrow must be larger. Consequently, job finding rates must be low today in order to create a larger pool of unskilled workers tomorrow.

4 Nominal Rigidities

Having characterized the flexible wage benchmark model, we now turn to our main objective: analyzing how temporary shocks affecting the economy can have permanent effects. In particular, we focus on how the conduct of monetary policy facilitates or hinders the adjustment in response to these temporary shocks. We assume that the monetary authority sets nominal interest rates i_t, subject to the zero lower bound $i_t \geq 0$. Because of risk neutrality, the real interest rate is fixed and equal to $r_t = \beta_t^{-1} - 1$, where we now allow β_t to be time varying. Inflation satisfies the Fisher equation:

$$\frac{P_{t+1}}{P_t} = \frac{1+i_t}{1+r_t} = \beta_t(1+i_t)$$

Given a fixed real interest rate, the zero bound puts a lower bound on inflation, i.e. an upper bound on date t prices, given date $t+1$ prices:

$$\beta_t \leq \frac{P_{t+1}}{P_t}$$

Of course, for monetary policy to be relevant in this economy, we need to introduce nominal rigidities in this economy. We do so by assuming that nominal wages are permanently fixed $W_t = W$. We also discuss how our results vary by relaxing this assumption to allow for nominal wages that are flexible upwards but rigid downwards, $W_t \geq W_{t-1}$.

4.1 Shocks

We analyze the response to 2 different scenarios for the economy. Throughout we consider unanticipated shocks which affect the economy at date 0. We assume that all shocks die out at date 1 and that all agents in the economy know this.

Demand Shocks The first shock we consider is a temporary increase in the households' desire to save. Formally, $\beta_0 > 1$, $\beta_t = \beta < 1$ for all $t > 0$. Such a shock has been employed in the standard NK literature to capture an increase in the supply of savings which puts downward pressure on the real interest forcing them to be negative.[8]

Supply Shocks Next, we consider a temporary fall in aggregate productivity. Formally, $A_0 < A$and $A_t = A$ for all $t > 0$. Because real wages are determined in a different fashion from the flexible wage benchmark, the dynamics of these two economies can differ markedly. While we have not shown how the flexible wage economy responds to these shocks, it is straightforward to show that the full employment steady state is stable in response to such shocks. In other words, starting from full-employment, temporary supply or demand shocks are incapable of causing permanent damage to the flexible price economy. However, as we show below, an economy with nominal wage rigidities and the ZLB need not inherit this stability.

4.2 Price Level Targeting

Most central banks have either explicit or implicit mandates of maintaining price stability. Motivated by this, the first monetary policy rule we consider is one in which the monetary authority tries to perfectly stabilize prices at some level P^*. In particular, we impose that P^* satisfies:

$$P^* = \frac{W_{-1}}{w^*_{fe}}$$

This monetary rule ensures that in the absence of shocks, the economy stays at the FESS. Under this **price level targeting** regime (PLT), the central bank sets prices at their full employment steady state level, whenever it is not constrained by the zero bound.[9] Formally:

$$P_t = \min\left\{P^*, \frac{P_{t+1}}{\beta_t}\right\}$$

[8] In a richer model such a shock can arise from a tightening of borrowing limits or an increase in precautionary savings (See for example Guerrieri and Lorenzoni (2015)).

[9] As is standard, any path of prices consistent with the zero lower bound can be implemented as a locally unique equilibrium with an appropriate Wicksellian policy rule governing the nominal interest rate paid on central bank balances (Woodford (2003)).

We consider price level targeting, rather than inflation targeting, because in an economy with rigid nominal wages, trend inflation would generate a trend in real wages, which makes it impossible to have a steady state in which real variables are constant.

Demand shocks Demand shocks cause the zero lower bound on nominal interest rates to bind, forcing prices to fall in order to generate inflation and a negative real interest rate. This fall in prices inflates real wages and reduces vacancy creation.

Lemma 7. *Suppose Assumption 3 holds. Then if $\beta_0 > 1$ and $\beta_t = \beta < 1$ for all $t > 0$, and if the economy returns to FESS after date 0, the zero lower bound binds at date 0 only, and J_0 is a decreasing function of β_0:*

$$\begin{aligned} J_0 &= A - \beta_0 w^*_{fe} + \beta_0 \gamma J_{min} \\ \frac{\partial J_0}{\partial \beta_0} &= -w^*_{fe} + \gamma J_{min} < 0 \end{aligned}$$

Define $\underline{\beta} := \dfrac{A-\kappa}{w^*_{fe} - \gamma J_{min}}$ as the largest possible demand shock that will still ensure that the economy returns to FESS after date 0. Then it is true that:

1. *If $\beta_0 < \underline{\beta}$, $\theta_0 = \dfrac{J_0}{\kappa} > 1$, $\mu_t = 0$ for all t.*
2. *If $\beta_0 > \underline{\beta}$, we have $\theta_t = 0$ for all t, $\mu_1 = \mu_R$, $\mu_t \to 1$.*

Proof. See Appendix A.7. □

Consider for simplicity the case where nominal wages are permanently fixed at some level W_{-1}. Following the demand shock, the ZLB will bind at date 0. Under price level targeting, the ZLB cannot bind at date 1, and so prices will return to their steady state level P^*. Thus prices must fall at date 0:

$$P_0 = \frac{P^*}{\beta_0} < P^*$$

The resulting rise in real wages reduces the value of the firm, which can be written as

$$J_0 = A - \frac{W_{-1}}{P_0} + \beta_0 \gamma J_{min} = A - \beta_0 w^*_{fe} + \beta_0 \gamma J_{min}$$

where w^*_{fe} denotes the FESS Nash wage. This expression shows that β_0 affects the value of a filled vacancy through two channels. First, a higher discount factor makes firms more willing to invest in vacancy creation. Second, a higher β_0 - a larger decline in real interest rates - requires higher inflation going forward, when nominal interest rates are pinned down by the ZLB. This means prices must fall today, so that they can rise and return to steady state. This increases real wages today, inducing firms to post fewer vacancies. Under Assumption 3, the second effect always outweighs the first, and J_0 is decreasing in β_0.

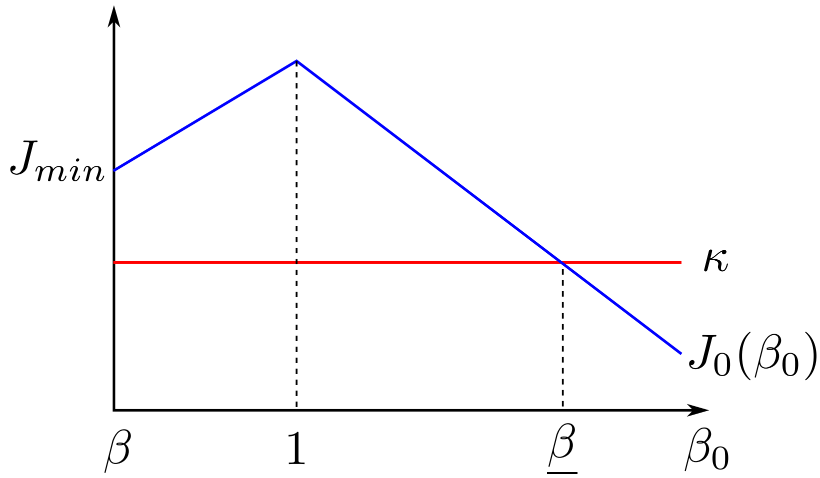


Figure 3. J_0 as a function of β_0

For small enough $\beta_0 > 1$, despite the fall in the firm's value, we still have $J_0 > \kappa$, and the firm still posts vacancies at date 0. Price level targeting succeeds in stabilizing the economy and averting a recession. Once the shock has passed, the economy returns immediately to the full employment steady state.

However, if β_0 is sufficiently large, we will have $J_0 < \kappa$, resulting in a hiring freeze at date 0. With no job creation, unemployment increases, and the skill composition of job seekers deteriorates:

$$\mu_1 = \mu_R > 0 = \mu_0$$

where $\mu_R = 1/(1+\gamma)$ denotes the fraction of unskilled job seekers after a one period hiring freeze.[10] Deterred by the higher training costs, firms are unwilling to post vacancies at the prevailing real wage; they would require a lower real wage. In a setting with fixed nominal wages, this adjustment must occur through a rise in prices. However, under a price level targeting regime, this cannot happen, so firms post zero vacancies even after the demand shock has dissipated, and the economy converges to zero employment. Notice that the situation would not be any different if instead of fixed nominal wages, we had considered a scenario in which nominal wages were rigid downwards.[11] Note also that in the flexible wage benchmark, the economy could never converge to a zero-employment steady state; this highlights again that the stability properties of the nominal economy can differ substantially from the flexible benchmark.

It is worth noting that the zero lower bound works through a different channel in this economy, compared to the standard New Keynesian (NK) model. In the standard NK model, the lower bound on nominal rates, together with an upper bound on expected inflation, render real interest rates

[10] See Lemma 4.

[11] This is clearly an extreme result, and relies on the particular matching function we have assumed. With a smoother matching function, the economy would converge to a small, but positive, rate of employment. Price level targeting leads to such extreme outcomes because it does not permit any rise in prices (i.e., any fall in real wages) even when economic conditions have worsened so that a lower real wage is necessary in order to sustain hiring.

too high, and make consumption growth too large. If long run consumption is pinned down (by the assumption that the economy returns to the unique steady state), a high consumption growth rate requires consumption to fall today. In our model with linear utility, real rates are fixed. The lower bound on nominal rates requires positive inflation to generate a lower real rate in response to demand shocks. If the long run price level is pinned down (by monetary policy), a high rate of inflation requires prices to fall today. With sticky nominal wages, this fall in prices raises real wages, and potentially reduces hiring.

In NK models of the ZLB, the goal of policy is to set the real interest rate equal to the Wicksellian rate r^* - either through fiscal policy, commitment to higher inflation and so forth. We abstract from this problem altogether by studying an environment in which this is *always* accomplished, thorough commitments to higher inflation. Our point is that even if policy succeeds in tracking the Wicksellian rate, this may not be enough to prevent a recession.

Supply shocks Under price level targeting, the economy responds in a broadly similar fashion to supply shocks.

Lemma 8. *Define*

$$\underline{A} \;:=\; w^*_{fe} + \kappa - \beta\gamma J_{min}$$

Then under supply shocks:

1. *If* $A_0 > \underline{A}$, $\theta_0 = \dfrac{J_0}{\kappa} > 1$, $\mu_t = 0$ *for all* t.
2. *If* $A_0 < \underline{A}$, *we have* $\theta_t = 0$ *for all* t, $\mu_1 = \mu_R$, $\mu_t \to 1$.

A temporary fall in productivity reduces the value of hiring. In the flexible wage benchmark, this fall in productivity must be compensated by a fall in real wages. In a setting with fixed nominal wages, this can only be accomplished by a rise in prices today. Since price level targeting stabilizes the price level at P^* at each date, the resulting real wages are too high, causing the value of filling a vacancy to fall and reducing the incentive of a firm to post vacancies. For a small enough fall in productivity, the value of a vacancy is still positive and the economy returns to full employment at date 1. However, for a substantially large shock, the value of filling a vacancy turns negative and causes the firm to stop hiring. As in the case with demand shocks, firms are only willing to post vacancies after a hiring freeze at lower wages which require higher prices than P^*: PLT prevents this from happening and thus does not let the economy return to full employment.

Thus, for both supply and demand shocks, a PLT regime perfectly stabilizes employment in response to small shocks. However, PLT is not accommodative enough in the face of larger shocks and permits even these temporary shocks to inflict permanent damage to the economy. Note that this depends crucially on the forces generating multiplicity: skill depreciation and the cost of training unskilled workers, $\chi > 0$.[12] If $\chi = 0$, the economy would immediately return to steady state after a temporary shock. Even though unemployment has increased, the rise in unemployment has no effect on training costs and does not necessitate lower real wages in order to sustain hiring.

4.3 Neutral monetary policy

A major flaw with the PLT regime above was that it did not react sufficiently to changing economic conditions. When economic conditions worsened and lower real wages were required to replicate allocations in the real economy, PLT did not permit such an adjustment. It is therefore natural to ask whether a class of policy rules that is more responsive to economic conditions does a better job.

Indeed a large literature argues that the goal of monetary policy should be to replicate allocations that would arise in an economy without nominal rigidities. Clearly, such a prescription would be optimal if nominal rigidities were the only distortion rendering equilibrium inefficient.[13] Following Goodfriend and King (1997), we label this **neutral monetary policy** (NMP).

Recall that with fixed nominal wages, the monetary authority effectively determines the level of real wages by determining prices. In the absence of the zero bound, we can simply define NMP as one which mimics the allocations in the flexible wage benchmark. However, the ZLB may limit the ability of the central bank to do this, and so it is necessary to specify how policy is conducted in this case.

Let M be a one period forward transition equation that describes how the economy transitions from μ_t to μ_{t+1}. Then equilibrium in the flexible wage benchmark can be described by policy functions $\mu_{t+1} = M^*(\mu_t)$, $w_t = w^*(\mu_t)$, and the equilibrium under NMP can be described by $M^B(\mu), w^B(\mu)$. It will not be possible to replicate the benchmark and set $w_t = w^*(\mu_t)$ at date t if

$$w^*(\mu_t) < \beta w^B(M^*(\mu_t))$$

In this case, if the ZLB binds, we must have

$$\kappa + \chi\mu_t = A - \beta_t w^B(\mu_{t+1}) + \beta_t\gamma[\kappa + \chi M^B(\mu_{t+1})]$$

Call the solution to this equation $\mu_{t+1} = M^{ZLB}(\mu_t)$. Then an equilibrium under NMP is described as follows:

Definition 2 (Neutral Monetary Policy)**.** *NMP sets interest rates to replicate allocations described*

[12]To be precise, what is important is not multiplicity per se, but the fact that training costs are large enough such that $\underline{\mu} < 1$.

[13]Indeed, such a policy may be optimal even in models with an inefficient real equilibrium. For example, the canonical New Keynesian model features inefficiencies due to both nominal rigidities and monopolistic competition.

by policy functions $w^B(\mu), M^B(\mu)$ *satsifying*

$$\begin{aligned} w^B(\mu) &= w^*(\mu), M^B(\mu) = M^*(\mu) && \text{if } w^*(\mu) \geq \beta w^B(M^*(\mu)) \\ w^B(\mu) &= \beta w^B(M^{ZLB}(\mu)), M^B(\mu) = M^{ZLB}(\mu) && \text{if } w^*(\mu) < \beta w^B(M^*(\mu)) \end{aligned}$$

Intuitively, NMP sets interest rates period by period to ensure that today's real wage takes the same value as in the flexible wage benchmark, as long as this does not violate the ZLB. If the ZLB binds, the central bank's ability to affect prices and the real wage is constrained. Accordingly, vacancy creation adjusts in response to the profitability associated with filled vacancies.

Demand shocks In what follows, it is convenient to make the following assumption:

Assumption 4. $\eta A + (1-\eta)b > \gamma[\kappa + \chi]$

The LHS of this expression is the lower bound on wages in the flexible benchmark while the RHS is proportional to the upper bound on the value of a filled vacancy at date 1.

This assumption ensures that wages are never too small, even at zero employment. Thus, this assumption ensures that $w_1^* > \gamma J_1$ which in turn ensures that the value of a filled vacancy is decreasing in β_0 at the ZLB. With this assumption in hand, the Proposition below characterizes the economy's response to a demand shock under NMP.

Proposition 2 (Demand Shocks and NMP). *Under Assumption 4, there exists* $\hat{\beta} > \underline{\beta} > 1$ *such that:*

1. *If* $\beta_0 \in (1, \underline{\beta}]$*, there exists an equilibrium with* $\theta^{FE} > \theta_0 \geq 1$ *and* $\mu_t = 0$ *for all* t*.*
2. *If* $\beta_0 \in (\underline{\beta}, \hat{\beta})$*, there exists an equilibrium with* $\theta_0 \in (0,1)$ *and* $\mu_1 \in (\underline{\mu}, \mu_R)$*.*
3. *If* $\beta_0 \geq \hat{\beta}$*, there exists an equilibrium with* $\theta_0 = 0$ *and* $\mu_1 = \mu_R$

In addition, in cases 2 and 3 above, if $\mu_1 < \widetilde{\mu}$*, then* $\lim_{T\to\infty} \mu_T = 0$ *and the economy returns to full employment. However, if* $\mu_1 \geq \widetilde{\mu}$*, then the economy never returns to full employment.*

Proof. See Appendix A.8. □

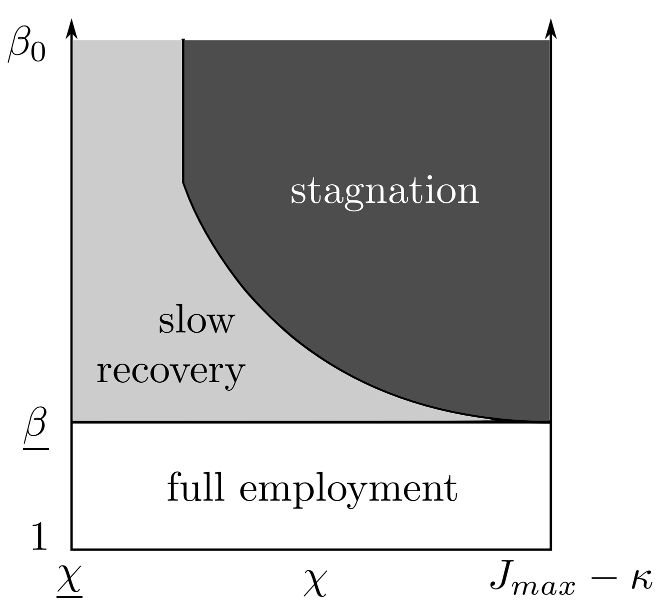
Regardless of the size of β_0, as long as it is above 1, the Fisher equation implies that the economy gets pushed to the ZLB at date 0. Because NMP keeps prices relatively high in the future, prices must fall today in order to generate inflation. Since nominal wages cannot fall, the fall in prices effectively raises the real wage which discourages vacancy creation.

Much like under the PLT regime, for small shocks, the increase in real wages is not substantial enough to induce firms to stop hiring, and the economy stays at full employment despite being constrained by the ZLB. In fact, the allocations attained under PLT and NMP are identical when the economy is hit by a small demand shock ($\beta_0 \in (1, \underline{\beta}]$).

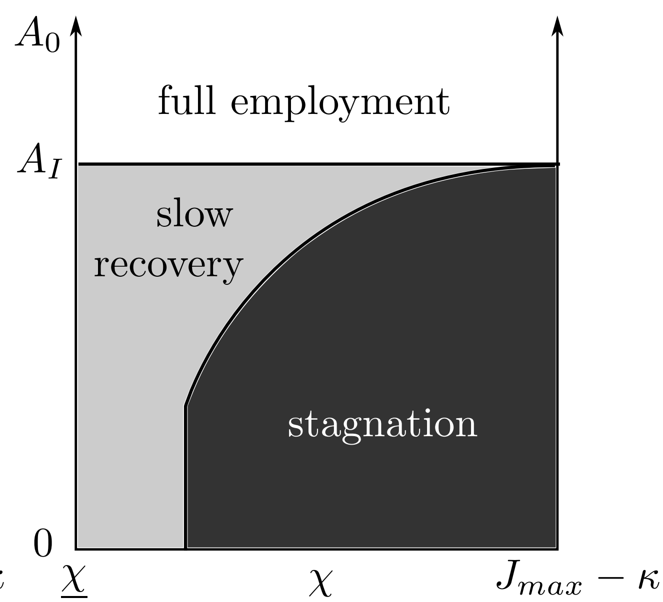
NMP fares better than PLT in dealing with larger demand shocks, for two reasons. First, if the economy is thrown into the convalescent or the stagnant region, NMP implements higher prices relative to PLT, in order to generate the lower real wages that would have obtained under Nash bargaining. This prevents the economy from being driven to zero employment. Second, the anticipation of higher prices once the shock has abated raises prices while the demand shock is still operative, mitigating the rise in real wages, potentially reducing the fall in hiring. Thus although a very large demand shock ($\beta \geq \hat{\beta}$) results in a hiring freeze at date 0, as was the case under PLT, for moderately large shocks ($\beta \in (\underline{\beta}, \hat{\beta})$), the fall in hiring is smaller under NMP compared to PLT.

Nonetheless, if a demand shock drives the economy into the convalescent region, it takes a long time to return to full employment. Worse still, if the economy gets pushed to the stagnant region, it never returns to full employment. Thus, even under NMP, transitory demand shocks can leave persistent or even permanent scars on the real economy.

What determines whether the economy ends up in the stagnant region or the convalescent region in response to a large shock? Intuitively, it depends on two factors. The first is the size of the demand shock - the larger the demand shock, the larger the decline in hiring and deterioriation in skills, increasing the likelihood the economy enters the stagnant region. The second factor is the strength of the forces generating multiple steady states in this economy. If χ - the training costs imposed by a less skilled work force - was equal to zero, there would be a unique steady state, and no adverse shock could ever prevent the economy from returning to full employment after the shock abates. For sufficiently high χ, there are multiple steady states as firms' job-creation decisions become sensitive to the cost of re-training unskilled workers. The higher is χ, the lower $\widetilde{\mu}$, and the more likely it is that the economy enters the stagnant region. Figure 4a illustrates all these possibilities.



(a) Demand Shocks



(b) Supply Shocks

Figure 4. Parameter Space for which demand and supply shocks lead to slow recoveries or stagnation.

Supply shocks We have just seen that NMP stabilizes the economy more than PLT in response to demand shocks. The same is true for supply shocks, albeit for a different reason. Supply shocks are inflationary whereas demand shocks are deflationary. NMP recognizes that, given that nominal wages can't fall, a temporary fall in productivity requires an increase in prices to reduce real wages. In contrast, price level targeting does not permit any rise in prices. However, given that NMP will bring the price level back down once productivity has returned to its steady state level, the zero lower bound constrains how much prices can rise today. The Lemma below summarizes the response of the economy under NMP to technology shocks of different sizes.

Proposition 3 (Supply Shocks and NMP)**.** *For β sufficiently close to 1, there exist $A_F < A_I < A_Z < A$ such that:*

1. *If $A_0 \in (A_Z, A)$, the ZLB does not bind, and $\theta_0 \geq 1$, $\mu_1 = 0$*
2. *If $A_0 \in [A_I, A_Z]$, the ZLB binds, and $\theta_0 \geq 1$, $\mu_1 = 0$*
3. *If $A_0 \in (A_F, A_I)$, the ZLB binds, and there exists an equilibrium with $\theta_0 \in (0, 1)$*
4. *If $A_0 \leq A_F$, the ZLB binds, and there exists an equilibrium with $\theta_0 = 0$, $\mu_1 = \mu_R$.*

Proof. See Appendix A.9. □

NMP can accommodate small enough productivity shocks, by raising prices and reducing real wages to sustain hiring. If the shock is very small ($A_0 > A_Z$), this rise in prices does not even cause the zero bound to bind. If the shock is slightly larger ($A_0 \in [A_I, A_Z]$), the required rise in prices is so great that it causes the zero lower bound to bind, preventing the central bank from reducing real wages to attain the frictionless wage benchmark allocation. Vacancy creation falls, but since the fall in productivity is not too large, the economy remains in the tight labor market regime. For even larger falls in productivity ($A_0 \in (A_F, A_I)$), the labor market becomes slack, and hiring falls. However, the anticipation of higher prices once the shock has abated reduces prices at date 0, and hiring does not fall all the way to zero. Finally, if the fall in productivity is larger still ($A_0 \leq A_F$) hiring falls to zero.

As was the case when we discussed demand shocks, if $\mu_1 < \widetilde{\mu}$, a large shock will push the economy into the convalescent region, but NMP will eventually return the economy to full employment. If $\mu_1 \geq \widetilde{\mu}$, however, a large shock pushes the economy into the stagnant region, and it never returns to full employment. Whether a large shock causes a slow recovery or permanent stagnation depends on the strength of the forces generating multiple steady states - in particular, on the value of χ, the cost of training an unskilled worker. Figure 4b illustrates these possibilities.

Much of the theory and practice of monetary policy aims to identify natural rates of interest, unemployment etc., and minimize the gap between these actual rates and their frictionless benchmarks. While this approach may perform well in models with a unique steady state, it can lead to perverse outcomes in economies featuring hysteresis and multiple steady states. Even though the term "natural rates" is misleading in such an environment, one could, loosely, say that these economies have multiple natural rates. Shocks leading to shifts in the composition of unemployed workers without any corrective policy could lead the economy to transition to a different steady state with a different natural rate of unemployment. As such, a mandate to keep both unemployment and inflation low suggests that policy should select the natural rate and not track it.

4.4 What is the role of multiplicity and the ZLB?

The take-away from the last two sub-sections is that in a world where temporary shocks can permanently scar economies, two apparently reasonable monetary policy regimes - price level targeting and neutral monetary policy - fail to prevent large transitory shocks from permanently damaging the economy. In standard models with nominal rigidities, however, very similar policy rules to PLT and NMP perform very well in stabilizing the economy.[14] What explains this discrepancy? The following Lemma shows that multiplicity is key.

Proposition 4 (No Multiplicity of Steady States)**.** *If $\chi < J_{min} - \kappa$, FESS is the only steady state. Furthermore, after either a temporary demand or supply shock, under both PLT and NMP, the economy returns to full employment by date 2 at the latest. Thus, temporary shocks only have short lived effects on the economy.*

Proof. First consider the PLT regime. By construction, PLT ensures that the price level at date 1 onwards is given by P^* (unless the ZLB binds). This ensures that the real wage from date 1 onwards is given by w^*_{fe}. As a result, it must be the case that at date 1, regardless of the level of μ_1, $J_1 = J(\mu_1) = J_{\min}$. Since the cost of re-training workers, χ, is small enough, firms are still willing to post a large number of vacancies no matter what the level of μ_1. This then leads us to similar dynamics as discussed in Lemma 3 albeit moved one period forward. It follows that $\theta_1 = \frac{J_1 - \chi\mu_1}{\kappa} > 1$ and hence $\mu_t = 0$ for all $t \geq 2$. This argument follows over to NMP as long as $w_t = w^*_{fe}$ for all $t \geq 1$. This is in fact true since in the flexible wage benchmark wages are always equal to w^*_{fe}. In other words in this economy with $\chi < J_{min} - \kappa$, the economy is always in the healthy region, i.e. $\underline{\mu} > 1$. □

Why do these monetary regimes perform poorly once multiplicity is introduced? For non-negligible values of χ, the firm's job-creation decision becomes sensitive to the extent of skill depreciation in the economy as the effective cost of hiring rises with the level of unemployment. Consequently, firms would only be willing to sustain hiring if wages were sufficiently low. PLT effectively ignores this changing cost of job creation and tries to keep wages at the level consistent with full-employment. However, if the skill composition of workers is sufficiently low, for large enough χ, the higher costs of job-creation reduces the firms' incentives for posting vacancies at that real wage, causing the economy to diverge away from the full employment steady state.

[14] They definitely do not lead to long-run declines in output in response to temporary shocks!

NMP does better than PLT because it is not insensitive to the changing costs of job-creation. However, it does not implement a price path (and hence a real wage path) which induces firms to hire enough so that the economy would quickly return to full-employment. Rather it tries to mimic the flexible wage benchmark which itself induces at best a slow recovery or at worst, stagnation.

It is important to realize that absent the ZLB, both PLT and NMP are perfectly capable of stabilizing the economy at full employment in response to demand shocks. An temporary increase in β_0 above 1 forces real interest rates to become negative. Absent the ZLB, negative real rates can easily be accommodated with negative nominal rates without a fall in prices. Thus, without the ZLB demand shocks lose their bite and the full employment steady state is stable in response to these shocks, both under PLT and NMP, as in the flexible wage benchmark.

The ZLB changes the stability properties of the model, and in particular of the full employment steady state. A negative real interest rates can no longer be accommodated by a negative nominal rate, and instead requires positive inflation, a fall in current prices, a rise in real wages and a contraction in hiring. This in turn leads to a slow recovery back to full employment or stagnation. Even absent the ZLB, if the economy started out near the low-pressure steady state, PLT and NMP might fail to bring unemployment back down. But it is only because of the ZLB that demand shocks can create unemployment, bringing multiplicity into play.

4.5 Alternative monetary policies

Recognizing the failures of PLT and NMP, we now ask whether there exists any monetary policy that can prevent transitory shocks from causing permanent damage.

PLT and NMP fail to prevent stagnation because they are insufficiently expansionary once the shock has abated. Monetary policy can do better by committing to create higher prices in the future, which both raises prices during the shock, and increases future profits, making firms more willing to hire. The following proposition describes such a policy for demand shocks.

Proposition 5 (Alternative Monetary Policy for Demand Shocks)**.** *Suppose $\beta_0 > \underline{\beta}$. For any T, define*

$$\omega_T = \frac{1-\gamma}{\beta^T\left(1-\gamma^{T+1}\right)}\left\{\left(\frac{\beta}{\beta_0}+\frac{\beta\gamma}{1-\beta\gamma}\right)A-\frac{(\beta\gamma)^{T+1}}{1-\beta\gamma}w^*_{fe}-\frac{\kappa\beta}{\beta_0}\right\}$$

There exists T such that $\omega_T \in (\beta w^*_{fe}, w^*_{fe})$. There is an equilibrium in which $\mu_t = 0$ for all t and the economy escapes from the zero lower bound at date $T+1$, with $w_T = \omega_T$, $w_t = \beta^{T-t}\omega_T$ for $t = 1, ..., T-1$, $w_0 = \beta_0 w_1$, $w_t = w^*_{fe}$ for $t \geq T+1$. This is implemented through a path of prices defined as: $P_t = W/w_t$.

Proof. See Appendix A.10. □

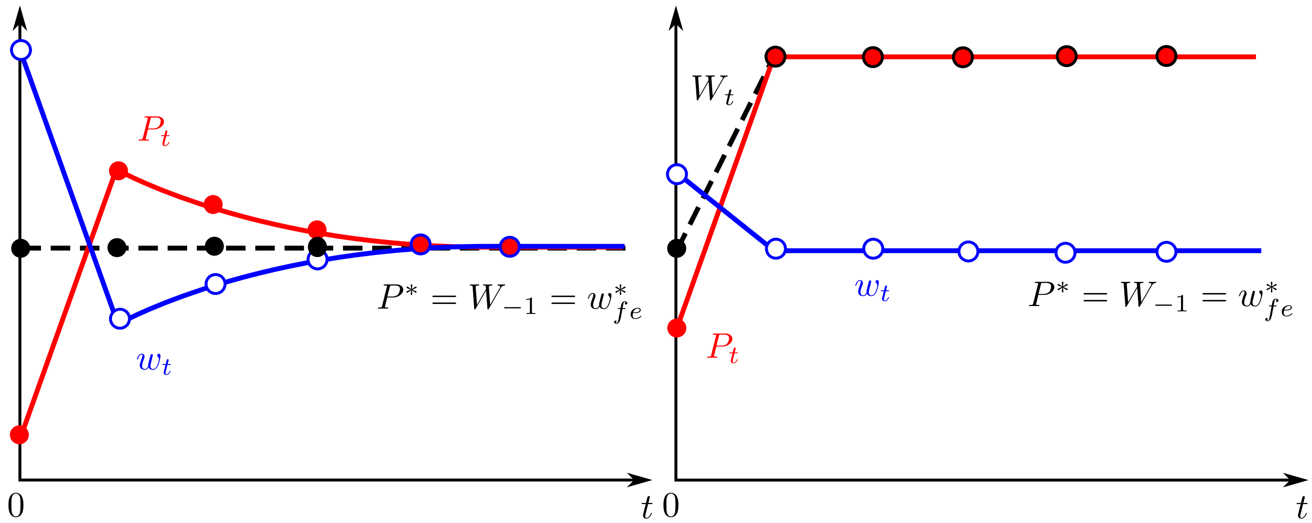
With rigid nominal wages, this monetary policy operates through two channels. First, recall that in this economy, an increase in β_0 requires a positive rate of inflation at the zero lower bound, in order to produce negative real interest rates. If future prices are pinned down, this in turn requires a fall in prices today, which raises real wages and causes a recession. If instead the monetary authority commits to higher prices in the future, starting at date 1 once the shock has abated, this prevents the price level from falling too much at date 0, mitigating the rise in real wages and the hiring freeze. Second, by committing to higher future prices, the monetary authority indirectly lowers future wages and raises future profits, which further increases hiring today.

Ideally, the monetary authority would like to commit to a short, sharp increase in prices at date 1, and then decrease prices back to their steady state level at date 2. This may not be possible, owing to the zero bound, which rules out a rapid deflation between dates 1 and 2. Instead, an increase in date 1 prices may necessitate a gradual deflation and a slow return to steady state, in which prices are higher and real wages lower than their steady state values for a number of periods. Nonetheless, it is always possible to prevent a recession, however large the date 0 demand shock, given a long enough commitment to higher future prices.

The second channel through which the alternative monetary policy operates crucially depends on the ability of the monetary authority to engineer booms after the shock has abated; this is only possible if nominal wages are unresponsive to prices. Suppose that instead of being perfectly rigid, nominal wages were only rigid downwards but could adjust upwards; then an attempt by the monetary authority to raise the price level in the future would be met by an increase in nominal wages by firms. Thus a commitment to higher future prices would be less effective in reducing real wages, weakening the second channel. In the limit where nominal wages can adjust upwards freely, the second channel is nullified. However, even in this case, the alternative monetary policy would still be effective through the first channel; a higher price at date 1 in such a scenario limits the fall in prices (and increase in real wages) at date 0, thus, insulating the economy from the shock.

Figure 5 depicts the monetary policy required to maintain full employment under both fixed nominal wages (left panel) and downward rigid nominal wage rigidity (right panel). The key thing to note is that the path of real wages differs across the two pictures. In the fixed nominal wages case, higher future prices result in lower real wages in the future. This reduces the present discounted value of wage costs for a firm, incentivizing more hiring. Thus, a relatively modest commitment to higher prices at date 1 is enough to sustain full employment through both the channels described above. In contrast if wages are only downward rigid, as depicted in the panel on the right, it is impossible to reduce wages below their frictionless value in the future. Higher prices at date 1 translate one-for-one into higher nominal wages leaving real wages unaffected. Since the second channel is not operative, a higher price commitment at date 1 is needed to ensure full employment through the first channel alone. As can be seen in the right panel, prices are higher in the long run in this scenario but this higher level has no allocative role as real wages are the same throughout.

While this result is reminiscent of the discussion of forward guidance at the ZLB in the New Keynesian literature, the mechanism is very different. That literature argues that committing to keep interest rates low in the future, once shocks have abated, can alleviate demand driven recessions.



(a) Fixed nominal wages

(b) Downwardly rigid wages

Figure 5. Trajectories under different assumptions on wage rigidity. The dashed black line plot the trajectory of nominal wages, the blue lines with hollow circles depict the path of real wages and the red line with filled circles denotes the price level. Trajectories are normalized such that $P^* = W_{-1} = w^*_{fe}$.

In NK models, this policy works through an intertemporal consumption smoothing channel. Low interest rates, once the shock has abated, encourage households to consume more in those future periods. Because of intertemporal substitution, this in turn encourages households to consume more during the recession, strengthening demand and reducing the severity of the contraction in output (Eggertsson and Woodford (2003), Werning (2011)).

There is an important debate about whether the strength of forward guidance in NK models is realistic (Del Negro et al. (2015), McKay et al. (2015)) and a related debate about the strength of the intertemporal substitution channel in reality (see e.g. Kaplan et al. (2016)). It is important to note that the power of commitment to higher future prices in our framework does not depend on the intertemporal substitution channel. Instead, here the rate of inflation is fixed by the ZLB; higher future prices raise the price level today, lowering real wages and encouraging hiring.

Naturally, a similarly accommodative policy can also neutralize supply shocks. The Proposition below characterizes such a policy.

Proposition 6 (Alternative Monetary Policy for Supply Shocks)**.** *Suppose $A_0 < A_Z$. For any T, define*

$$\omega_T = \frac{1-\gamma}{\beta^T\left(1-\gamma^{T+1}\right)}\left\{\frac{(\beta\gamma)^{T+1}}{1-\beta\gamma}w^*_{fe} + A_0 + \frac{\beta\gamma A}{1-\beta\gamma} - \kappa\right\}$$

There exists T such that $\omega_T \in (\beta w^*_{fe}, w^*_{fe})$. There is an equilibrium in which $\mu_t = 0$ for all t and the economy escapes from the zero lower bound at date $T+1$, with $w_T = \omega_T$, $w_t = \beta^{T-t}\omega_T$ for $t = 0, ..., T-1$, $w_t = w^*_{fe}$ for $t \geq T+1$. This is implemented through a path of prices defined as: $P_t = W/w_t$.

Proof. The proof is identical to the proof of Proposition 5. □

4.6 Escaping Unemployment Traps?

The alternative monetary policy prevents adverse shocks from causing a recession, and the associated decline in skill composition of the workforce. Suppose however, that monetary policy has been insufficiently accommodative in the past and the unemployment rate has increased leading to a substantial deterioration of the skill composition. Are these scars permanent or can monetary policy heal them?

The answer to this question depends crucially on whether monetary policy is capable of creating a boom in the economy. First consider the case in which the economy is characterized by rigid nominal wages and suppose that the economy has reached the stagnant region following an adverse shock in the past. Monetary policy can temporarily raise prices; since nominal wages are fixed, this lowers real wages, compensating firms for the additional costs of training a largely unskilled workforce. This creates a hiring boom, returning the economy to full employment. Similarly, if the economy is struggling amidst a slow recovery in the convalescent region, the monetary authority can engineer lower effective real wages and speed up recovery by making hiring cheaper.

Proposition 7 (Monetary Policy in a Recession)**.** *Suppose the economy us characterized by positive levels of unemployment at date 0 ($\mu_0 > 0$). For any T, define*

$$\omega_T = \frac{1-\gamma}{\beta^T\left(1-\gamma^{T+1}\right)}\left\{\frac{(\beta\gamma)^{T+1}}{1-\beta\gamma}w^*_{fe} + \frac{A}{1-\beta\gamma} - \kappa - \chi\mu_0\right\}$$

There exists T such that $\omega_T \in (\beta w^*_{fe}, w^*_{fe})$. There is an equilibrium in which $\mu_t = 0$ for all t and the economy escapes from the zero lower bound at date $T+1$, with $w_T = \omega_T$, $w_t = \beta^{T-t}\omega_T$ for $t = 0, ..., T-1$, $w_t = w^*_{fe}$ for $t \geq T+1$. This is implemented through a path of prices defined as: $P_t = W/w_t$.

Proof. The proof is identical to the proof of Proposition 5. □

The Proposition above presents a monetary policy rule which repairs the economy. Notice that the policy prescription is very similar to the alternative monetary policy rule. Indeed, it works through the second channel through which the alternative monetary policy stabilized the economy. This should not be surprising since a high level of unemployment increases the effective costs of creating jobs, which can be though of as an (endogenous) adverse supply shock.

This policy may involve a commitment to keep prices elevated for an extended period of time. Such a commitment is not costless. Higher prices in the full employment steady state imply lower real wages, and more vacancy posting. In general this may be inefficient ex-post as it exacerbates congestion in the labor market. In fact, given our choice of the matching technology, total hires is fixed and equals δ, so these extra vacancies posted are a drain on resources, and reduce aggregate consumption. As in standard New Keynesian models of the zero lower bound (e.g. Eggertsson and Woodford (2003)), such policies therefore involve commitment on the part of the monetary authority.

Also, note that monetary policy is extremely powerful in this setting with fixed nominal wages as the monetary authority can engineer a boom whenever it wishes to by effectively lowering real wages. Monetary policy may not be so powerful in reality. To consider this possibility, suppose that nominal wages are downwardly rigid instead of being fixed: $W_t \geq W_{t-1}$.[15] A large literature argues this case is empirically more realistic. In this case, monetary policy can make real wages too high, creating recessions, but can never reduce real wages below their value in the frictionless benchmark and create booms. Thus, it follows immediately that while preemptive monetary policy can prevent adverse shocks from causing a rise in unemployment, if this rise in unemployment (skill deterioration in the workforce) has already taken place, monetary policy is powerless to escape from the trap.

Abstracting from the specifics of our model for a moment, there are multiple channels through which recessions reduce the productive capacity of the economy in reality. Recessions reduce capital accumulation, reduce labor force participation, slow productivity growth, raise the natural rate of unemployment, reduce unemployed workers' human capital, and so on. Some of these effects may be physically irreversible, in the sense that no ex post policy could undo the effects of the recession. For example, Wee (2016) shows that recessions can permanently change the search behavior of young workers, causing them to stay in careers in which they have a comparative disadvantage but have accumulated sufficiently high human capital, instead of searching for a career in which they have a comparative advantage. Even after the recession has passed, the cost of discarding accumulated human capital in search of a career in which they have comparative advantage dissuades workers from searching further. To the extent that hysteresis is irreversible, accommodative monetary policy ex post, once the damage has be done, will not be desirable.[16]

If monetary policy is indeed unable to reverse the damage caused by a deep recession, it is all the more important to prevent severe downturns in the first place through sufficiently accommodative monetary policy. Indeed, in this scenario the risks associated with monetary easing are asymmetric. As Brad De Long has argued, the costs of keeping rates low for too long are small because this mistake can be reversed, while the costs of raising rates prematurely are large because the resulting damage may be irreversible.[17]

At the other extreme, some of the effects of recessions can be reversed by accommodative monetary policy, as Ball (2015) argues:

[15] To be precise, we assume that nominal wages are set to $W_t = w_t^* P_t$ whenever possible, where w_t^* is the real wage in the flexible wage benchmark. However, if $W_{t-1} > w_t^* P_t$, then $W_t = W_{t-1}$.

[16] See remarks made by (Cowen, 2010) on the case for higher inflation today to prevent the advent of hysteresis once workers are laid off.

[17] See arguments made by (DeLong, 2015) on how the appropriate monetary policy is to keep rates depressed rather than prematurely raising them and inflicting irreparable damage to the economy.

> ...common sense suggests that the damage from the Great Recession is reversible. If a recession leaves workers discouraged and detached from the labor force, a high-pressure economy with plentiful job opportunities could draw them back in. If low investment during a recession reduces the economy's capital stock, a high-pressure economy could spur investment to rebuild it. (Ball (2015))

In between these two extremes, recessions may have some lasting effects that cannot be reversed by monetary policy, but can nevertheless be reversed (perhaps at some cost) by other measures. For example, in our setup a hiring subsidy or a subsidy to firm for training workers could always return the economy to full employment.

Our point is that to the extent that the damage from recessions can be reversed (in particular, by monetary policy), apparently reasonable policy regimes which focus on price stability, or which treat the productive capacity of the economy as exogenously given, may fail to reverse the damage. How far the damage is in fact reversible is an empirical question. It should be noted that most of the empirical literature on hysteresis is unable to answer this question, since it looks at whether recessions permanently lower the productive capacity of the economy given current policy and cannot inform us whether this effect would persist given alternative policy.

5 Conclusion

We presented a model to study the conduct of monetary policy when temporary shocks can scar the economy. If recessions reduce the productive capacity of the economy, reasonable policies - which focus on price stability, or which try to replicate allocations in the flexible wage economy - may lead to undesirable outcomes. Given large enough shocks, such policies generate at best a slow recovery; at worst, they may lead to permanent stagnation. Accommodative monetary policy can prevent this from happening. If monetary policy has been insufficiently accommodative in the past, accommodative monetary policy may be able to undo the damage. Our model is deliberately highly stylized, allowing us to analytically characterize monetary policy in an economy with multiple steady states. Our belief is that the predictions of this model would carry over to a richer environment.

We focused on one particular mechanism through which temporary shock can have permanent effects. In reality, other mechanisms may also be important: recessions reduce capital accumulation, reduce labor force participation, slow productivity growth, raise the natural rate of unemployment, reduce unemployed workers' human capital, and so on. Our general point is that if these mechanisms are operative, a neutral monetary policy which seeks to track measured natural rates - of unemployment u^*, of interest rates r^*, and so forth - might be insufficiently accommodative to actually engineer a full and quick recovery. The reason such policies fall short is that in a world with hysteresis, these "natural" rates are endogenous. Policies should aim to to set these rather than track these.[18]

Of course, whether or not the mechanisms described above are strong enough to support such a conclusion is an empirical question. An influential alternative to the view we have sketched here, which emphasizes hysteresis, is the "secular stagnation" hypothesis. While this literature shares our concern with long run outcomes, it hypothesizes that stagnation occurs because of permanent changes in natural rates and thus, would warrant very different policy prescriptions.

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[18]Unrelated, but nevertheless a hat tip to http://www.alessonislearned.com/

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Appendix

A Proofs

A.1 Wages and Nash Bargaining

Recall that the value of an employed worker and of an unemployed worker, respectively, are defined by the recursions (3) and (4). Also, the value of a filled vacancy to a firm is given by equation (12). We can then define the surplus of a match between a worker and a firm as:

$$\mathcal{S}_t = J_t + \mathbb{W}_t - \mathbb{U}_t$$

Wages are determined by Nash bargaining. Denoting workers' bargaining power by η, wages solve

$$\max_{w_t} J_t^{1-\eta}(W_t - U_t)^\eta$$

implying

$$\eta J_t = (1-\eta)(W_t - U_t)$$

Notice that the match surplus can be rewritten as:

$$\begin{aligned}
\mathcal{S}_t &= J_t + \mathbb{W}_t - \mathbb{U}_t \\
&= A - b + \beta(1-\delta)J_{t+1} + \beta(1-\delta)(1-q_{t+1})(\mathbb{W}_{t+1} - \mathbb{U}_{t+1}) \\
&= A - b + \beta(1-\delta)(1-q_{t+1})S_{t+1} + \beta(1-\delta)q_{t+1}J_{t+1}
\end{aligned}$$

Using the fact that $W_t - U_t = \eta S_t$ in the equation above, we have:

$$w_t = \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1}J_{t+1}$$

A.2 Proof of Lemma 2

Steady states solve

$$\frac{a}{1-\beta\gamma(1-\eta(1-\mu))} = \kappa + \chi\mu$$

Define $x = \frac{\chi}{J_{min}}$, dividing through by J_{min}, this becomes

$$\frac{1}{1-e\mu} = k + x\mu$$

Assumptions 1 and 2 impose that $k < 1$ and $\dfrac{1}{1-e} > k + x$. Since $e \in (0,1)$, $\dfrac{1}{1-e\mu}$ is an increasing, strictly convex function. Starting from $x = 0$, as we increase x, either the intersection of these two functions first occurs at $\mu \in (0,1)$, in which case a slightly higher x would give us multiplicity, or the first intersection has $\mu \geq 1$. Consider the knife edge case in which the first intersection of these two curves is at $\mu = 1$. Then the curves must be tangent and equal to each other at $\mu = 1$, i.e.

$$\frac{e}{(1-e)^2} = k, \frac{1}{1-e} = k + x$$

which implies $k = \frac{1-2e}{(1-e)^2}$. In order to have multiple intersections in $(0,1)$, then, it is clear graphically that we need $k > \frac{1-2e}{(1-e)^2}$. Assumption 3 is sufficient (but not necessary) to ensure this, since it implies that $e > 0.5$. If this is true, and if x is just large enough that there is a single slack steady state, then the polynomial

$$1 - (k + x\mu)(1 - e\mu) = 0$$

has a unique solution, i.e. its discriminant equals zero:

$$(x - ek)^2 - 4xe(1-k) = 0$$
$$x^2 - 2e(2-k)x + e^2k^2 = 0$$

Graphically, it is clear that this will have two solutions, the larger of which corresponds to $\mu \in (0,1)$; choosing this one we have

$$x^* = e(2-k) + \sqrt{e^2(2-k)^2 - e^2k^2} = e[2 - k + 2\sqrt{1-k}]$$

Thus there will be multiple steady states if $x > x^*$.

A.3 Proof of Lemma 3

Suppose $\mu_0 = 0$. Then, note that $\mu_t = 0$ (which implies $n_t = 1$) is consistent with equation 16, since in the tight labor market regime $q_t = 1$, and $n_t = 1, \mu_{t+1} = 0$. Next we show that we cannot have $\theta_0 < \theta^{FE}$ given $\mu_0 = 0$. (Since $\theta^{FE} \geq 1$ by Assumption 1, this implies in particular that we cannot have $\theta_0 < 1$.) In any equilibrium, equation 15 must be satisfied, which means that

$$J_{min} \leq a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1 - \eta \min\{\theta_\tau, 1\}) \leq \kappa \max\{\theta_t, 1\}$$

where the first inequality comes because the left hand side is decreasing in θ_τ. Since we know that $J_{min} > \kappa$ from Assumption 1, it is immediate that this inequality can only be satisfied if $\theta \geq \theta^f \geq 1$.

Finally, we show that we cannot have $\theta_0 > \theta^f$. We have shown that

$$a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1-\eta \min\{\theta_\tau, 1\}) = \kappa\theta_0$$

in any equilibrium, and that this expression is satisfied by $\theta_t = \theta^{FE}$, $\forall t \geq 0$. If $\theta_0 > \theta^{FE}$, it follows that $\theta_t < \theta^{FE}$ for some $t > 0$. Let T be the first date at which this is true. Then up to that date, since the labor market has been tight, $\mu_T = 0$. This is a contradiction, since we have already shown that if $\mu_T = 0$, $\theta_T \geq \theta^{FE}$. It follows that the unique equilibrium has $\theta_t = \theta^{FE}$ for all $t \geq 0$. The proof for any $\mu_0 \in (\mu_0, \underline{\mu})$ is similar and follows from the fact that $q_0 = 1$ which implies that all workers are employed in period 0.

A.4 Proof of Lemma 4

We prove the Lemma by proving the contrapositive. The first thing to note is that $\mu_R := \frac{1}{1+\gamma} > 0.5$ since $0 < \gamma < 1$. Recall that $\underline{\mu} = \frac{J_{min}-\kappa}{\chi}$. Suppose $\underline{\mu} \geq \mu_R$. This implies that $\underline{\mu}$ must also be greater than 0.5. In this case, no interior steady state can exist. Recall that any interior steady state solves:

$$\begin{aligned}
\kappa + \chi\mu &= \mathcal{Q}(\mu) \\
&= \frac{a}{1-\beta\gamma[1-\eta(1-\mu)]} \\
&= \frac{a}{1-\beta\gamma(1-\eta)} \frac{1-\beta\gamma(1-\eta)}{1-\beta\gamma[1-\eta(1-\mu)]} \\
&= J_{min} \frac{1}{1-e\mu}
\end{aligned}$$

where, as before $e = \frac{\beta\gamma\eta}{1-\beta\gamma(1-\eta)}$.

Thus interior steady states solve:

$$\Omega(\mu) := \frac{J_{min}}{1-e\mu} - \kappa - \chi\mu = 0$$

We show that this is not possible if $\underline{\mu} > \mu_R$. In particular, we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$. First, we show that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. Notice that e can also be rewritten as:

$$e = \frac{1}{1 + \frac{1-\beta\gamma}{\beta\gamma\eta}} > \frac{1}{1+\frac{\eta}{\eta}} = \frac{1}{2}$$

where the inequality follows since $\eta > \frac{1-\beta\gamma}{\beta\gamma}$ by Assumption 3. Thus, $e > \frac{1}{2}$. To see that $\chi <$

$2(J_{min} - \kappa)$, note that from the definition of $\underline{\mu}$:

$$\chi = \frac{J_{min} - \kappa}{\underline{\mu}} < 2(J_{min} - \kappa)$$

since $\underline{\mu} > 0.5$ by assumption.

Fix $\kappa \in [0, J_{min})$, $\mu \in [0,1]$. Even though we have shown above that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$, for a moment, set $e = 1/2$, $\chi = 2(J_{min} - \kappa)$. We claim that

$$\mathcal{Q}(\mu) = \frac{J_{min}}{1 - e\mu} \geq \kappa + \chi\mu = \kappa + 2(J_{min} - \kappa)\mu$$

with strict inequality unless $\kappa = 0$ and $\mu = 1$, in which case the expression holds with equality. When $\kappa = 0$, the RHS becomes $2J_{min}\mu$, and the LHS and RHS are only equal for $\mu = 1$. For any $\mu < 1$, the LHS is larger. When $\kappa > 0$, the RHS is strictly lower for any $\mu > 1/2$. Thus for any $\kappa \in [0, J_{min}]$, the inequality holds for all $\mu \in [0,1)$. Finally, for any $\mu \leq 1/2$, the inequality clearly holds since the LHS is greater than J_{min}, and the RHS smaller than J_{min}.

Next, suppose $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. If $\mu = 0$, this does not change the inequality, which still holds strictly (since $\mu \neq 1$). If $\mu > 0$, this strictly increases the LHS and strictly decreases the RHS. Thus the expression is still satisfied with strict inequality. Thus we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$, and there is no interior steady state. Since we have shown that $\underline{\mu} \geq \mu_R$ implies there exists no interior steady state, it follows that if there exist multiple interior steady states, we must have $\underline{\mu} < \mu_R$. □

A.5 Proof of Lemma 5

For $\mu_0 = \underline{\mu}$, the value of an employed worker for a firm is given by J_{min}. To see this, notice that $J_0 \leq \kappa + \chi\underline{\mu}$ as long as labor markets are slack, $\theta_0 \leq 1$. In this case, by definition, $J_0 \leq J_{min}$ and by definition this relationship has to hold with equality. If labor markets are tight, $\theta_0 > 1$, then $J_0 = \kappa\theta_0 + \chi\underline{\mu} > J_{\min}$ since $\theta_0 > 1$. This is a contradiction since if $\theta_0 > 1$, $\mu_1 = 0$ from Lemma 3 and $J_0 = J_{min}$ from Lemma 3. Furthermore, from Lemma 1, it follows that $J_1 = J_{min}$ and $\theta_1 \geq 1$.

The contradiction above shows that $\theta_0 \leq 1$. We now need to show that $\theta_0 > 1 - \underline{\mu}$. Suppose that $\theta_0 < 1 - \underline{\mu}$. Then μ_1 is given by:

$$\mu_1 = \frac{1 - \theta_0}{1 + \gamma[1 - \theta_0 - \underline{\mu}]} > \underline{\mu}$$

This is a contradiction since

$$J_1 = J_{min} = \kappa + \chi\underline{\mu} < \kappa + \chi\mu_1$$

which requires that $\theta_1 = 0$. Thus, we have shown that $\theta_0 \in [1 - \bar{\mu}, 1]$. The rest follows from equation (24) and the previous Lemmas.

A.6 Proof of Proposition 1

Definition 3. *Define the functions* $\Theta^1 : I^1 \to [0,1]$, $F^1 : I^1 \to \mathbb{R}_+$, $M^1 : I^1 \to \{\underline{\mu}\}$ *as:*

$$\begin{aligned} \Theta^1(\mu_{T-1}) &:= 1 - \frac{\underline{\mu}}{1-\gamma\underline{\mu}}(1-\gamma\mu_{T-1}) \\ F^1(\mu_{T-1}) &:= \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1-\eta\Theta^1(\mu_{T-1}))(\kappa + \chi\mu_{T-1})\right] \\ M^1(\mu_{T-1}) &:= \underline{\mu} \end{aligned}$$

where $I^1 = [\underline{\mu}, \mu^1]$ *and* $\mu^1 := F^1(\underline{\mu})$.

Intuitively, at any date t, for any $\mu_t \in I^1$, $\Theta^1(\mu_t)$ describes the job-finding rate that ensures that the economy reaches $\underline{\mu}$ at date $t+1$. $F^1(\mu_t)$ describes the unique value that μ_{t-1} can have in period $t-1$ such that $\mu_t \in I^1$ and also $\mu_{t+1} = \underline{\mu}$. In other words, given market tightness at date t, $\Theta^1(\mu_t)$, one can compute the value of a filled vacancy at date $t-1$ and zero and by no-arbitrage, this pins down the value of μ_{t-1} for which firms would have been willing to post the requisite number of vacancies. $M^1(\mu)$ is just a constant function which by definition describes where any $\mu \in I^1$ ends up.

Corollary 2. *It must be true that* $\mu^1 < \widetilde{\mu}$.

By the definition of μ^1, it must be true that

$$\begin{aligned} \mu^1 &= \frac{1}{\chi}\left[a - \kappa + \beta\gamma\left[1 - \eta(1-\underline{\mu})\right](\kappa + \chi\underline{\mu})\right] \\ &< \frac{1}{\chi}\left[a - \kappa + \beta\gamma\left[1 - \eta(1-\widetilde{\mu})\right](\kappa + \chi\widetilde{\mu})\right] \\ &= \widetilde{\mu} \end{aligned}$$

Lemma 9. *For β sufficiently close to 1, F^1 is increasing in μ for $\mu \in [\underline{\mu}, \widetilde{\mu})$*

Proof. Since $F^1(\mu)$ is composed of constants and a concave part, it suffices to consider the concave polynomial $\xi(\mu) = \left[1 - \eta\Theta^1(\mu)\right](\kappa + \chi\mu)$. This function is increasing in μ for

$$\mu < \frac{1}{2}\left[\frac{\left(1-\gamma\underline{\mu}\right)(1-\eta)}{\eta\gamma\underline{\mu}} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right] \tag{25}$$

It is thus sufficient to show that $\widetilde{\mu}$ satisfies this inequality. Recall that $\widetilde{\mu}$ satisfies

$$\mathcal{Q}(\widetilde{\mu}) = \frac{a}{1 - \beta\gamma\left(1 - \eta + \eta\widetilde{\mu}\right)} = \kappa + \chi\widetilde{\mu}$$

Since the left hand side is convex and the right hand side linear, since $\widetilde{\mu}$ is the smaller of two solutions to this equation, then

$$\mathcal{Q}'(\widetilde{\mu}) = \frac{a\beta\gamma\eta}{\left[1-\beta\gamma\left(1-\eta+\eta\widetilde{\mu}\right)\right]^2} < \chi$$

In other words, the LHS cuts the RHS from above. Next, dividing the first equality by the second inequality, we have

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1-\beta\gamma}{\beta\gamma\eta} + 1 - \frac{\kappa}{\chi}\right] \tag{26}$$

Define:

$$\Xi = \frac{1}{2}\left\{\frac{(1-\gamma\underline{\mu})(1-\eta)}{\eta\gamma\underline{\mu}} - \frac{1-\beta\gamma}{\eta\beta\gamma} + \frac{1}{\gamma} - 1\right\}$$

Assuming that $\beta > \frac{\widetilde{\mu}}{\eta\widetilde{\mu}+1-\eta}$, it can be shown that $\Xi > 0$.[19] Thus, as required:

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1-\beta\gamma}{\beta\gamma\eta} + 1 - \frac{\kappa}{\chi}\right] + \Xi = \frac{1}{2}\left[\frac{\left(1-\gamma\underline{\mu}\right)(1-\eta)}{\eta\gamma\underline{\mu}} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right]$$

□

It was already clear that given a $\mu_{t+1} \in I^1$, there exists a unique μ_t which could have led there. In addition, this Lemma shows that given any μ_t, there exists at most one $\mu_{t+1} \in I^1$ is consistent with equilibrium.

Corollary 3. *Let $I^2 = F^1(I^1)$ and let $M^2(\mu)$ be the inverse of this function. Then $M^2(\mu^1) = M^1(\mu^1) = \underline{\mu}$.*

Since F^1 is increasing and continuous, its inverse M^2 exists and is increasing and continuous. Consequently, $F^1(I^1)$ maps into an interval $(\mu^1, \mu^2]$. Further since $\mu^1 = F^1(\underline{\mu})$, then $M^2(\mu^1) = \underline{\mu}$.

Lemma 10. $\mu^2 = F^1(\mu^1) < \widetilde{\mu}$

Proof. Since $\Theta^1(\underline{\mu}) = 1 - \underline{\mu}$ and Θ^1 is increasing, we have $\Theta^1(\mu^1) > 1 - \underline{\mu} > 1 - \widetilde{\mu}$. It follows that:

$$\frac{1}{\chi}\left[a - \kappa + \beta\gamma(1-\eta\Theta^1(\mu^1))(\kappa+\chi\widetilde{\mu})\right] \quad < \quad \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1-\eta(1-\widetilde{\mu}))(\kappa+\chi\widetilde{\mu})\right]$$

[19] Note that this assumption is a condition on an endogenous variable, $\widetilde{\mu}$ and can be rewritten as $\widetilde{\mu} < \dfrac{1-\eta}{\beta^{-1}-\eta}$. Nonetheless, it is a weak condition: for any $\widetilde{\mu} < 1$, it is satisfied for β sufficiently close to 1.

Then, from Corollary 2, since $\mu^1 < \widetilde{\mu}$:

$$\begin{aligned} F^1(\mu^1) &= \frac{1}{\chi}\left[a-\kappa+\beta\gamma(1-\eta\Theta^1(\mu^1))(\kappa+\chi\mu^1)\right] \\ &< \frac{1}{\chi}\left[a-\kappa+\beta\gamma(1-\eta(1-\widetilde{\mu}))(\kappa+\chi\widetilde{\mu})\right] \\ &= \widetilde{\mu} \end{aligned}$$

□

Lemma 11. *Define* $\Theta^2(\mu) : I^2 \to [0,1]$ *as:*

$$\Theta^2(\mu) := 1 - M^2(\mu)\frac{1-\gamma\mu}{1-\gamma M^2(\mu)}$$

Then,

$$\frac{\partial\Theta^2(\mu)}{\partial\mu} \leq \frac{\gamma M^2(\mu)}{1-\gamma M^2(\mu)}$$

Proof.

$$\begin{aligned} \frac{\partial\Theta^2(\mu)}{\partial\mu} &= M^2(\mu)\frac{\gamma}{1-\gamma M^2(\mu)} - \frac{\partial M^2(\mu)}{\partial\mu}\left[1+\frac{\gamma\left(1-\gamma\mu\right)M^2(\mu)}{\left[1-\gamma M^2(\mu)\right]^2}\right] \\ &\leq M^2(\mu)\frac{\gamma}{1-\gamma M^2(\mu)} \end{aligned}$$

where the inequality comes because $M^2(\mu)$ is increasing and the expression in square brackets is positive. □

We are now ready to characterize equilibrium in the entire convalescent region.

Lemma 12 (Induction Step)**.** *Suppose the functions* $\Theta^n(\mu)$*,* $M^n(\mu)$ *are defined on some interval* $I^n = [\mu^{n-1}, \mu^n]$ *and* $M^{n-1}(\mu_{T-n+1})$ *is defined on an interval* $I^{n-1} = [\mu^{n-2}, \mu^{n-1}]$*, with* $\underline{\mu} < \mu^{n-2} < \mu^n < \widetilde{\mu}$*, and that these functions satisfy*

$$\begin{aligned} \Theta^n(\mu) &= 1 - M^n(\mu)\frac{1-\gamma\mu}{1-\gamma M(\mu)} \\ \frac{\partial\Theta^n(\mu)}{\partial\mu} &< \frac{\gamma M^n(\mu)}{1-\gamma M^n(\mu)} \\ M^n(I^n) &= I^{n-1} \\ M^n(\mu^{n-1}) &= M^{n-1}(\mu^{n-1}) = \mu^{n-2} \end{aligned}$$

Then, for β *sufficiently close to 1, we have the following results:*

1. *The function*

$$F^n(\mu) := \frac{1}{\chi}\left[a-\kappa+\beta\gamma(1-\eta\Theta^n(\mu))(\kappa+\chi\mu)\right]$$

is monotonically increasing in μ for $\mu \leq \widetilde{\mu}$.

2. *Let $I^{n+1} = F^n(I^n)$ and let $M^{n+1}(\mu)$ be the inverse of this function. Then $M^{n+1}(\mu^n) = M^n(\mu^n) = \mu^{n-1}$.*

3. *$I^{n+1} = [\mu^n, \mu^{n+1}]$ with $\mu^{n+1} < \widetilde{\mu}$.*

4. *Define $\Theta^{n+1}(\mu)$ on I^{n+1} by*

$$\Theta^{n+1}(\mu) = 1 - M^{n+1}(\mu)\frac{1-\gamma\mu}{1-\gamma M^{n+1}(\mu)}$$

The derivative of this function satisfies

$$\frac{\partial \Theta^{n+1}(\mu)}{\partial \mu} < \frac{\gamma M^{n+1}(\mu)}{1-\gamma M^{n+1}(\mu)}$$

Proof. (1.) The derivative of $F^n(\mu)$ is

$$\begin{aligned}\frac{\partial F^n(\mu)}{\partial \mu} &= \frac{\beta\gamma}{\chi}\left[-\eta\frac{\partial\Theta^n(\mu)}{\partial\mu}(\kappa+\chi\mu) + \chi(1-\eta\Theta^n(\mu))\right] \\ &> \frac{\beta\gamma}{\chi}\left[-\eta\frac{\gamma M^n(\mu)}{1-\gamma M^n(\mu)}(\kappa+\chi\mu) + \chi(1-\eta\Theta^n(\mu))\right]\end{aligned}$$

Substituting in the definition of Θ^n and rearranging, we see that this expression will be positive provided that

$$\mu < \frac{1}{2}\left[\frac{1-\eta}{\eta}\frac{(1-\gamma M^n(\mu))}{\gamma M^n(\mu)} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right]$$

By the same logic as in Lemma 9, for β sufficiently close to 1, this is satisfied for any $\mu \leq \widetilde{\mu}$, since we have $M^n(\mu) \leq \widetilde{\mu}$. So $F^n(\mu)$ is increasing, and hence invertible, for $\mu < \widetilde{\mu}$. Let $M^{n+1}(\mu)$ be the inverse of this function.

(2.) We have

$$\begin{aligned} M^n(\mu^{n-1}) &= M^{n-1}(\mu^{n-1}) \\ \Theta^n(\mu^{n-1}) &= \Theta^{n-1}(\mu^{n-1}) \\ F^n(\mu^{n-1}) &= F^{n-1}(\mu^{n-1}) = \mu^n \text{ by definition of } \mu^n \\ M^{n+1}(\mu^n) &= M^n(\mu^n)\end{aligned}$$

(3.) Since F^n is a continuous, increasing function, the image of the interval $[\mu^{n-1}, \mu^n]$ under F^n must be an interval $[\mu^n, \mu^{n+1}]$. (We have already shown that $F^n(\mu^{n-1}) = \mu^n$.) We need to show that $\mu^{n+1} = F^n(\mu^n) < \widetilde{\mu}$. We know that $\widetilde{\mu} \geq M^n(\mu^n)$. Then, it must be true that

$$
\begin{aligned}
1-\widetilde{\mu} &< 1-M^n(\mu^n) \\
&= 1-M^n(\mu^n)\frac{1-\gamma\mu^n}{1-\gamma\mu^n} \\
&< 1-M^n(\mu^n)\frac{1-\gamma\mu^n}{1-\gamma M^n(\mu^n)} \\
&= \Theta^n(\mu^n)
\end{aligned}
$$

Then, by the same logic as in Lemma 10 we have $F^n(\mu^n) < \widetilde{\mu}$. So we have shown that $I^{n+1} \subset [\underline{\mu}, \widetilde{\mu}]$.

(4.) The bound on the derivative is established in the same way as Lemma 11.

□

Lemma 13. $\lim_{n\to\infty} \mu^n \to \widetilde{\mu}$.

Proof. We have shown that $\{\mu^n\}$ is an increasing sequence bounded above by $\widetilde{\mu}$; thus by the Monotone Convergence Theorem, its limit μ^∞ exists, and $\mu^\infty \leq \widetilde{\mu}$. Suppose by contradiction that $\mu^\infty < \widetilde{\mu}$. Then μ^∞ must be a steady state. But by definition, $\widetilde{\mu}$ is the smallest slack steady state. So we must have $\mu^\infty = \widetilde{\mu}$. □

A.7 Proof of Lemma 7

The full employment steady state Nash wage equals

$$
w^* = \frac{\eta}{1-\beta\gamma(1-\eta)}A + \frac{(1-\beta\gamma)(1-\eta)}{1-\beta\gamma(1-\eta)}b
$$

So $\frac{\partial J}{\partial \beta_0} = -w^* + \gamma J_{min}$ will be negative provided that

$$
\begin{aligned}
-\frac{\eta}{1-\beta\gamma(1-\eta)}A - \frac{(1-\beta\gamma)(1-\eta)}{1-\beta\gamma(1-\eta)}b + \gamma\frac{(1-\eta)(A-b)}{1-\beta\gamma(1-\eta)} &< 0 \\
A\left[\gamma - \frac{\eta}{1-\eta}\right] - (2-\delta-\beta(1-\delta))b &< 0
\end{aligned}
$$

By assumption 3, both terms are negative, so this condition is satsified. If $\beta_0 > \underline{\beta}$, $J_0 < \kappa$, thus no vacancies will be posted, and we have

$$
\mu_1 = \frac{1-\theta_0}{1+\gamma[1-\theta_0-\mu_0]} = \frac{1}{1+\gamma} := \mu_R
$$

We already know that under a price level targeting rule, the economy converges to zero employment if $\mu_1 > \underline{\mu}$. Since $\mu_R > \underline{\mu}$, by Lemma 4, this is indeed what happens.

A.8 Proof of Proposition 2

It is first necessary to prove two lemmas. The first states that wages are lower in the convalescent region than at full employment. We need this result in order to show that prices will be higher in the convalescent region.

Lemma 14. *$w(\mu) < w^*$ if $\mu \in (\underline{\mu}, \tilde{\mu})$.*

Proof. We know that $M(\mu) < \mu$ if $\mu \in (\underline{\mu}, \tilde{\mu})$.

$$\begin{aligned} w(\mu) &= A - (\kappa + \chi\mu) + \beta\gamma[\kappa + \chi M(\mu)] \\ &= A - \beta\gamma\chi(\mu - M(\mu)) - (1 - \beta\gamma)(\kappa + \chi\mu) \\ &< A - (1 - \beta\gamma)(\kappa + \chi\mu) \\ &< A - (1 - \beta\gamma)(\kappa + \chi\underline{\mu}) = w^* \end{aligned}$$

□

Lemma 15. *Under Assumption 4, $w^B(\mu) > \gamma[\kappa + \chi\mu]$ for all $\mu \in [0, 1]$.*

Proof. We know that $w^B(\mu) \geq w^*(\mu)$, so it suffices to show that $w^*(\mu) > \gamma[\kappa + \chi\mu]$ for all $\mu \in [0, 1]$. In the flexible wage benchmark we have

$$w_t = \eta A + (1 - \eta)b + \beta\gamma q_{t+1} J_{t+1} \geq \eta A + (1 - \eta)b$$

□

Finally, we need to characterize dynamics of the economy starting at date 1, once the shock has abated. Under neutral monetary policy, if the ZLB never binds, allocations are (by definition) equal to those in the flexible wage benchmark. The following is immediate.

Lemma 16. *If $\mu_1 \geq \widetilde{\mu}$, the economy never reaches the full employment steady state.*

Proof. If the ZLB never binds, allocations are equivalent to those in the flexible wage benchmark, and we know that the economy never returns to steady state. It only remains to show that the ZLB can never help the economy converge to the FESS. Suppose by contradiction that the economy converges to the FESS. Let μ_t^R, μ_t^N denote allocations in the flexible wage benchmark and in the nominal economy, respectively, given the initial condition $\mu_1 \geq \widetilde{\mu}$. Let $T \geq 1$ be the first date at which $\mu_t^N < \mu_t^R$ (there must be some such date, since in the long run $\mu_t^N = 0, \mu_t^R > 0$, by assumption). Then we have

$$\begin{aligned} J_{T-1}^N = \kappa + \chi\mu_{T-1}^N = \kappa + \chi\mu_{T-1}^R = J_{T-1}^R \\ J_T^N = \kappa + \chi\mu_T^N < \kappa + \chi\mu_T^R = J_T^R \end{aligned}$$

This implies that real wages are higher at date $T-1$ in the flexible wage benchmark than in the nominal economy:

$$J_{T-1}^N = J_{T-1}^R$$
$$A - w_T^N + \beta\gamma J_T^N = A - w_T^R + \beta\gamma J_T^R$$
$$w_t^R - w_t^N = \beta\gamma(J_T^R - J_T^N) > 0$$

This in turn means that prices are higher in the nominal economy than in the flexible wage benchmark. But this is a contradiction, since given the monetary policy rule, prices can only ever be lower in the nominal economy than in the flexible wage benchmark. Thus the economy cannot converge to the FESS. □

We are now ready to prove Proposition 2. Part 1. follows for the same reasons as in the previous lemmas.

Define the function

$$B(\mu) = \frac{A-\kappa}{w^B(\mu) - \gamma[\kappa + \chi\mu]}$$

on $[\underline{\mu}, \mu_R]$. It is straightforward to show that w^B is continuous, and thus B is continuous. We have $B(\underline{\mu}) = \underline{\beta}$.

Define $\hat{\beta} := B(\mu_R)$. If $\beta_0 > \hat{\beta}$, then if $\mu_1 = \mu_R$, we have

$$J_0 = A - \beta_0 w^B(\mu_R) + \beta_0\gamma[\kappa + \chi\mu_R] < \kappa$$

thus $\theta_0 = 0$, which is consistent with $\mu_1 = \mu_R$. If instead $\beta_0 \in (\underline{\beta}, \hat{\beta})$, then there exists $\mu \in (\underline{\mu}, \mu_R)$ such that $B(\mu) = \beta_0$, and a corresponding $\theta_0 = 1 - \dfrac{\mu_1}{1-\gamma\mu_1}$. Then we have

$$J_0 = \kappa = A - \beta_0 w^B(\mu_1) + \beta_0\gamma[\kappa + \chi\mu_1]$$

and firms are indifferent between posting any number of vacancies; thus $\theta_0 \in [0, 1]$ can indeed be an equilibrium.

The last part of the Proposition is immediate given the analysis in Section 3.

A.9 Proof of Proposition 3

We start by constructing the intervals specified in the Lemma. Assuming that the economy returns to full employment at date 1, date 0 wages in the frictionless benchmark are given by:

$$w_0^* = \eta A_0 + (1-\eta)b + \beta\gamma\eta J_{min} < w_{fe}^*$$

Define A_Z such that

$$\eta A_Z + (1-\eta)b + \beta\gamma\eta J_{min} = \beta[\eta A + (1-\eta)b + \beta\gamma\eta J_{min}]$$

For any $A_0 < A$, there exists β sufficiently close to 1 that $w_0^* < \beta w_{fe}^*$. Given fixed nominal wages, this implies that attaining the frictionless benchmark would require a negative nominal interest rate, which would violate the ZLB. Thus, A_Z must be positive. Define

$$A_I = \kappa + \beta w_{fe}^* - \beta\gamma J_{min}$$

and suppose that $A_0 \in [A_I, A_Z]$. Then we have

$$J_0 = A_0 - \beta w_{fe}^* + \beta\gamma J_{min} \geq \kappa$$

and there is an equilibrium with $\mu_1 = 0$, $\theta_0 = \dfrac{J_0}{\kappa} \geq 1$.

Define the continuous function $A(\mu) := \kappa + \beta w^B(\mu) - \beta\gamma[\kappa + \chi\mu]$ on the interval $[\underline{\mu}, \mu_R]$. By construction, $A(\underline{\mu}) = A_I$, since $w^B(\underline{\mu}) = w_{fe}^*$. Define $A_F = A(\mu_R)$. Let $A_0 \leq A_F$, and suppose that $\mu_1 = \mu_R$. Then

$$J_0 = A_0 - \beta w^B(\mu_R) + \beta\gamma[\kappa + \chi\mu_R] \leq \kappa$$

and so $\theta_0 = 0$, which is consistent with $\mu_1 = \mu_R$. If instead $A_0 \in (A_F, A_I)$, then there exists $\mu_1 \in (\underline{\mu}, \mu_R)$ such that $A(\mu_1) = A_0$, and a corresponding $\theta_0 = 1 - \dfrac{\mu_1}{1-\gamma\mu_1}$. Then we have

$$J_0 = \kappa = A_0 - \beta_0 w^B(\mu_1) + \beta\gamma[\kappa + \chi\mu_1]$$

the ZLB binds, and firms are indifferent between posting any number of vacancies; thus $\theta_0 \in [0,1]$ can indeed be an equilibrium.

A.10 Proof of Proposition 5

By construction, if wages satisfy $w_T = \omega_T$, $w_t = \beta^{T-t}\omega_T$ for $t = 1, ..., T-1$, $w_0 = \beta_0 w_1$, $w_t = w^*$ for $t \geq T+1$, then the value of a firm at time 0 is

$$\begin{aligned} J_0 &= A - w_0 + \beta_0\gamma \sum_{t=1}^{T} (\beta\gamma)^{t-1}[A - w_t] + \beta_0\beta^T\gamma^{T+1} J_{min} \\ &= A - \beta_0\beta^{T-1}\omega_T + \beta_0\gamma \sum_{t=1}^{T} (\beta\gamma)^{t-1}[A - \beta^{T-t}\omega_T] + \beta_0\beta^T\gamma^{T+1} J_{min} \\ &= \kappa \end{aligned}$$

so firms are willing to post vacancies at date 0. If $w_T = \omega_T < w^*$, then we will have $w_t = \beta^{T-t}\omega_T < w^*$ and $J_t > J_{min}$ for $t = 1, \ldots, T-1$. Thus there will be a tight labor market at these dates. Finally, to show that $\omega_T \in (\beta w^*, w^*)$ for some T, note that we could alternatively write the above equation as

$$\kappa = A - \beta_0 w_1 + \sum_{t=1}^{\infty} (\beta\gamma)^t [A - \min\{\beta^{1-t} w_1, w^*\}]$$

The right hand side is a continuous function of w_1. Thus for any β_0, this equation is satisfied for some w_1. Let T be the largest integer such that $\beta^{1-t} w_1 < w^*$. Then we have $\omega_T := \beta^{1-T} w_1 \in (\beta w^*, w^*)$, as required.