College Pricing and Income Inequality

Zhifeng Cai
U of Minnesota and FRB Minneapolis

Jonathan Heathcote
FRB Minneapolis

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Introduction

- Context:
  - Price of college tuition in U.S. has been outstripping inflation
  - Income inequality has been rising
- Questions:
  1. What determines tuition pricing?
  2. How does rising income inequality impact:
     - Equilibrium college pricing
     - College attendance
- Key feature of theory: College is a club good
  - Quality (desirability) of a given college depends on attributes (e.g. academic ability) of the students who attend

Related Literature

- Theory:
  - Rothschild & White (1995), Ellickson, Grodal, Scotchmer, Zame (1999), Cole & Prescott (1997), Caucutt (1999),
- Applied:
  - Epple & Romano (1998), Caucutt (2002), Epple, Romano & Sieg (2006, 2013), Gordon & Hedlund (2016), Jones & Yang (2014)
- Distinctive features of our framework:
  - Perfectly competitive environment
    - Large number of clubs of each type
  - No lotteries over club membership
    - Large number of members in each club
    - Large number of possible club types

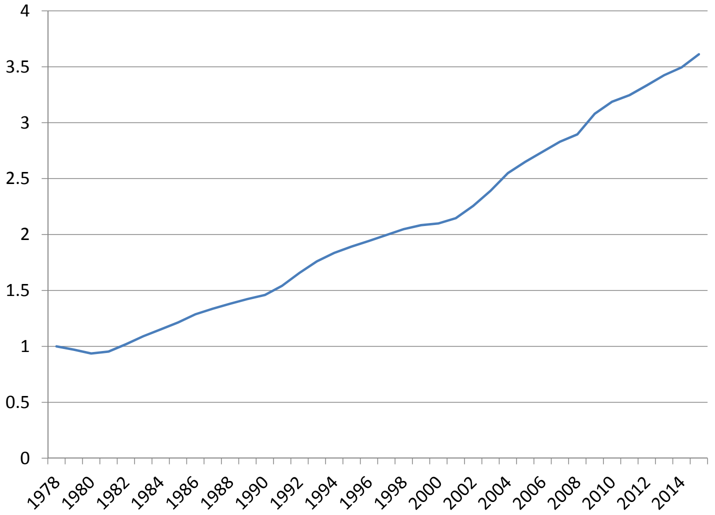
Trends in College Tuition

1. Average tuition has been rising

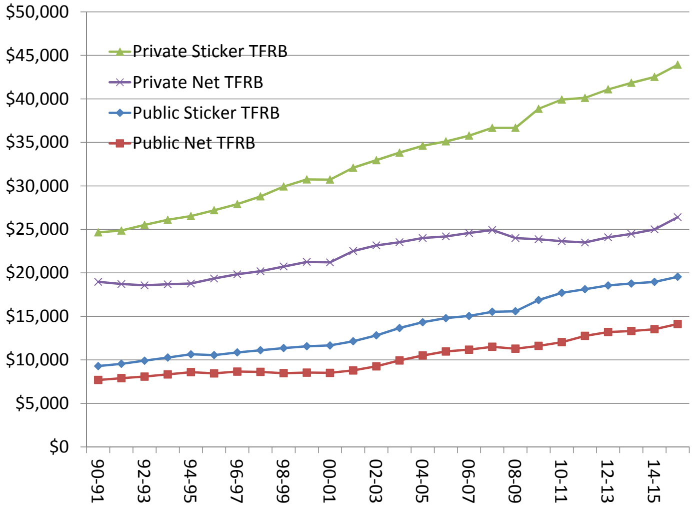
2. Tuition dispersion within schools has been rising

    - High ability & low income students often pay less than sticker price

Tuition and Fees / All Items Price Index (BLS)



Tuition, Fees, Room & Board (College Board $2015)



Model: Households

- Continuum of measure 2 of households, each containing a parent and a college-age child
- Heterogeneous wrt: (i) income y, (ii) student ability a
- Two ability levels, indexed $i \in \{l.h\}$, $a_l < a_h$, measure 1 of each level
- Continuous distribution for income, CDF $F^i(y)$
- Utility from non-durable consumption c and quality q of the college the child attends

$$u(c,q) = \log c + \varphi \log(\kappa + q)$$

Household Problem

- Take as given tuition functions $t^i(q; y)$
- Let $\mathcal{Q}$ denote the set of college qualities available
- Assume $q = 0 \in \mathcal{Q}$, with $t^h(0; y) = t^l(0; y) = 0$
- Given idiosyncratic state (y, i), solve

$$\max_{\{c, q \in \mathcal{Q}\}} u(c, q)$$
$$s.t.$$
$$c + t^i(q; y) = y$$

- Solution: $c^i(y)$, $q^i(y)$

Model: Colleges

- CRS technology for producing education of a given quality
- Quality (per student) reflects:

  (i) average ability of student body

  (ii) consumption good input (per student) e (faculty etc)

$$q = (\eta a_h + (1-\eta)a_l)^{\theta} e^{1-\theta}$$

  where η is share of student body that is high ability

- Fixed consumption cost R&B ϕ per student admitted

College Problem

- Colleges profit maximize
- Observe income y and child's ability type i, take as given tuition functions
- Choose quality level(s) to deliver, size, and input mix
- Input mix sub-problem for supplying mass 1 spots at $q > 0$

$$\max_{y^h, y^l, \eta, e} \left\{ t^h(q; y^h)\eta + t^l(q; y^l)\left(1-\eta\right) - e - \phi \right\}$$
$$s.t.$$
$$q = \left(\eta a_h + \left(1-\eta\right) a_l\right)^{\theta} e^{1-\theta}$$

- Let $y^h(q)$, $y^l(q)$, $\eta(q)$, $e(q)$ denote solution, and $\pi(q)$ per capita profit
so when there is another schools with

College Problem (cont.)

- Define $t^i(q) = \max_y t^i(q;y)$: no admissions at lower prices
- First-order conditions:

$$\eta(q)a_h + (1-\eta(q))\,a_l = q\left(\frac{(1-\theta)\left(t^l(q)-t^h(q)\right)}{\theta\Delta a}\right)^{\theta-1}$$

$$e(q) = q\left(\frac{(1-\theta)\left(t^l(q)-t^h(q)\right)}{\theta\Delta a}\right)^{\theta}$$

$$\frac{e(q)}{\eta(q)a_h + (1-\eta(q))\,a_l} = \frac{(1-\theta)\left(t^l(q)-t^h(q)\right)}{\theta\Delta a}$$

- Optimal size at each q:

$$\begin{cases} 0 & \text{if} \quad \pi(q) < 0 \\ [0,\infty] & \text{if} \quad \pi(q) = 0 \\ \infty & \text{if} \quad \pi(q) > 0 \end{cases}$$

Equilibrium

Define $\chi(Q)$: measure of students in colleges with $q \in Q \subset \mathcal{Q}$. Equilibrium is set of functions $\chi(q)$, $t^i(q)$, $\eta(q)$, $e(q)$, $c^i(y)$, $q^i(y)$ s.t.

1. Given $t^i(q)$, $q^i(y)$ and $c^i(y)$ solve household's problem
2. Given $t^i(q)$, $\eta(q)$ and $e(q)$ solve college's problem
3. Zero profits: $\forall Q$, $\pi(q) \leq 0 \ \forall q \in Q$ and
$$\int_Q \pi(q) d\chi(q) = 0$$
4. Market clearing:
$$\sum_{i=h,l} \int c^i(y) dF^i(y) + \int e(q) d\chi(q) + (2 - \chi(0))\phi = \sum_{i=h,l} \int y dF^i(y)$$
$$\int 1_{\{q^h(y) \in Q\}} dF^h(y) = \int_Q \eta(q) d\chi(q) \qquad \forall Q$$
$$\int 1_{\{q^l(y) = Q\}} dF^l(y) = \int_Q (1 - \eta(q)) d\chi(q) \qquad \forall Q$$

General Properties

- Tuition is independent of income: $t^i(q; y) = t^i(q)$
  - Profit-maximizing firms don't want to give unnecessary subsidies (and could not afford to anyway)
- At each quality level, $t^h(q) < t^l(q)$
  - Otherwise colleges would strictly prefer high ability students
- Tuition is increasing in quality: $q_1 > q_2 \Rightarrow t^i(q_1) > t^i(q_2)$
  - Otherwise no students would choose lower quality college
- Assortative matching
  - Holding fixed ability, higher income households will choose higher quality colleges (education a normal good)

Parametric Example

- Pure club good model: $\theta = 1 \Rightarrow q = \eta a_h + (1-\eta)a_l$
  - Households sell and buy ability in college market

- Set $\varphi = 1 \Rightarrow u(c,q) = \log c + \log(\kappa + q)$

- No R&B: $\phi = 0$

- Uniform income distribution:

$$y \sim U\left[\mu_y - \frac{\Delta_y}{2}, \mu_y + \frac{\Delta_y}{2}\right]$$

$$F^h(y) = F^l(y)$$

- Let $\mu_a = \frac{a_h + a_l}{2}$, $\Delta_a = a_h - a_l$

Questions

1. What are $\chi(q)$, $t^h(q)$, $t^l(q)$?
2. How do these objects depend on Δ_y?
3. How does market for college differ from market for fish?

Digression: Modeling College Like Fish

- Households endowed with a_l or a_h units of ability
- Sell and buy ability at centralized market at per unit price p
- Household problem:

$$\max_{c,q} \{\log(c) + \log(\kappa + q)\}$$
$$s.t.$$
$$c + pq = y + pa_i$$

- Market clearing

$$\sum_{i=h,l} \int c^i(y)dF^i(y) = \sum_{i=h,l} \int ydF^i(y)$$
$$\sum_{i=h,l} \int q^i(y)dF^i(y) = a_h + a_l$$

Standard (Non-Club Good) Model

- Household FOC:

$$q^i(y) = \frac{y + pa_i - p\kappa}{2p}$$

- Market clearing:

$$p = \frac{\mu_y}{\mu_a + \kappa}$$

- "Tuition" (net price) function:

$$t_F^i(q) = pq - pa_i = (q - a_i)\,\frac{\mu_y}{\mu_a + \kappa}$$

1. Dist. of ability demand depends on dist. of income
2. Net price functions are linear in q, and
3. Price function does not depend on income inequality Δ_y

The Club Good Model

- College distribution: $\forall Q \subset (a_l, a_h)$

$$\chi(Q) = \frac{2}{\Delta_a}\left(\frac{2}{4+\pi}\right)\int_Q \left[(1-\eta(q))^2 + \eta(q)^2\right]^{-2} dq$$

$$\chi(a_h) = \chi(a_l) = \frac{2}{4+\pi} = 0.28$$

- Tuition functions:

$$t^i(q) = \mu_y\left(\frac{q-a_i}{\kappa+q}\right)\left[1-\left(\frac{2}{4+\pi}\right)\frac{\Delta_y}{\mu_y}\arctan\left(1-2\eta(q)\right)\right]$$

- Competitive equilibrium is **Pareto efficient**

1. Distribution of quality independent of $(\mu_y, \Delta_y, \kappa)$
2. Price functions **non-linear** in q
3. Price functions **depend on** Δ_y

Sketch of Solution Method

1. Given any college distribution $\chi(q)$, derive income of households attending college q: $y^i(q;\chi(.))$
2. Given $y^i(q;\chi(.))$, household's FOC gives an ODE that pins down the college tuition function: $t^i(q;\chi(.))$

$$\frac{dt^i(q;\chi(.))}{dq}\frac{1}{y^i(q;\chi(.))-t^i(q;\chi(.))}=\frac{1}{\kappa+q}$$

3. Given $t^i(q;\chi(.))$, derive a college profit function:

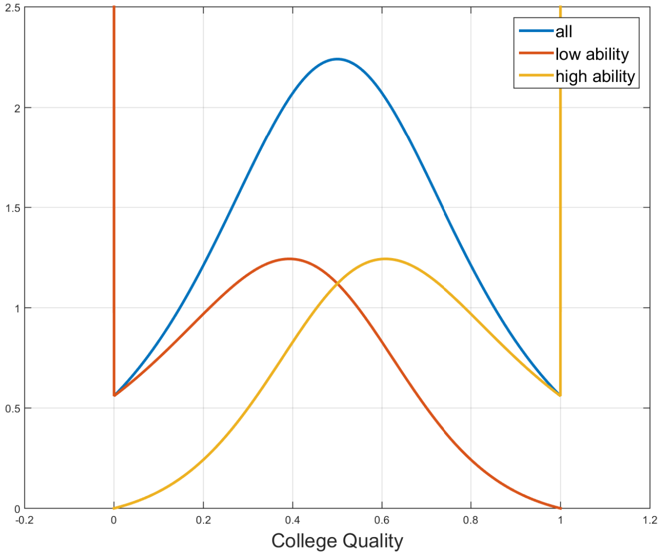
$$\pi(q;\chi(.))=\eta(q)t^h(q;\chi(.))+(1-\eta(q))t^l(q;\chi(.))$$

4. Solve for $\chi(q)$ from the functional equation

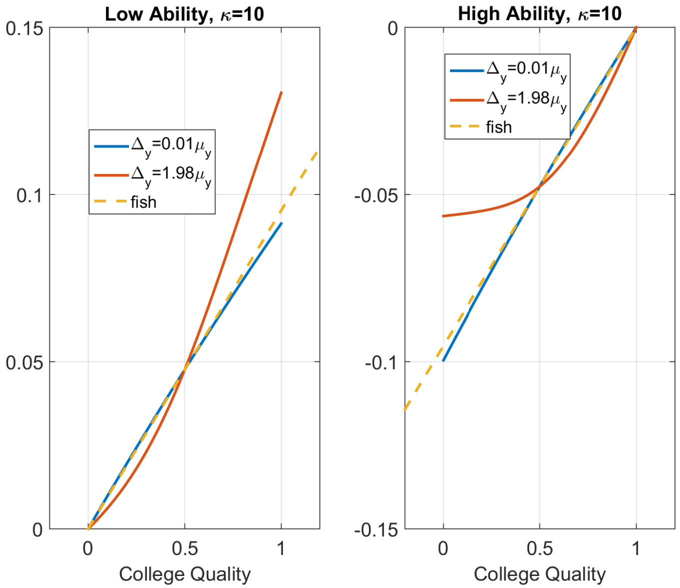
$$\pi(q;\chi(.))=0$$

   - This is a Volterra integral equation of the second kind with degenerate kernels, which has an analytical solution

College Distribution

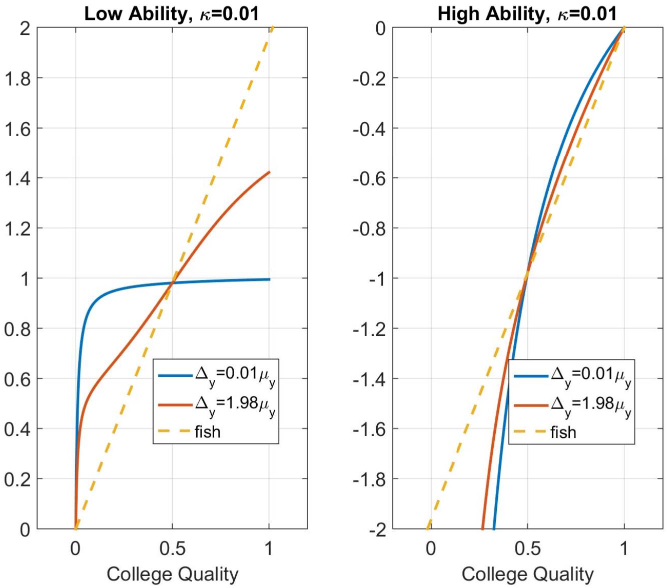


Tuition



Low Ability, κ=10
0.15
0.1
0.05
0
0
0.5
1
College Quality
Δy=0.01μy
Δy=1.98μy
fish
High Ability, κ=10
0
-0.05
-0.1
-0.15
0
0.5
1
College Quality
Δy=0.01μy
Δy=1.98μy
fish

Tuition



Low Ability, κ=0.01
High Ability, κ=0.01
College Quality
College Quality
Δy=0.01μy
Δy=1.98μy
fish

More Properties of Club Good Equilibrium

1. Perfectly-mixed & single-ability schools set "fish" tuition:

$$
\begin{aligned}
t^i(\mu_a) &= t_F^i(\mu_a) = \mu_y \left(\frac{\mu_a - a_i}{\mu_a + \kappa} \right) \\
t^h(a_h) &= t_F^h(a_h) = 0 \\
t^l(a_l) &= t_F^l(a_l) = 0
\end{aligned}
$$

2. Increasing Δ_y:
   - raises (lowers) $t^l(q)$ for $q \geq (\leq)\, \mu_a$
   - lowers (raises) $t^h(q)$ for $q \geq (\leq)\, \mu_a$
   - raises tuition differential for high q, lowers diff. for low q
3. At any quality level $q \in (0, 1)$ colleges will have 2 types of customer: (i) high ability with relatively low income, and (ii) low ability with relatively high income

Quantitative Example: Calibration

- Income distribution: Pareto Log-Normal:

$$\ln y \sim EMG(\mu^i, \sigma^2, \alpha)$$

- $\sigma^2 = 0.4117$ (SCF, 2007)
- $\alpha = 1.8$ (Piketty-Saez, 2014)
- μ^i s.t.

$$\frac{E[y|_{i=h}]}{E[y|_{i=l}]} = \frac{\$67,000}{\$45,000}$$

- (avg. family income conditional on child's AFQT score being above / below median, 1997 NLSY).

Preferences and College Technology

Preferences (φ, κ), Technology: (θ, ϕ)

1. Enrollment: $31.9\% \Rightarrow \kappa = 0.0361$
2. Tuition spending to Agg. Cons.: $2.14\% \Rightarrow \varphi = 0.0409$
3. Room and Board vs. Tuition and Fees $\Rightarrow \phi = 0.0319$
4. Peers vs. goods equally important in quality $\Rightarrow \theta = 0.5$

(target total 4 yr enrollment, tuition targets for private schools)

Preferences and College Technology

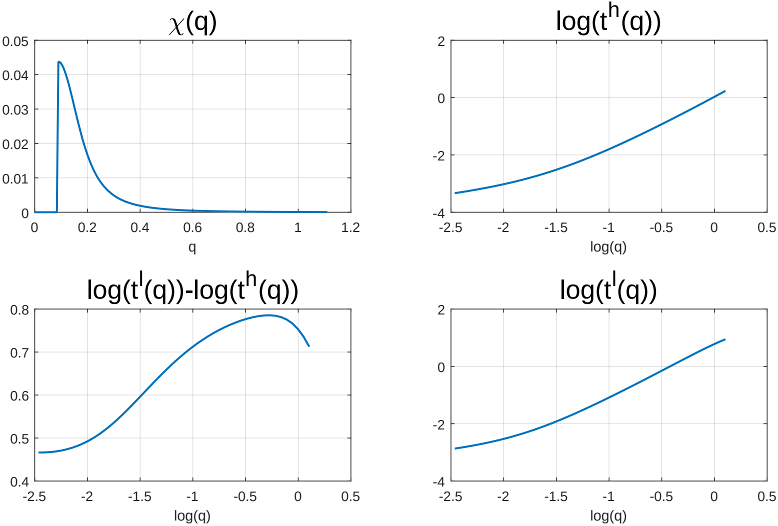
5. Ability gap $a_h - a_l$ drives within-school tuition dispersion

   - (i) avg. sticker tuition + R&B = $\$41,770$
     (ii) avg. price paid = $\$22,900$
     (iii) avg. financial aid = $\$12,380$ institutional, $\$6,490$ other
   - Assume everyone gets "other" aid, all institutional aid goes to high ability

$$\frac{\text{ave. net low ability tuition}}{\text{ave. net tuition}} = \frac{\$41,770 - \$6,490}{\$22,900}$$

   - $\Rightarrow a_l = -0.0216 \; (a_h = 1)$

Calibrated Model Results



Calibrated Model Results

AGGREGATE LEVEL		
	Model	Data
Enrollment rate	31.9%	31.9%
Tuition per student / GDP pc	6.7%	6.7%
Share high ability in college	48.8%	
Share low ability in college	15.0%	

INDIVIDUAL LEVEL CORRELATIONS	
	Model
corr.(college(0,1), a_i)	0.363
corr.(college(0,1), log(y))	0.666
corr.(log(q), a_i)	0.185
corr.(log(q), log(y))	0.905
corr.(log(t), a_i)	-0.344

Model-Data Comparison

Rich data on college characteristics from College Scorecard (US Dept Education)

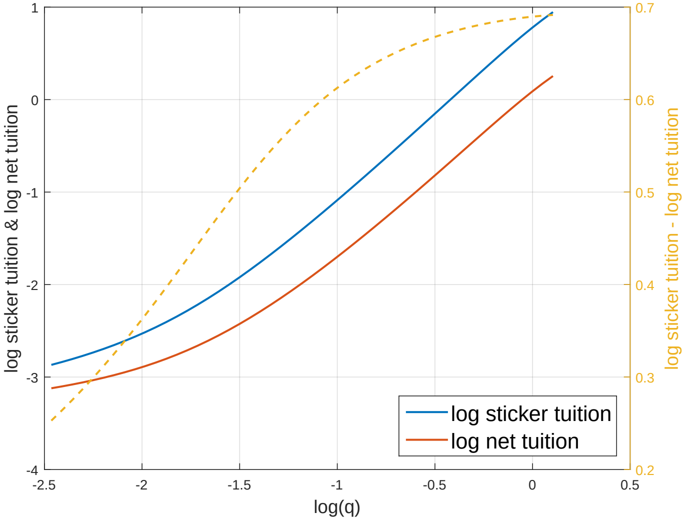
- Sticker price tuition
- Avg. net price paid
- Avg. family income
- Avg. SAT score
- Avg. earnings 10 years after graduation

Focus on 4 year private non-profit sector

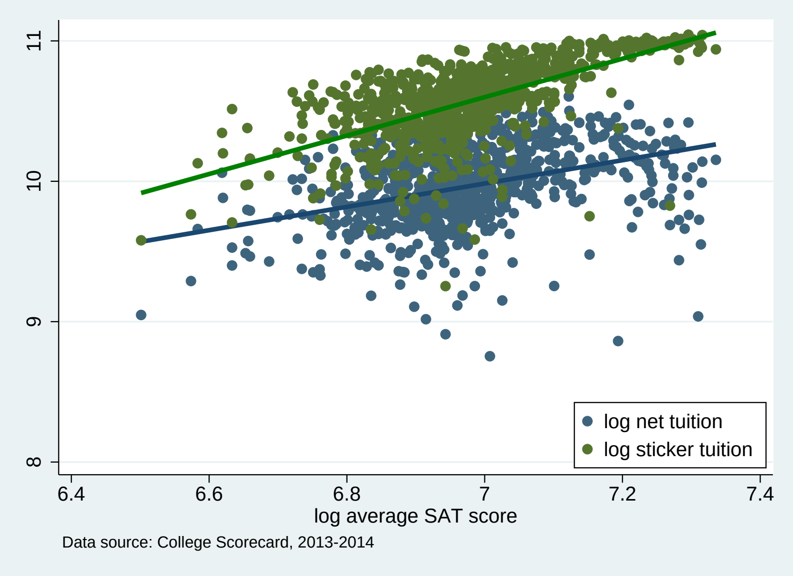
Model Versus College Scorecard

COLLEGE LEVEL STATISTCS		
	Model	Data
var.(log avg. net tuition)	0.132	0.108
var.(log sticker tuition)	0.205	0.128
var.(log avg. fam income)	0.312	0.115
var.(log avg. SAT)	0.015	0.018
corr.(log net tuition, log income)	0.988	0.590
corr.(log net tuition, log SAT)	0.759	0.330
corr.(log income, log SAT)	0.844	0.580
elast. tuition wrt income	0.839	0.330

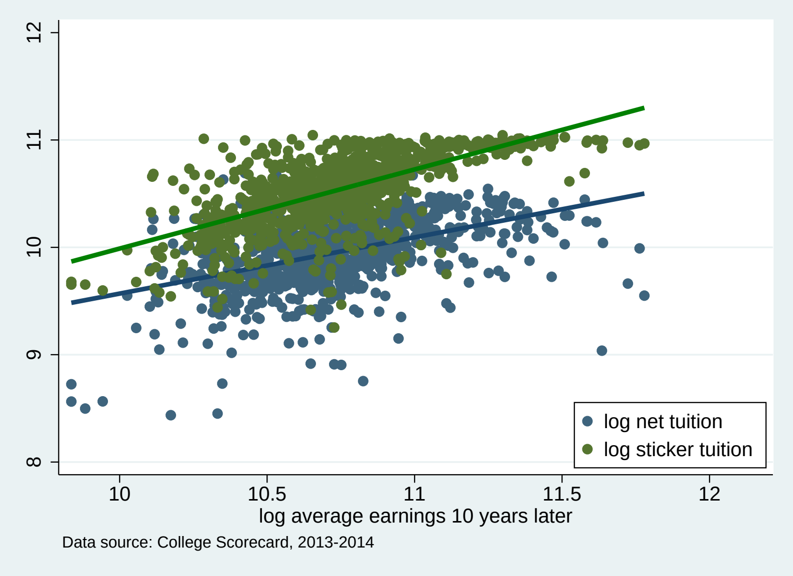
Sticker Price vs. Net Price Paid: Model



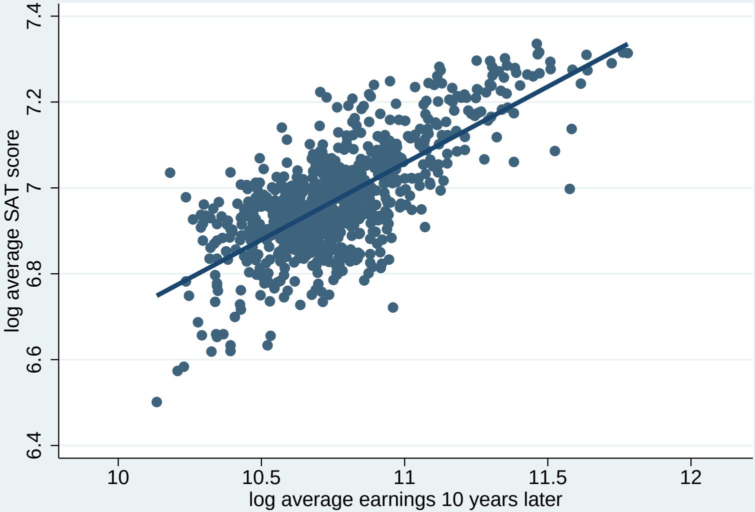
Sticker Price vs. Net Price Paid: Data



Alternative Measure of Quality



SAT versus Earnings Later in Life



Data source: College Scorecard, 2013-2014

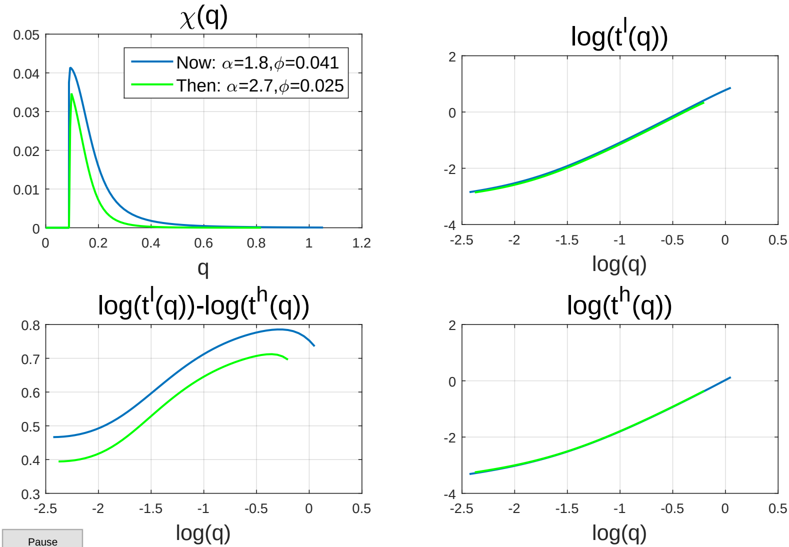
Changing Income Inequality

- Conduct experiment where we move back from income distribution of 2014 to the one of 1984:
  - $\alpha^{1984} = 2.7$ instead of $\alpha^{2014} = 1.8$
  - Adjust μ to hold average income fixed

- Also adjust φ to replicate 1984 enrollment rate (19.1%)
  - $\varphi^{1984} = 0.025$ instead of $\varphi^{2014} = 0.041$

- How large an increase in tuition does the model generate?
  - Is higher tuition driven primarily by (i) stronger taste for college, or (ii) greater income inequality?

Calibrated Model Results



Stop Pause

Changes Over Time

	MODEL			
	1984	$\Delta\varphi$	$\Delta\alpha$	2014
Enrollment rate	19.1%	38.0%	16.6%	31.9%
Tuition per student / GDP pc	5.6%	6.0%	6.3%	6.7%
Share high ability in college	31.4%	58.9%	26.3%	48.8%
var.(log avg. net tuition)	0.051	0.072	0.112	0.132
var.(log sticker tuition)	0.089	0.121	0.177	0.205
var.(log avg. fam income)	0.159	0.200	0.161	0.313
corr.(college(0,1), a_i)	0.314	0.430	0.261	0.363
corr.(log(q_i), a_i)	0.197	0.234	0.164	0.185

Key Findings

- 12.8 pct.pt. increase in enrollment reflects increased demand for college
- 19.7% increase in tuition mostly due to increased inequality
- More demand for college increases correlation between ability and college attendance / quality
- Higher income inequality reduces correlation between ability and college attendance /quality

Role of Increased Income Inequality

- Why does higher inequality increase tuition so much (+19.7%)?

  1. rich demand higher quality colleges $\Rightarrow$ average college quality goes up (+11.4%)
  2. marginal high ability are poorer and get priced out $\Rightarrow$ high ability students become scarcer and more expensive $\Rightarrow$ increased cost of producing quality

Conclusions

- Shape of income distribution affects tuition schedule in club-good model of college
- Rising income inequality an important factor underlying rise in average tuition
- Future work: make the model dynamic:

$$\begin{gathered}(y_t, a_{i,t}) \rightarrow q_t \\ (q_t, a_{i,t}) \rightarrow y_{t+1} \\ \Gamma(a_{i,t+1} | a_{i,t})\end{gathered}$$

  - Experiment with increasing the return to quality, trace out inter-generational dynamics of inequality