Team Incentives with Non-Verifiable Performance Measures: A Multi-period Foundation for Bonus Pools[1]

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Abstract

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A common means of incorporating non-verifiable performance measures in compensation contracts is via bonus pools. We study a prinicpal-multi-agent relational contracting model in which the optimal contract resembles a bonus pool. It specifies a minimum joint bonus the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to both increase the size of the pool and to allocate the pool to the agents. The joint bonus floor is useful because of its role in motivating the agents to mutually monitor each other (team incentives). Even when incentive schemes without positive bonus floors could be used to provide team incentives, contracts with positive bonus floors can be less costly because they do a better job of creating strategic complementarity in the agents' payoffs, which is a desirable property of incentive schemes designed to motivate mutual monitoring. When team incentives are not optimal, the contract must be collusion proof. Even when contracts without positive bonus floors would prevent tacit collusion, contracts with joint bonus floors can be less costly because they facilitate strategic independence in the agents' payoffs, which is a desirable property of collusion proof incentives.

1 Introduction

This paper studies discretionary rewards based on non-verifiable performance measures. A concern about discretionary rewards is that the evaluator must be trusted by the evaluates (Anthony and Govindarajan, 1998). In a single-period model, bonus pools are a natural economic solution to the "trust" problem (MacLeod, 2003; Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009; Ederhof, 2010). A bonus pool can be seen as a special case of a relational contract. While explicit contracts are enforced by the courts, a relational (implicit) contract must be self-enforcing. In a multi-period setting, relational contracts can be enforced by threats of retaliation by one party when the other(s) renege on their promises (MacLeod and Malcomson, 1989). In a single-period setting with multiple agents, the only self-enforcing contract is bonus pool, which leaves the principal discretion only in allocating the fixed pool to the agents.

We study a multi-period, prinicpal-multi-agent relational contracting model in which the optimal contract resembles a bonus pool with added discretion to increase the size of the bonus pool. It specifies a minimum joint bonus (hereafter bonus floor) the principal is required to pay out to the agents and gives the principal discretion to use non-verifiable performance measures to both increase the size of the pool and to allocate the pool to the agents. Such discretion is common in practice.[1] As an example, UBS explains that the company's board of directors has full discretion to determine the size of the bonus pool and, in doing so, takes into account both quantitative and qualitative factors such as collaboration within the firm.[2] Empirically, Murphy and Oyer (2003) find that 42% of their sample of 262 firms gave the compensation committee discretion in determining the size of the executive bonus pool, while 70% had discretion in allocating the bonus pool to individual executives.[3]

In the relational contracting literature, Levin (2002) distinguishes between bilateral and

[1]Umbrella plans or "inside/outside" plans are often used to ensure the incentive compensation is tax deductible per IRS Rule 162(m). Effectively, the inside plan can be used to specify a bonus floor, while the outside plan specifies a bonus cap.

[2]UBS Group AG Compensation Report 2015.

[3]For evidence on discretion in individual bonus plans, see Bushman, Indjejikian, and Smith (1996).

multilateral contracts. Our relational contracts are multilateral in that they rely on the agents observing each other's pay and punishing the principal when the principal breaks any promise she made to either of the agents. The bonus floor is an explicit contract but is also multilateral: it also relies on the agents observing each other's pay in order to enforce the contract when the sum of the two payments made to the agents is less than the floor specifies.

The joint bonus floor is useful because of its role in motivating the agents to mutually monitor each other (team incentives). Even when incentive schemes without positive bonus floors could be used to provide team incentives, contracts with positive bonus floors can be less costly because they do a better job of creating strategic complementarity in the agents' payoffs, which is a desirable property of incentive schemes designed to motivate mutual monitoring.[4] When team incentives are not optimal, the contract must be collusion proof. Even when contracts without positive bonus floors would prevent tacit collusion, contracts with joint bonus floors can be less costly because they facilitate strategic independence in the agents' payoffs, which is a desirable property of collusion proof incentives.

The demand for mutual monitoring using implicit (relational) contracts in our model is the same as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001).[5] The agents work closely enough that they observe each other's actions, while the principal observes only individual performance measures that imperfectly capture those actions. The key idea in those papers is to replace the agents' Nash incentive constraints, which imply that individual performance evaluation (*IPE*) is optimal, with group incentive constraints, which imply that joint performance evaluation (*JPE*) is optimal. *JPE* is a means of setting the stage for the agents

[4]Strategic complementarities, which mean that each agent's marginal return to his action is increasing in the other agent's action, have been widely studied in economics (e.g., Milgrom and Roberts, 1990). Empirically, Gibbs, Merchant, Van der Stede, and Vargus (2004) and Bushman, Dai, and Zhang (2016) document evidence that firms incorporate synergistic relationships among executives in designing compensation contracts. Baiman and Baldenius (2009) study a model in which the stronger the interdependencies across divisions, the more the firm uses non-financial information.

[5]There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh's (1993) model of explicit side-contracting can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

to mutually monitor each other. As Milgrom and Roberts (1992, p. 416) write: "[g]roups of workers often have much better information about their individual contributions than the employer is able to gather...[g]roup incentives then motivate the employees to monitor one another and to encourage effort provision." We are agnostic about the type/level of employees our model applies to, other than they should work closely enough to be able to monitor each other's actions better than their superior(s) can. As a simplifying assumption, we (and the existing literature on relational team incentives) assume(s) the mutual monitoring is perfect.

Like our paper, Che and Yoo (2001) study an infinitely repeated relationship. The principal needs only to ensure that, from the agents' perspective, both working is preferred to both shirking (a group incentive constraint) and that the punishment of playing the stage equilibrium of shirking in all future periods is larger than the one-time benefit of free-riding on the other agent's effort. The difference between our paper and Che and Yoo (2001) is that, since the performance measures are *non-verifiable* in our model, the principal too has to rely on a relational contract.

The non-verifiable performance measures in our model can be interpreted as the principal's imperfect assessment of each agent's effort. The non-verifiability of the performance measures does not entirely rule out explicit contracts. In particular, the principal can specify a joint bonus floor that does not depend on the non-verifiable performance measures. In a single period version of our model, the principal's ability to make promises is so limited that she would renege on any promise to pay more than the minimum, so the joint bonus floor becomes a simple bonus pool arrangement (as in Macleod, 2003; Rajan and Reichelstein 2006; 2009). Under repeated play, relational contracts allow the principal to use discretion not only in allocating the bonus pool between the agents but also to increase its size above the joint bonus floor.

In our model, all players share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important.

When the principal's ability to commit is strong, the optimal contract emphasizes team incentives. *JPE*, which rewards the agents when both do well, emerges as an optimal means of setting the stage for the agents to mutually monitor each other. *JPE* provides the agents with incentives to monitor each other (by creating strategic complementarity in the agents' payoffs) and a means of disciplining each other (by creating a punishing equilibrium – a stage game equilibrium with lower payoffs than the agents obtain on the equilibrium path). As the principal's reneging constraint becomes a binding constraint, rewarding (joint) poor performance via a positive bonus floor can be a feature of the optimal compensation arrangement. The role of rewarding poor performance is that it enables the principal to keep incentives focused on mutual monitoring by using *JPE*. The alternative is to use relative performance evaluation (*RPE)* to partially replace mutual monitoring incentives with individual incentives. Here, the potential benefit of *RPE* is not in removing noise from the agents' performance evaluation as in Holmstrom (1979) but rather in relaxing the principal's reneging constraint, since *RPE* has the principal making payments to only one of the two agents. The problem with *RPE* is that it undermines mutual monitoring by reducing the strategic complementarity in the agents' payoffs. Therefore, if the principal attempts to replace team incentives with individual incentives using *RPE*, even more individual incentives are needed to makeup for the reduced team incentives. As it turns out, substituting individual incentives (i.e., stage Nash incentives) with *RPE* is optimal only when mutual monitoring's advantage over individual incentives is small, which occurs when the agents' ability to commit (to mutually monitoring each other) is relatively weak.

When individual rather than team incentives are optimal, the principal would use *RPE* if she did not have to prevent tacit collusion between the agents. The unappealing feature of *RPE* is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has them alternating between (*work, shirk*) and (*shirk, work*).[6] The collusion threat is particularly severe if the agents' ability to commit is strong, in which case paying for poor performance via a bonus floor is again optimal because it creases a strategic independence in payoffs. When the agents' ability to commit is instead relatively weak, *RPE* is optimal since the cost of preventing collusion is relatively small.

The relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on subjective/non-verifiable performance measures (e.g., Baker, Gibbons, and Murphy, 1994; Ederhof, Rajan and Reichelstein, 2011), but this literature has mostly confined attention to single-agent settings.[7] In this paper, we explore optimal discretionary rewards based on subjective individual performance measures in a multi-period, principal-multi-agent model. The multi-period relationship creates the possibility of trust between the principal and the agents, since the agents can punish the principal for reneging behavior. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, opportunities for both team incentives/mutual monitoring and collusion between the agents.

Kvaløy and Olsen (2006) also study team incentives in a multilateral relational contracting setting. The most important difference between our paper and theirs is that they do not allow for a commitment to a joint bonus floor, which is the focus of our paper. While the principal cannot write a formal contract on the non-verifiable performance measures, it seems difficult to rule out contracts that specify a joint bonus floor. After all, this idea is at the heart of bonus pools. Kvaløy and Olsen also restrict attention to stationary strategies for the agents, while we allow agents to play arbitrary history-dependent strategies.

Our paper is also closely related to Baldenius, Glover, and Xue (2016). In their model, (i) the principal perfectly observes the agents' actions and (ii) there is a verifiable joint performance measure (e.g., firm-wide earnings) that can be explicitly contracted on. Because the

[6]Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

[7]One exception is Levin (2002), which examines the role multilateral contracting can have in bolstering the principal's ability to commit if the principal's reneging on a promise to any one agent means she will loose the trust of both agents. We make the same assumption. Levin does not study relational contracting between the agents.

principal observes the agents' actions, there is no role for mutual monitoring/team incentives in their model. When mutual monitoring is not optimal, our results are similar to theirs in that strategic independence emerges as an optimal response to agents' collusion. Our modeling adds the realism of imperfect non-verifiable individual performance measures at the cost of leaving out the verifiable joint performance measure.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies implicit side-contracting between the agents. Section 4 specifies the principal's optimization problem and characterizes the optimal contract. Section 5 concludes.

2 Model

A principal contracts with two identical agents, $i = A, B$, to perform two independent and ex ante identical projects (one project for each agent) in an infinitely repeated relationship, where t is used to denote the period, $t = 1, 2, 3, \ldots$. All parties are risk neutral. Each agent chooses a personally costly effort $e^i \in \{0, 1\}$ in period t: the agent chooses either "*work*" ($e^i_t = 1$) or "*shirk*" ($e^i_t = 0$). Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to one. Each agent's effort e^i_t generates a stochastic output $x^i_t \in \{H, L\}$ in the current period, with $q_1 = \Pr(x^i_t = H | e^i_t = 1), q_0 = \Pr(x^i_t = H | e^i_t = 0)$, and $0 < q_0 < q_1 < 1$. (Whenever it does not cause confusion, we drop sub-and superscripts.) The outputs (x^i_t, x^j_t), which are also the performance measures, are individual rather than joint measures in the sense that agent i's effort does not affect agent j's probability of producing a high output. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e^i_t = 1$) in every period. (Sufficient conditions are provided in the appendix.) The principal's problem is to design a contract that motives both agents to work in every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort in each period. As in Che and Yoo (2001), we assume that the communication from the agents to the principal is blocked and, therefore, the outcome pair (x^i, x^j) is the only signal on which the agents' wage payment can depend. As in Kvaløy and Olsen (2006), we assume the performance measures (x^i, x^j) are unverifiable. Therefore, the principal cannot directly incorporate the performance measures into an explicit contract. Instead, the performance measures can be used in determining compensation only via a self-enforcing *relational* (implicit) contract. We will introduce an objective/verifiable team measure in Section 5.

The relational contract governs the parties' entire (infinite) horizon relationship, specifying the agents' actions in each period, the equilibrium payments the principal makes to the agents, and what the parties will do in retaliation if any of them reneges on their promises. At the beginning of each period, the principal promises the agents a wage scheme that depends on the non-verifiable performance measures and, hence, must be self-enforcing. Because the production technology is stationary and the principal induces high efforts in each period (a stationary effort policy), we know from Theorem 2 in Levin (2003) that the same wages will be offered in each period on the equilibrium path. In our setting, any promise to punish the agent in the future based on the current period's performance (and the history of performance that arises after that) can instead be built into current period payments by taking expectations.[8] This rules out the kind of history-dependent equilibrium punishment phases that arise in other games of imperfect monitoring (Mailath and Samuelson, 2006). Since a stationary wage scheme is optimal, we omit the time subscript from wages.

Denote by w^i_{mn} the wage agent i expects to receive according to the *relational* contract if his (performance) outcome is $x^i = m$ and his peer's outcome is $x^j = n$, with $m, n \in \{H, L\}$. For tractability, we assume the ex ante identical agents are offered the same wage schemes,

[8] The basic idea is that the agent's incentives come from two sources: performance measure contingent payments in the current period and a (possible) change in continuation payoffs that depend on the current period's performance measures. Because of the limited liability constraints, even the lowest of these continuation payoffs must be nonnegative. Since all parties are risk neutral and have the same discount rate, any credible promise the principal makes to condition continuation payoffs on current period performance measures can be converted into a change in current payments that replicates the variation in continuation payoffs without affecting the principal's incentives to renege on the promise or violating the limited liability constraints on payments. The new continuation payoffs are functions of future actions and outcomes only, removing the history dependence. The new contract is a short-term one. If instead the effort level to be motivated is not stationary, then, of course, non-stationary contracts can be optimal.

i.e., $w^i_{mn} = w^j_{nm}$, $i \neq j$. As a result, we can drop the agent superscript and denote by $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ the implicit contract the principal promises the agents.[9] The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

$$w_{mn} \geq 0, \forall m, n \in \{H, L\}. \qquad \text{(Non-negativity)}$$

We assume each agent's best outside opportunity provides him with a payoff of 0 in each period. Therefore, any contract that satisfies the limited liability constraints will also satisfy the agents' individual rationality constraints, since the cost of low effort is zero. The individual rationality constraints are suppressed throughout our analysis.

In addition to the implicit promises $\mathbf{w}$, we assume the parties can write an *explicit* contract as long as the explicit contract does not depend on the non-verifiable performance measures (x^i, x^j). This effectively limits the explicit contract to a *bonus floor* $\underline{w} \geq 0$ – a minimum *total* bonus to paid to both agents in the current period independent of the non-verifiable performance measures. Given the multilateral nature of the contracting relationship, it seems difficult to rule out such explicit contracts, since they require only that a court be able to verify whether or not the contractual minimum $\underline{w}$ was paid out. Throughout the paper, we will say that the contract has a bonus pool (BP)-type feature if it specifies a positive total bonus floor, i.e., $\underline{w} > 0$. Our BP-type contracts can be thought of as discretionary bonus pools that allow the principal to pay the agents more than the contractually agreed upon minimum $\underline{w}$. (There will always be some room for such discretion in our model because the model is a dynamic one.) To better connect with prior literature, we present in the end of the paper a setting that assumes the principal cannot commit to a joint bonus floor.

It is easy to see that the minimum joint bonus $\underline{w}$ satisfies $\underline{w} \leq \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$

[9] Confining attention to symmetric contracts is a restrictive assumption in that asymmetric contracts can be preferred by the principal, as in Demski and Sappington's (1984) single period model. As we will show in Section 4, restricting attention to symmetric contracts greatly simplifies our infinitely repeated contracting problem by reducing the infinite set of possibly binding collusion constraints to two. Without the restriction to symmetric contracts, we know of no way to simplify the set of collusion constraints into a tractable programming problem.

in any optimal contract.[10] That is, the explicit bonus floor is never triggered as long as the principal honors her implicit promises $\mathbf{w}$. Denote by $\pi(k, l; \mathbf{w})$ the expected wage payment of agent i if he chooses an effort level $k \in \{1, 0\}$ while the other agent j chooses effort $l \in \{1, 0\}$, assuming the principal honors the implicit contract $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$. The discussion above implies that

$$\pi(k, l; \mathbf{w}) = q_k q_l w_{HH} + q_k(1 - q_l)w_{HL} + (1 - q_k)q_l w_{LH} + (1 - q_k)(1 - q_l)w_{LL}. \tag{1}$$

To motivate the principal to honor her implicit promises $\mathbf{w}$ with the agents, we consider the following (grim) trigger strategy played by the agents: both agents behave as if the principal will honor the implicit promises $\mathbf{w}$ until the principal reneges, after which the employment relationship reverts to punishment phase – the agents shirk in all future periods, and the principal offers them a fixed salary of zero. Such a grim trigger strategy is also used in Baker, Gibbons, and Murphy (1994) and Kvaløy and Olsen (2006) and is without loss of generality, since this is the harshest punishment the agents can impose on the principal. The principal will not renege if

$$\frac{2\left[q_1 H - \pi(1, 1; \mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n \in \{H,L\}} \{w_{mn} + w_{nm} - \underline{w}\}. \tag{Principal's IC}$$

The constraint above assures the principal will abide by the promised wage scheme $\mathbf{w}$ rather than renege and payout the minimum joint bonus floor $\underline{w}$. The left hand side is the cost of reneging, which is the present value of the production loss in all future periods net of wage payment. The right hand side of this constraint is the principal's benefit of paying out only the minimum joint bonus. If instead a bonus floor is not allowed, the right hand of the principal's incentive constraint becomes $\max_{m,n}\{w_{mn} + w_{nm} - 0\}$, i.e., the principal can

[10] Suppose by contradiction that $\underline{w} > w_{m,n} + w_{n,m}$ when the agents' outcome pair is (m, n). Since the court will enforce $\underline{w}$, it will allocate the difference $\Delta = \underline{\mathrm{w}} - (w_{m,n} + w_{n,m})$ between the two agents according to a certain allocation rule. The principal can directly give the agents the same payments the courts would impose by increasing the payments so that $(w'_{m,n} + w'_{n,m}) = \underline{w}$. The principal can do better by optimizing over possible $(w'_{m,n} + w'_{n,m}) = \underline{w}$.

always breach her promise and pay zero to both agents in the current period.

All parties in the model share a common discount rate r, capturing the time value of money or the probability the relationship will end at each period (the contracting horizon). Denote by H_t the history of all actions and outcomes before period t, including whether the principle has ever reneged her implicit promises. Denote by P_t the public profile before period t — the history without the agents' actions. The principal's strategy is a mapping from the public profile P_t to period t wages. Each agent's strategy maps the entire history H_t to his period t effort choice. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Among the large set of PBE's, we choose the one that is best for the principal subject to collusion-proofness. To be collusion-proof, there can be no other PBE that has only the agents changing their strategies and provides each agent with a higher payoff (in a Pareto sense) than their equilibrium payoff.

3 Implicit Contracting between the Agents

The fact that agents observe each other's effort choice, together with their multi-period relationship, gives rise to the possibility that they use implicit contracts to motivate each other to work (mutual monitoring) as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). Consider the following trigger strategy used to enforce (*work, work*): both agents play work until one agent i deviates by choosing shirk; thereafter, the agents play (*shirk, shirk*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w})-1\right] \geq \pi(0,1;\mathbf{w})+\frac{1}{r}\pi(0,0;\mathbf{w}). \qquad \text{(Mutual Monitoring)}$$

Such mutual monitoring requires two conditions. First, each agent's expected payoff from playing (*work, work*) must be at least as high as from playing the punishment strategy (*shirk, shirk*). In other words, both working must Pareto dominate both shirking from the agents' point of view in order for (*shirk, shirk*) to be perceived as a punishment. That is,

$$\pi(1,1;\mathbf{w}) - 1 \geq \pi(0,0;\mathbf{w}). \qquad \text{(Pareto Dominance)}$$

Second, the off-equilibrium punishment strategy (*shirk, shirk*) must be self-enforcing in that playing (*shirk, shirk*) is a stage game Nash equilibrium. That is,

$$\pi(0,0;\mathbf{w}) \geq \pi(1,0;\mathbf{w}) - 1. \qquad \text{(Self-Enforcing Shirk)}$$

As we will show in Section 4, contracts that motivate agents to mutually monitor each other (i.e., team incentives) are naturally collusion proof: there is no other PBE in which the agents can receive a higher payoff in the Pareto sense by deviating from the (work, work) equilibrium. Despite its desirable feature in preventing collusion between the two agents, exploiting team incentives can be costly because of the two added constraints above that accompany (Mutual Monitoring). When motivating team incentives is too costly for the principal, she can ignore the three constraints specified above, and instead provide the individual incentive by ensuring that (*work, work*) is a stage game Nash equilibrium.

$$\pi(1,1;\mathbf{w}) - 1 \geq \pi(0,1;\mathbf{w}). \qquad \text{(Static NE)}$$

The Nash constraint may not be sufficient to motivate the agents to act as the principal intends because they may find other implicit contracting more desirable – that is, they may tacitly collude against the principal. To sustain a collusive strategy, the agents adopt a grim trigger strategy in which agent i punishes agent j for deviating from the collusive strategy by playing the stage game equilibrium that gives j the lowest payoff. As we show in the proof of Lemma 1, for any contract that satisfies (Static NE), the grim trigger strategy calls for the agents to revert to the stage game equilibrium (*work, work*) in all future periods if any agent deviates from the collusive strategy. That is, when the principal chooses to provide individual

Nash incentives, playing *(work,work)* is the most severe punishment the agents can impose on each other from deviating a collusive strategy.

The principal needs to prevent all collusive strategies. Given the infinitely repeated relationship, the space of potential collusions between the two agents is rich. Nonetheless, Lemma 1 below shows that we can confine attention to two intuitive collusive strategies when constructing collusion-proof contracts. First, the contract has to satisfy the following condition to prevent "*Joint Shirking*" strategy in which agents collude on playing (*shirk, shirk*) in all periods:

$$\pi(1,0;\mathbf{w}) - 1 + \frac{\pi(1,1;\mathbf{w}) - 1}{r} \geq \frac{1+r}{r}\pi(0,0;\mathbf{w}). \qquad \text{(No Joint Shirking)}$$

In addition, the contract has to satisfy the following condition to prevent "*Cycling*" collusive strategy in which agents collude on alternating between (*work, shirk*) and (*shirk, work*):

$$\frac{1+r}{r}\left[\pi(1,1;\mathbf{w}) - 1\right] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;\mathbf{w}) + \frac{(1+r)}{r(2+r)}\left[\pi(1,0;\mathbf{w}) - 1\right]. \qquad \text{(No Cycling)}$$

The left hand side of the two constraints is the agent's expected payoff if he unilaterally deviates by choosing work when he is supposed to shirk in some period t and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff the agent derives from the collusive strategy – either *Joint Shirking* or *Cycling.* Lemma 1 provides a necessary and sufficient conditions for any contract that provides individual Nash incentives to be collusion proof. (Lemma 1 is borrowed from Baldenius, Glover, and Xue (2016) but is repeated here for completeness.)

Lemma 1 *For contracts that provide individual incentives and hence satisfy* (Static NE)*, the necessary and sufficient condition to be collusion-proof is that: either both* (No Joint Shirking) *and* (No Cycling) *are satisfied, or both* (No Cycling) *and* (Pareto Dominance) *are satisfied.*

Proof. All proofs are provided in an appendix. ■

The intuition for the lemma is that all other potential collusive strategies can only provide some period t' shirker with a higher continuation payoff than under *Joint Shirking* or *Cycling* if some other period t'' shirker has a lower continuation payoff than under *Joint Shirking* or *Cycling*. Hence, if the contract motivates all potential shirkers under *Joint Shirking* and *Cycling* to instead deviate to work, then so will the period t'' shirker under the alternative strategy.

4 The Principal's Problem

The principal designs an *explicit* joint bonus floor $\underline{w}$ and an *implicit* wage scheme $\mathbf{w} = \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$ to ensure (*work, work*) in every period is a collusion-proof equilibrium. The implicit promises $\mathbf{w}$ must satisfy the *Principal's IC* constraint in order to make the promise self-enforcing, since the courts cannot be used to enforce the implicit scheme. When designing the optimal contract, the principal can choose to motivate mutual monitoring between agents if it is worthwhile (and feasible). Alternatively, she can implement a static Nash equilibrium subject to collusion proof constraints (which is always feasible). The following integer program summarizes the principal's problem. It is an integer program because the variable T (short for team incentives) takes a value of either zero or one: $T = 1$ means the principal designs the contract to induce team incentives by motivating mutual monitoring, while $T = 0$ represents individual incentives.

$$\textbf{Program P:} \quad \min_{T\in\{0,1\},\underline{w}\geq 0,w_{mn}\geq 0} \pi(1,1)$$

$s.t.$

Principal's IC

$T \times$ Mutual Monitoring (2)

$T \times$ Pareto Dominance

$T \times$ Self-Enforcing Shirk

$(1-T) \times$ Static NE

$(1-T) \times$ No Joint Shirking

$(1-T) \times$ No Cycling

Two features of the program deserve further discussion. First, the team incentives case ($T = 1$) does not include any collusion-proof constraints, which will be shown to be optimal after Proposition 1. The idea is, even without collusion-proof constraints, the solution under $T = 1$ is such that playing (*work, work*) in the repeated game is Pareto optimal for the agents and hence collusion proof by definition. Second, while Lemma 1 specifies the necessary and sufficient condition for contracts providing individual incentives ($T = 0$) to be collusion proof, once we endogenize the choice of T, it is without loss of generality to incorporate only the sufficient condition that both (No Joint Shirking) and (No Cycling) are satisfied. This is because, under the alternative collusion proof condition shown in Lemma 1, that is, (Pareto Dominance) and (No Cycling), the individual-incentive case $T = 0$ results in a strictly smaller feasible set than the team-incentive case.[11] Intuitively, if the contract already satisfies (Pareto Dominance), the principal will be better-off by providing team incentives ($T = 1$) than by providing individual incentives that requires additional collusion-proof conditions.

[11]To see this, note that (Pareto Dominance) and (Static NE) together imply (Mutual Monitoring). This, however, means the $T = 0$ case has more constraints than the $T = 1$ case (noting that the (Self-Enforcing Shirk) constraint never binds in the $T = 1$ case).

The following lemma links the explicit joint bonus floor $\underline{w}$ and the implicit promises $\{w_{LL}, w_{LH}, w_{HL}, w_{HH}\}$, and therefore further simplify the program **P** by reducing the number of control variables.

Lemma 2 *It is optimal to set* $\underline{w} = \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$.

That is, the principal optimally sets the contractible joint bonus floor equal to the minimum total compensation specified by the principal's implicit contract with the agents. The result is fairly obvious. If the bonus floor is set lower than the minimum promised total compensation, then the principal can relax her *IC* constraint by increasing the bonus floor. If the bonus floor is greater than the minimum promised compensation, then the agents will use the courts to enforce the bonus floor. That is, the actual minimum compensation will not be the promised minimum but instead the bonus floor. The same equilibrium payments can be achieved by revising the principal's implicit contract so that the minimum promised compensation is equal to the bonus floor.

Lemma 2 allows us to rewrite the (Principal's IC) constraint as follows:

$$\frac{2\left[q_1 H - \pi(1,1;\mathbf{w})\right] - 2q_0 H}{r} \geq \max_{m,n,m',n' \in \{H,L\}} \{w_{mn} + w_{nm} - (w_{m',n'} + w_{n',m'})\}.$$

As the reformulated Principal's IC constraint suggests, the explicit bonus floor in our model is equivalent to an explicit contract that specifies the range of possible payments and asks the principal to *self-report* the non-verifiable performance measures.[12] The courts would then be used to enforce payments that are consistent with some realizations of the performance measures but not verify the actual realizations, since they are non-verifiable by assumption.

We solve Program **P** by the method of enumeration and complete the analysis in two steps. In Sections 4.1 and 4.2, we solve Program P while setting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T in Section 4.3.

[12] Deb, Li, and Mukherjee (2016) uses a similar interpretation in studying relational contracts with subjective measures.

4.1 Team Incentives

This subsection takes team incentives ($T = 1$) as given and solve for the optimal solution. Following the literature, we say that a wage scheme exhibits individual performance evaluation (IPE) if $w_{HH} = w_{HL}$ and $w_{LH} = w_{LL}$, relative performance evaluation (RPE) if $w_{HL} \geq w_{HH}$ and $w_{LL} \geq w_{LH}$ (with one $>$), and joint performance evaluation (JPE) if $w_{HH} \geq w_{HL}$ and $w_{LH} \geq w_{LL}$ (with one $>$). We say that the contract has a bonus pool (BP) feature if it specifies a strictly positive total bonus floor, i.e., $\underline{w} > 0$. The proposition below provides a complete characterization of the optimal contract, as a function of the common discount rate r. that is, we express the results as a function of discount rate cutoffs, which are themselves functions of other exogenous parameters.

Proposition 1 *Given $T = 1$, the solution to Program P is one of the following (with $w_{LH} = 0$ and $w_{HH} > 0$ in all cases):*[13]

(i) JPE1: w_{HH} is the only positive payment for $r \in (0, r^A]$ and it increases in r;

(ii) BPC1: $w_{HL} = 2w_{LL} = \underline{w} > 0$ for $r \in (r^A, \min\{r^L, r^C\}]$;

(iii) JPE2: $w_{HH} > w_{HL} > 0, w_{LL} = \underline{w} = 0$ for $r \in (\max\{r^L, r^A\}, r^B]$;

(iv) BPC2: $w_{HL} > 2w_{LL} = \underline{w} > 0$ for $r \in (\max\{r^L, r^B\}, r^C]$;

(v) infeasible otherwise,

where closed-form expressions for all solutions are given in the appendix and $r^L = \frac{q_1+q_0-1}{(1-q_1)^2}$, and r^A, r^B, and r^C are increasing functions of H and are also specified in the appendix.

Denote by $U_{k,l} = \pi(k, l; \mathbf{w}) - C(e^i = k)$ agent i's expected utility if he chooses an effort $k \in \{1, 0\}$ while agent j chooses effort $l \in \{1, 0\}$. It is easy to check that all the solutions create a strategic payoff complementarity, i.e., $U_{1,1} - U_{0,1} > U_{1,0} - U_{0,0}$ (denoted by the C in the BP-type contracts). Moreover, the payoff satisfies $2U_{1,1} = \max\{2U_{1,1}, 2U_{0,0}, U_{1,0} + U_{01}\}$, which means all solutions in Proposition 1 are collusion-proof because no other action profile

[13] Given a set of primitives such as H, q_1 and q_0, not all of these solutions may exist: the corresponding interval of r can be an empty set.

can Pareto-dominate the equilibrium strategy (*work, work*).[14]

Strategic complementarity in agents' payoffs is a desirable property in motivating mutual monitoring, since the benefit of free-riding on the other agent's effort is decreasing in the complementarity. This explains why *JPE1* is optimal whenever feasible ($r \in (0, r^A]$): it generates the strongest payoff complementarity by rewarding agents only if the outcomes from both agents are high. Our *JPE1* contract is the same as the "Joint Performance Evaluation" contract studied in Che and Yoo (2001). As r increases in the region, the principal must increase w_{HH} to incentivize the increasingly impatient agents to mutually monitor each other. The principal too becomes more impatient for higher r, and, therefore, perceives the cost of reneging (in the reduction of production in all future periods) less costly. The principal is eventually unable to credibly promise *JPE1* to the agents at $r = r^A$.

For higher discount rate, the principal has to lower the difference between w_{HH} and other payments to make her promise credible. Proposition 1 Parts (ii) and (iii) provide two options of restoring the principal's credibility. BPC1 *has the principal paying for poor performances $w_{LL} > 0$ in order to further increase w_{HH} and keep the focus on team incentives.* JPE2 *has the principal substituting individual incentives for team incentives by setting $w_{HL} > 0$ and holding w_{LL} at 0 – i.e., increasing her reliance on relative performance evaluation (*RPE*) in which an agent is rewarded for producing the only high outcome.*[15] *It is tempting to think that the principal would never use* BPC1 *when its alternative* JPE2 *is feasible, because paying for bad performance w_{LL} does not seem to provide any incentives. However, the corollary below shows that this intuition is incorrect: $BPC1$ is sometimes optimal when* JPE2 *is also feasible.*

Corollary 1 *Given $T = 1$, paying for poor performances w_{LL} is optimal if (i) the agents' ability to enforce mutual monitoring is strong and (ii) L is not too informative about "shirk". That is,* BPC1 *follows* JPE1 *if* $H \leq H^* \doteq \frac{(q_1+(q_1-1)q_1)\left(q_0+(q_1-1)\left((q_1-1)q_1^2+1\right)\right)}{(q_1-1)^2(q_0-q_1)^2\left((q_0-1)q_1^2-q_0q_1+q_0+q_1^3\right)}$ *and* $q_0 > 1 - q_1$.

[14] The strategic payoff complementarity result, together with (Pareto Dominance) required for team incentives, implies $2U_{1,1} = \max\{2U_{1,1}, 2U_{0,0}, U_{1,0} + U_{01}\}$.

[15] Here, the potential benefit of *RPE* is not in removing noise from the agents' performance evaluation as in Holmstrom (1979) but rather in relaxing the principal's reneging constraint, since *RPE* has the principal making payments to only one of the two agents.

Compared to JPE2 *that does not pay agents for poor performances,* BPC1 *has the benefit of keeping the focus on the team incentives while has the cost of rewarding for performance that has the lowest likelihood ratio* $\frac{q_0}{q_1}$*. The condition* $q_0 > 1 - q_1$ *in the corollary above limits the opportunity cost of rewarding for low likelihood ratios, whereas the requirement* $H \leq H^*$ *assures that the agents' credibility to enforce the mutual monitoring side contract (hence the benefit for the principal to promot team incentives) is strong. To gain some intuition for the requirement* $H < H^*$*, notice that the principal's credibility to honor subjective measures is enhanced by a high value of* H*, as the punishment the agents by can bring to bear on the principal by shirking is more severe. Therefore, the region over which the principal can commit to the ideal* JPE1 *contract expands for higher* H*. If* H *is sufficiently large (i.e.,* $H > H^*$*), the principal's credibility is so strong that by the time she looses her credibility at* r^A*, the agents have already lost their patience to mutually monitor each other. Without the benefit of mutual monitoring, however, rewarding for poor performances* w_{LL} *is not justified and* JPE2 *is instead optimal.*

Finally, Proposition 1 shows that team incentives is infeasible when players are sufficiently impatient: there is a conflict between principal's desire to exploit the agents' mutual monitoring and her ability to make credible promises.[16]

4.2 Collusion

This subsection characterizes the solution to Program P, taking individual incentives ($T = 0$) as given. Unlike the team incentives case ($T = 1$), collusion-proof consideration plays an important role in the analysis.

Proposition 2 *Given* $T = 0$*, the solution to the Program P is one of the following (*$w_{LH} = 0$ *and* $w_{HH} > 0$ *in all cases):*

(i) IPE: $w_{HH} = w_{HL} = \frac{1}{q_1 - q_0}, w_{LL} = \underline{w} = 0$ *for* $r \in (0, r^B]$*;*

[16] Mathematically, the intersection of (Mutual Monitoring) and (Principal's IC) is an empty set for sufficiently large r.

(ii) BPI: $2w_{LL} = \underline{w} > 0$, $w_{HL} = w_{HH} + w_{LL}$.*;*

(iii) RPE: $w_{HL} > w_{HH} > 0$, $w_{LL} = \underline{w} = 0$ *for* $r \in (\max\{r^B, r^H\}, r^D]$*;*

(iv) BPS: $w_{HH} > 0$, $w_{HL} = 2w_{HH} > 2w_{LL} = \underline{w} > 0$ *for* $r > \max\{r^H, r^D\}$*;*

where closed-form expressions for all solutions are given in the appendix and $r^H = \frac{2q_1-1}{(1-q_1)^2}$ *and* r^B, r^D *are increasing functions of* H*, which are also specified in the appendix.*

We use "S", and "I" to denote a strategic payoff substitutability and strategic independence, respectively. The strategic inter-dependence in the two agents' payoffs is critical in determining which collusive strategy is more appealing to the agents, and, therefore, the optimal contract. Rewrite (No Joint Shirking) and (No Cycling) as $f^{SHK} \leq 0$ and $f^{CYC} \leq 0$, respectively. One can show that

$$a \times f^{SHK} - b \times f^{CYC} = [\pi(1,1) - 1 - \pi(0,1)] - [\pi(1,0) - 1 - \pi(0,0)], \quad (3)$$

where $a = \frac{r}{1+r}$ and $b = \frac{r(2+r)}{(1+r)^2}$ are two positive numbers. We can show that colluding by *Cycling* (or *Joint Shirking*) is more attractive to the agents for contracts that create strategic substitutability (or complementarity) in payoffs.[17] Moreover, the two collusive strategies are equally attractive when the agents payoff exhibit strategic payoff independence.

In Proposition 2, individual performance evaluation (*IPE*) is optimal whenever feasible. This is not surprising given the assumed independent production technologies, because treating agents independent is an efficient way of creating strategic payoff independence and therefore discouraging collusion. As r increases beyond r^B, *IPE* is no longer feasible because the impatient principal has incentive to renege when the output pair is (H, H). The gap between w_{HH} and w_{LL} must be decreased in order to prevent the principal from reneging: the principal can either increase w_{LL} (as in *BPI*), or decrease w_{HH} and increase w_{HL} to provide incentives (as in *RPE*). While *BPI* pays for bad performance and, therefore, seems to be inferior to

[17] For contracts that create strategic substitutability in payoffs (i.e., the RHS of (3) is negative), we know $f^{SHK} < \frac{b}{a} f^{CYC}$, and $f^{CYC} \leq 0$ implies $f^{SHK} \leq 0$. Similar argument can show that if a contract creates strategic complementarity in payoffs (i.e., the RHS of (3) is positive), $f^{SHK} \leq 0$ implies $f^{CYC} \leq 0$.

RPE, the corollary below shows that *BPI* is sometimes optimal when *RPE* is also feasible.

Corollary 2 *Given $T = 0$, paying for poor performance w_{LL} is optimal if the agents' ability to enforce agent-agent collusion is strong. That is, as discount rate r increases,* BPI *immediately follows* IPE *if $q_1 > \frac{1}{2}$ and $H \leq H^{**} \doteq \frac{q_1(3-(2-q_1)q_1)-1}{(1-q_1)^2(q_1-q_0)^2}$.*

The benefit of having a positive bonus floor in combatting collusion is the key to understand the result. Note that RPE *creates a strategic substitutability in the agents' payoffs, which, given the discussion following Proposition 2, implies that the* Cycling *collusive strategy is more difficult to upset than* Joint Shirking*. In contrast, by increasing $w_{LL} > 0$,* BPI *creates a payoff strategic independence and makes the two collusion constraints equally costly to upset.* BPI *generalizes the* IPE *solution by extending the region over which agents' payoffs exhibit strategic independence, a desirable property of collusion proof incentives. Since the threat of the* Cycling *collusion is the reason the principal chooses to reward for poor performance w_{LL} in* BPI*, it is optimal only if the agents' ability to collude on the side contract is strong. The condition $q_1 > \frac{1}{2}$ in Corollary 2 is intuitive: the agents' ability to collude on the* Cycling *collusive strategy is stronger for higher q_1, for a higher q_1 directly increases the likelihood for the (only) working agent to collet the payment w_{HL}. To see this condition $H \leq H^{**}$, recall from the discussion following Corollary 1 that a higher output H strengthens the principal's credibility relative to the agents. If the condition $H \leq H^{**}$ is violated, the principal's credibility is so strong that the agents' ability to collude on the side contract is already weak by the time the principal looses her credibility at r^B, and the principle will use* RPE *to avoid paying for poor performances.*

The role of *BPS* in Proposition 2 is to expand the feasible region. The variation of wage payment is extremely limited under *BPS*, because both parties are sufficiently impatient ($r > \max\{r^H, r^D\}$). As the discount rate becomes arbitrarily large, the optimal incentive scheme converges to a bonus pool with a fixed total payment to the agents. This coincides with the traditional view that bonus pools without discretion in determining the total payout will eventually come into play because they are the only self-enforcing compensation form in such cases.

4.3 Overall optimal contract

The following proposition endogenizes the principal's choice of team incentives $T = 1$ and individual incentives $T = 0$. As the result suggests, whenever *JPE1*, *BPC1*, or *JPE2* in Proposition 1 are optimal when team incentives are exogenously imposed, they remain optimal when the choice between team and individual incentives is endogenous. In contrast, $BPC2$ in Proposition 1 is never uniquely optimal once we endogenize the choice of $T \in \{0, 1\}$.

Proposition 3 *The overall optimal contract is:*

(i) JPE1 for $0 < r \leq r^A$*;*

(ii) BPC1 for $r^A < r \leq \min\{r^L, r^C\}$*;*

(iii) JPE2 for $\max\{r^L, r^A\} < r \leq r^B$*;*

(iv) BPI for $\max\{r^B, \min\{r^L, r^C\}\} < r \leq r^H$*;*

(v) RPE for $\max\{r^B, r^H\} < r \leq r^D$*;*

(vi) BPS for $r > \max\{r^H, r^D\}$.

The following numerical examples are intended to provide an intuitive interpretation of the conditions given in Proposition 3. Fix $q_0 = 0.53$ and $q_1 = 0.75$ for all examples to ease comparison. In the first example shown in Figure 1, set $H = 200$. The principal's ability to commit is high because the expected production she forgoes after reneging is large. In comparison, the agents' ability to enforce their implicit side contract – whether it is mutual monitoring or collusion – is limited, so relational contracting between the two agents is never the driving determinant of the form of the optimal compensation arrangement. Once the discount rate becomes large enough ($r > 7.3$) that *JPE1* is not feasible, the agents ability to make commitments is so limited that the principal optimally substitutes individual incentives for team incentives, making *JPE2* optimal. As the discount rate becomes even larger ($r > 8.9$), the principal switches entirely to individual incentives (i.e., $T = 0$) and *RPE* rather than *BPI* is optimal because collusion is not costly to deal with. Once the discount rate is large enough ($r > 10.5$), the optimal contract gradually converges to a proper bonus pool under which a fixed total payment is paid to the agents.[18]

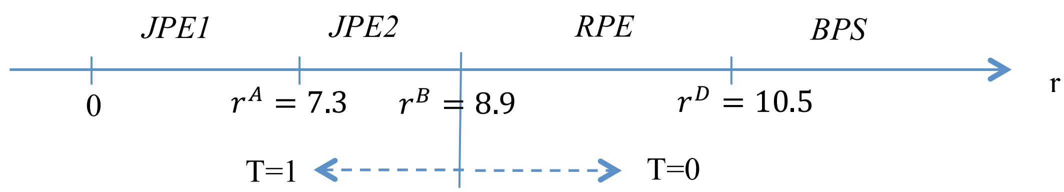

Figure 1: Optimal contract when the principal's relative credibility is *high* ($H = 200$).

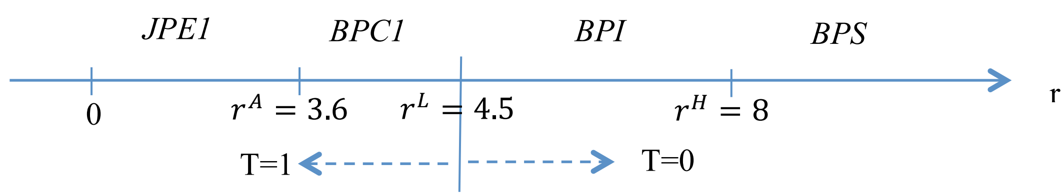

Figure 2: Optimal contract when the principal's relative credibility is *low* ($H = 100$).

In the second example shown in Figure 2, $H = 100$. The principal's ability to commit is now low relative to the agents', so the benefit of having a positive bonus floor (i.e., $2w_{LL} > 0$) in dealing with the agents' relational contracting (either mutual monitoring or collusion) can outweigh its cost of paying for joint bad performance. In particular, as the discount becomes large enough ($r > 3.6$) that *JPE1* is no longer feasible, the agents' ability to make commitments is still strong, making mutual monitoring highly valuable and *BPC1* preferred to *JPE2*. Individual incentives ($T = 0$) are optimal for $r > 4.5$, and *BPI* that creates strategic payoff independence is optimal because its benefit in upsetting collusion. For $r > 8$, the principal's ability to commit is so limited that *BPS* is the only feasible solution.[19]

To further illustrate the second case (Figure 2), we present the optimal contract and the payoff matrix of the stage game of the two bonus pool type contract for the intermediate discount rate r. For $r = 4$, $JPE2$ with $\mathbf{w} \equiv \{w_{LL}, w_{LH}, w_{HL}, w_{HH}\} = (0, 0, 3.84, 4.66)$ is feasible

[18] Compared to Proposition 3, the missing cutoffs in Figure 1 are $r^C = 11.3, r^L = 4.5$, and $r^H = 8$.

[19] Compared to Proposition 3, the missing cutoffs in Figure 2 are $r^B = 4.1, r^C = 5.1$, and $r^D = 4.4$.

but is not optimal. Instead, *BPC1* is the optimal wage scheme with $\mathbf{w} = (0.68, 0, 1.36, 5.35)$. The stage game payoff matrix follows.

BPC1 ($r = 4$)

A/B	0	1
0	(1.99,1.99)	(2.39,1.69)
1	(1.69,2.39)	(2.31,2.31)

The benefit of free-riding of 2.39 - 2.31 = 0.08 is exactly equal to the punishment of reverting to the stage game Nash equilibrium of (2.31 - 1.99)/4 = 0.08.

For $r = 5$, team incentives are no longer optimal. The optimal means of preventing collusion is *BPI* with $\mathbf{w} = (1.09, 0, 5.82, 4.73)$. The payoff matrix follows.

BPI ($r = 5$)

A/B	0	1
0	(3.02,3.02)	(2.78,3.06)
1	(3.06,2.78)	(2.82,2.82)

The principal provides any shirking agent with a benefit of 0.04 for instead working and upsetting collusion on either *Joint Shirking* or *Cycling*. It is easy to verify that the shirking agent's continuation payoff under *Joint Shirking* of $\frac{3.02}{5} = 0.604$ is the same as his continuation payoff under *Cycling*. Under the equilibrium strategy (*work,work*), each agent's continuation payoff is $\frac{2.82}{5} = 0.564$. Therefore, the difference in continuation payoffs of $0.604 - 0.564 = 0.04$ is exactly equal to the benefit an agent receives for upsetting collusion by playing *work* instead of *shirk* in the current period. That is, both (No Joint Shirking) and (No Cycling) hold as equality – a feature of the contract that creates payoff strategic independence.

Given Propositions 1 - 3, it is straightforward to derive the optimal contract in the setting that assumes the principal cannot commit to a joint bonus floor, that is, $\underline{w} = 0$ by assumption. We present the optimal contract in this alternative setting as follows, and, to better connect with prior literature, highlight the differences caused by the joint bonus floor.

Proposition 4 *When the principal cannot commit to a joint bonus floor, that is,* $\underline{w} = 0$ *by assumption, the optimal contract is (i) JPE1 for* $r \in (0, r^A]$*, (ii) JPE2 for* $r \in (r^A, r^B]$*, (iii) RPE for* $r \in (r^B, r^D]$*, and infeasible otherwise. In comparison, the ability to commit to a positive bonus floor* $\underline{w} > 0$ *results in*

(i) more relational contract in general:

- *when* $\underline{w} = 0$ *by assumption, there is no feasible solution for* $r > r^D$*;*
- *otherwise, relational contract is feasible for all* r *otherwise.*

(ii) more team incentives:

- *when* $\underline{w} = 0$ *by assumption, team incentives are optimal if and only if* $r < r^B$;
- *otherwise, team incentives are optimal if and only if (1)* $r < r^B$ *or (2)* $r < \min\{r^L, r^C\}$ *in the case of* $r^L \geq r^B$*.*

(iii) more strategic payoff independence $\Delta U = 0$*:*

- *when* $\underline{w} = 0$ *by assumption,* $\Delta U = 0$ *holds at the singular discount rate* r^B*;*
- *otherwise,* $\Delta U = 0$ *is optimal for a range of discount rates, all with* $\underline{w} > 0$*.*

The optimal contract essentially reproduces the results of Kvaløy and Olsen (2006).[20] Part (i) of the proposition above is straightforward, as the ability to commit to a joint bonus floor strengthens the principal's credibility. Inspecting Program **P** shows that the joint bonus floor relaxes the (Principal's IC) constraint in both the team incentive case (Proposition 1) and individual incentive case (Proposition 2), it is unclear, a priori, whether such a commitment results in more or less team incentives once team incentives is a choice variable. Part (ii) of the proposition shows that the region in which team incentives are optimal expands once the bonus floor is introduced. Part (iii) highlights the role of the bonus floor in combating agents' collusion. When the principal cannot commit to such floor, $JPE2$ and RPE converges to

[20] We say "essentially" because they restrict attention to stationary strategies, while we do not.

independent performance evaluation at the singular discount rate r^B (i.e., $w_{HH} = w_{HL} > 0$, and $w_{LH} = w_{LL} = 0$), which creates strategic independence $\Delta U = 0$. As we have shown in Proposition 2 and Corollary 2, when the principal can commit to a joint bonus floor, BPI (with a bonus floor) creates $\Delta U = 0$ and is optimal for a range of discount rate r even though RPE is also feasible.

5 Extension: Incorporating an Objective Team-based Performance Measure

In a typical bonus pool arrangement, the size of the bonus pool is based, at least in part, on an objective/verifiable joint performance measure such as group or divisional earnings (Eccles and Crane, 1988). Suppose that such an objective measure $y \in \{H, L\}$ exists and define $p_1 = \Pr(y = H|e^A = e^B = 1)$, $p = \Pr(y = H|e^A \neq e^B)$, and $p_0 = \Pr(y = H|e^A = e^B = 0)$ with $p_1 > p > p_0$. Agent j's subjective performance measure is $x_j \in \{0, 1\}$, and $q_1 = \Pr(x^j = 1|e^j = 1)$ and $q_0 = \Pr(x^j = 1|e^j = 0)$ are defined the same way as in the main model. Denote by $\mathbf{w} \equiv \{w^y_{mn}\}$ the implicit contract the principal promises to pay to Agent i if the objective team measure is $y \in \{H, L\}$ and agent i and j's subjective individual measures are m and n, respectively. Assuming the principal honors the subjective measures m, n truthfully, we can rewrite agent i's expected wage payment if he chooses effort k and agent j chooses l as:

$$\pi(k, l; \mathbf{w}) = \mathbb{E}_{y \in \{H,L\}} \left[q_k q_l w^y_{HH} + q_k(1 - q_l) w^y_{HL} + (1 - q_k) q_l w^y_{LH} + (1 - q_k)(1 - q_l) w^y_{LL} \right], \quad (4)$$

where the expectation is taken over the team measure y given effort k, l.

The principal's problem is same as Program **P** in the main model after substituting the expected payment $\pi(k, l; \mathbf{w})$ everywhere by (4) shown above. The other change made to

Program **P** is to redefine *Principal's IC* for each realized team measure y as follows:

$$\frac{2\left[\frac{p_1}{\min\{p_1-p,\,p-p_0\}}-\pi(1,1;\mathbf{w})\right]}{r} \geq \max_{m,n\in\{H,L\}}\{w^y_{mn}+w^y_{nm}-w^y_{m'n'}-w^y_{n'm'}\},\ \forall y\in\{H,L\}. \tag{Principal's new IC}$$

Introducing the objective team measure y qualitatively changes the fallback contract triggered by the principal's reneging. In the main model, the principal looses agents' trust upon reneging and can only expect agents to shirk in perpetuity on the off-equilibrium path. With an objective team measure, however, the reneging principal can continue to induce (work, work) on the off-equilibrium path by replying solely on the verifiable team measure. That is, the reneging principal can ensure (work, work) as the *unique* stage game equilibrium, which requires paying $\frac{p_1}{\min\{p_1-p,\,p-p_0\}}$ to each agent.[21]

Before we characterize the analytical solution, we first demonstrate the robustness of our results in the main model via a numerical example. We use the same parameters as in Figures 1 and 2 to ease comparison, that is, $q_0 = 0.53$ and $q_1 = 0.75$. The objective measure y satisfies $p_1 = 0.6, p = 0.5$, and $p_0 = 0.4$ in Figure 3. In this example, the optimal contract satisfies $w^L_{mn} = 0, \forall m, n$, that is, the payment is zero whenever the team measure is low. Given the high team measure $y = H$, the way the optimal contact varies with the discount rate r is qualitatively same to that in Figure 2. The moment discount rate is high enough ($r > 0.8$) so that the principal cannot credibly commit to rewarding the increasingly higher w^H_{11}, the principal pays for *poor individual performances* w^H_{00} in order to continue raising w^H_{11}, which has a benefit of fostering agents' mutual monitoring. Individual incentive is optimal for higher

[21] We assume throughout that the agent's effort is valuable enough so that, following reneging (out-of-equilibrium), the principal chooses to induce "work" by using the verifiable measure. Denote by X the incremental value that a working agent brings to the principal than a shirking agent. Our out-of-equilibrium specification requires $(q_1 - q_0)X \geq \frac{p_1}{\min\{p_1-p,\,p-p_0\}}$. If the condition is violated, the out-of-equilibrium stage game triggered by reneging will have both agent shirking and the principal paying nothing. As a result, the LHS of the (Principal's new IC) will be $\frac{2[q_1X-\pi(1,1;\mathbf{w})]-2q_0X}{r}$. We verify that the two main results in the section persist if the maintaining assumption $(q_1 - q_0)X \geq \frac{p_1}{\min\{p_1-p,\,p-p_0\}}$ is violated. First, to promote team incentives, the principal rewards agents for poor team performance w^L_{11} the moment pure JPE w^H_{11} is infeasible. Second, optimal contract sometimes creates strategic payoff independence (to combat collusion) when individual incentives $T = 0$ is optimal.

discount rate $r > 3.6$, and BPI is optimal for $r < 8$ because it has the benefit of creating strategic independence, a desirable feature to combat agents' collusion. We use the the name developed in the main model to label contracts in Figure 3 not only to facilitate comparison with Figure 2, but also because the optimal contract w^H_{mn} (i.e., given the team output $y = H$) are qualitatively similar to solutions in Propositions 1 and 2 in terms of having a positive bonus floor and their strategic payoff interdependencies.

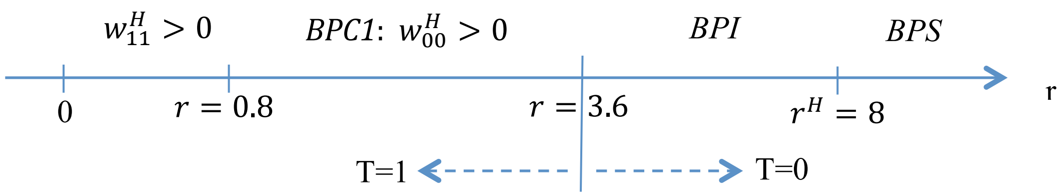

Figure 3: Optimal contract with a team measure ($p_0 = 0.4, p = 0.5, p_1 = 0.6$).

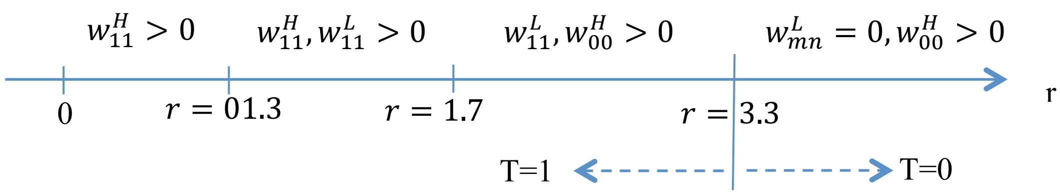

Figure 4: Optimal contract with a less informative team measure ($p_0 = 0.45, p = 0.5, p_1 = 0.55$).

Continuing with the same example but lower the informativeness of the public disclosure so that $p_1 = 0.55, p = 0.5$, and $p_0 = 0.45$. Figure 4 demonstrates a new type of "pay for poor performance". That is, when the principal cannot credibly commit to relying solely on w^H_{11} at $r = 1.3$, she starts to pay for *poor team performance* w^L_{11} as long as both agents' subjective measures are good. For even higher discount rate $r > 1.7$, the principal rewards both for poor team measure w^L_{11} and for poor individual measures w^H_{00} to encourage agents' incentive to mutually monitor each other. For $r > 3.3$, the optimal contract instead provides individual incentives ($T = 0$), in which case poor team measure is never rewarded, i.e., $w^L_{mn} = 0$.

The reminder of the extension characterizes the optimal solution analytically. To maintain tractability, we further assume that the objective team measure satisfies $p_1 = \frac{1}{2} + P$, $p = \frac{1}{2}$, and $p_0 = \frac{1}{2} - P$ for $P \in [0, 0.5]$; while each agent's subjective performance measure satisfies $q_1 = \frac{1}{2} + Q$ and $q_0 = \frac{1}{2} - Q$ for $Q \in [0, 0.5]$. That is, the informativeness of the team measure y (or the individual measure x_j) are captured by the single parameter P (or Q), respectively. Consistent with the finding in Figure 4, the lemma below shows that paying for poor team measure can be part of the optimal contract.

Lemma 3 *Given team incentives T=1, the optimal contract for $r \leq r1$ satisfies that w_{11}^H is the only positive payment and Principal's IC does not bind. As r increases from $r1$, there exists a unique P^* such that the principal*

- *pays for "bad" team performance (i.e., $w_{11}^L > 0$) while keeping $w_{10}^H = 0$ for $P \leq P^*$;*
- *increases w_{10}^H without paying for bad team performance (i.e., $w_{mn}^L = 0$) for $P > P^*$.*

Part (ii) is at odds with the insights derived from standard results on individual incentives: the standard informativeness argument suggests that paying for w_{10}^H provides stronger (individual) incentive than rewarding w_{11}^L for *any* $P > 0$. However, compared to paying w_{10}^H that has the feature of *RPE*, paying for "bad" team measure w_{11}^L has the benefit of keeping the focus on providing team incentives. The principal trades off the benefit of providing team incentives (via w_{11}^L) and the benefit of rewarding for higher likelihood ratio (via w_{10}^H). Since the advantage of w_{10}^H in likelihood ratios is stronger when the team measure y is more informative (higher P), Part (ii) shows that the benefit of focusing on team incentives dominates if the loss of likelihood ratio is not particularly strong. Given Lemma 3 and our focus on team incentives, we confine our attention to

$$P < \bar{\Delta} \doteq \frac{Q}{2(1+Q)}$$

in the remainder, which is sufficient to assume that paying for "bad" team performance (i.e., Part ii of the lemma) is always part of the optimal contract.[22]

[22]We impose this sufficient condition to maintain tractability when we later endogenize team incentives $T = \{0, 1\}$, in which each programing has 15 constraints (excluding 8 non-negativity constraints for control variables w_{mn}^y).

The lemma below summarizes several features of the optimal contract given individual incentives $T = 0$, highlighting the similarities to Proposition 2 in the main model. In particular, the principal creates strategic payoff independence for the intermediate discount region to combat agent-agent collusion. Because agents' ability to collude diminishes for large discount rate, the contract relies more on the RPE and, therefore, creates strategic payoff substitution; while the solution converges to standard bonus pools in the limit.

Lemma 4 *Given individual incentive T=0, the optimal contract satisfies that*

i. neither the principal's IC nor the agents' collusion constraints binds for low r.

ii. it creates strategic payoff independence $\Delta U = 0$ for intermediate discount $r \in [\underline{r}, \bar{r}]$, which is a non-empty interval if and only if the fall-back objective measure is not too noisy (hence the principal's credibility is not too strong), i.e., $P \geq \underline{\Delta}$.[23]

iii. it creates payoff substitution $\Delta U < 0$ for large discount rate r. As $r \to \infty$, the contract converges to a conditional symmetric bonus pool in that $w^L_{mn} = 0, w^H_{11} = w^H_{00} = \frac{1}{2}w^H_{10}$.

The condition $P \geq \underline{\Delta}$ in Part (ii) of the lemma reinforces another finding in the main model: the optimal contract depends on the principal's credibility *relative to* agents'. To see this, note that the principal's credibility is stronger the lower the P is because, upon reneging, the principal's off-equilibrium option of relying solely on the objective measure is costlier for smaller P. The worse off-equilibrium contract, in turn, strengthens the principal's credibility to hornor the subjective measures on equilibrium. If $P \geq \underline{\Delta}$ is violated, the principal's credibility will be so strong that by the time she looses her credibility, the agents are already so impatient that their ability to collude is weak, therefore, creating $\Delta U = 0$ to combat collusion is unnecessary.

Our next result endogenizes the use the team incentives $T = \{0, 1\}$ to characterize the overall optimal contract.

[23] The threshold $\underline{\Delta}$ is implicitly characterized in the Appendix. A sufficient condition is $P > 0.025$.

Proposition 5 *Assuming $\underline{\Delta} \leq P \leq \bar{\Delta}$, the overall optimal contract depends on r as follows:*

i. $T = 1$ for $r \leq r1$, $w_{11}^H > 0$ is the only positive payment, or Pure JPE;

ii. $T = 1$ for $r \in (r1, r2]$. As r increases, w_{11}^H decreases while the payment for "bad" team output w_{11}^L increases till $w_{11}^L = w_{11}^H$ at $r = r2$. Other payments are zero.

iii. $T = 1$ for $r \in (r2, \tilde{r}]$, w_{10}^H increases from zero and w_{11}^L decreases as r increases.

iv. $T = 0$ for $r \in (\tilde{r}, r3]$, the contract creates strategic payoff independence and $w_{00}^H > 0$.

v. $T = 0$ for $r > r3$, the contract creates payoff strategic substitute $\Delta U < 0$, and converges to conditional symmetric bonus pool as $r \to \infty$.

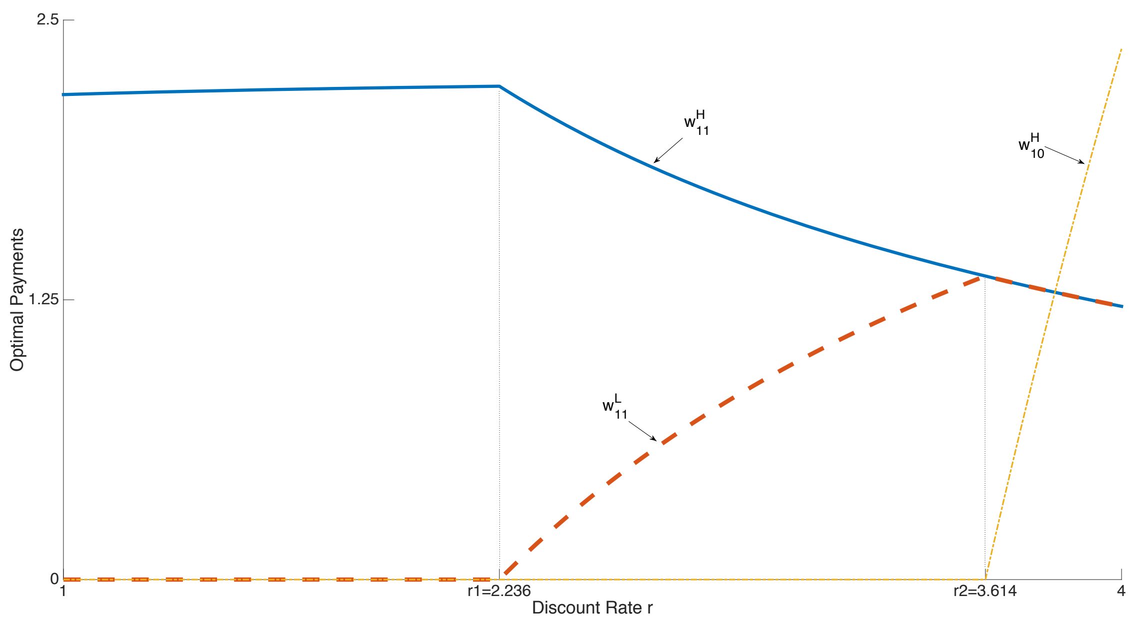

Figure 3: Optimal contract with an objective team measure $y = \{H, L\}$. $(Q = 0.4, P = 0.1)$

Comparing Proposition 5 to Proposition 3 confirms the insights derived without the team measure. In both cases, team incentive is optimal if and only if the discount rate r is not too large, and the limiting contract is either Pure JPE (for $r \to 0$) or stage game bonus pools (for $r \to \infty$). More central to our main message, Part (ii) of both propositions share the feature of paying for "bad" performance measures to foster team incentives even though increasing other payments (with higher likelihood ratio) is feasible. Finally, part (iv) of the two propositions are similar in that both pay for "bad" individual measures (either w_{LL} or w^H_{LL}) to facilitate strategic independence, which is desirable in combating agent-agent collusion.

Figure 3 illustrates the provision of *team* incentives - the focus of the paper – via an example in which $Q = 0.4$ and $P = 0.1$ (given $Q = 0.4$, the assumption $P \in [\underline{\Delta}, \bar{\Delta}]$ in Proposition 5 is $P \in [0.0048, 0.143]$). The pure *JPE* (only $w^H_{11} > 0$) is optimal if the principal's credibility of horning subjective performance measures is strong, i.e., $r < 2.236$. As the principal cannot hornor pure JPE for higher r, it is clear that she *reduces* her reliance on the objective measure while *increases* the reliance on subjective individual measures. In fact, the optimal contract completely ignores the team measure (i.e., $w^H_{mn} = w^L_{mn}$) and provides team incentives solely through the subjective measures at $r = r2 = 3.614$. It is the benefit of paying for "bad" team measure w^L_{11} in fostering the team incentive that makes it optimal, even though paying for w^H_{10} is feasible and has strictly higher likelihood ratio. The principal starts to increase w^H_{10} for $r > r2$ because agents' ability to enforce mutual monitoring (hence the benefit of using w^L_{11}) is weak when the discount rate is high. The principal optimally tunes down team incentives for $r \in [3.614, 4.13]$ (by increasing w^H_{10} and lowering w^L_{11}) and eventually switch to individual incentives $T = 1$ for $r > \tilde{r} = 4.13$.

6 Conclusion

Stepping back from the principal's commitment problem, the general theme of team (mutual monitoring) vs. individual incentives seems to be under-explored. For example, the models of team-based incentives typically assume the agents are symmetric (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001). With agents who have different roles such as top executives, static models predict the agents would be offered qualitatively different compensation contracts. Yet, in practice, the compensation of executives are often similar. As an extreme example, Apple pays the same base salary, annual cash incentive, and long-term equity award to each of its executive officers other than its CEO.[24] [25] We conjecture that compensating productively different agents similarly can be rationalized by a team-based model of dynamic incentives (with a low discount rate/long expected tenure).

If we apply the team-based model to thinking about screening, we might expect to see compensation contracts that screen agents for their potential productive complementarity, since productive complementarities reduce the cost of motivating mutual monitoring. A productive substitutability (e.g., hiring an agent similar to existing ones when there are overall decreasing returns to effort) is particularly unattractive, since the substitutability makes it appealing for the agents to tacitly collude on taking turns working. We might also expect to see agents screened for their discount rates. Patient agents would be more attractive, since they are the ones best equipped to provide and receive mutual monitoring incentives. Are existing incentive arrangements such as employee stock options with time-based rather than performance-based vesting conditions designed, in part, to achieve such screening?

Even if screening is not necessary because productive complementarities or substitutabilities are driven by observable characteristics of agents (e.g., their education or work experience), optimal team composition is an interesting problem. For example, is it better to have one team with a large productive complementarity and another with a small substitutability or to have two teams each with a small productive complementarity?

Another natural avenue for future research is peer evaluation (or 360-degree evaluation) under relational side-contracting. We assumed communication from the agents to the principal is blocked, as in Arya, Fellingham, and Glover (1997), Che and Yoo (2001), and Kvaløy

[24] According to Apple Inc.'s 2016 Proxy Statement: "Our executive officers are expected to operate as a team, and accordingly, we apply a team-based approach to our executive compensation program, with internal pay equity as a primary consideration. This approach is intended to promote and maintain stability within a high performing executive team, which we believe is achieved by generally awarding the same base salary, annual cash incentive, and long-term equity awards to each of our executive officers, except [CEO] Mr. Cook."

[25] For related observations about cash bonuses paid to CEOs that are similar to bonuses paid to other executives, see Core, Guay, and Verrecchia (2003) and Guay, Kepler, and Tsui (2016), which finds that, for approximately 75% of their sample, the CEO and the fifth-highest-paid executive have an identical set of performance targets for their cash bonuses.

and Olsen (2006). Ma (1988) studies a single-period, multi-agent model of moral hazard without side-contracting under unblocked communication (peer reports) from the agents to the principal. In Ma (1988), if the agents perfectly observe each other's actions and the principal contracts on the agents' peer reports, the principal can implement the first-best if every action pair induces a unique distribution over performance measures.[26] Baliga and Sjostrom (1998) study a similar setting but allow for explicit side-contracting between the agents and show that side-contracting greatly limits the role of peer reports. Deb, Li, and Mookerherjee (2016) study peer evaluation under relational contracting between the principal and the agents but without (relational or explicit) side-contracts between the agents, showing that peer evaluations are sparingly incorporated in determining compensation. As far as we are aware, there have been no studies of peer evaluation under relational side-contracting.

[26] Towry (2003) initiated an important experimental literature that contrasted the Arya, Fellingham, and Glover (horizontal) and Ma (vertical) views of mutual monitoring, emphasizing the important role of team identity.

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Appendix

Proof of Lemma 1. We start with a preliminary result that specifies the harshest punishment supporting the collusion.

Claim: It is without loss of generality to restrict attention to collusion constraints that have the agent punishing each other by playing $(1,1)$ (i.e., *work, work*) forever after a deviation from the collusion strategy.

Proof. We first argue that the off-diagonal action profiles $(0,1)$ and $(1,0)$ cannot be punishment strategies harsher than $(1,1)$. We illustrate the argument for $(0,1)$; similar logic applies to $(1,0)$:

		Agent B	
		L	H
Agent A	L	U_{00}, U_{00}	U_{01}, U_{10}
	H	U_{10}, U_{01}	U_{11}, U_{11}

If playing $(0,1)$ were a harsher punishment for Agent A (i.e., $U_{01} < U_{11}$), he would be able to profitably deviate to $(1,1)$. That is, playing $(0,1)$ is not a stage-game equilibrium and thus cannot be used as a punishment strategy. If $(0,1)$ were a harsher punishment for Agent B (i.e., $U_{10} < U_{11}$), we would need $U_{10} \geq U_{00}$ to prevent Agent B from deviating from $(0,1)$ to $(0,0)$. However, $U_{10} < U_{11}$, $U_{10} \geq U_{00}$, and (Static NE) together imply $U_{11} \geq \max\{U_{10}, U_{00}, U_{01}\}$, which means there is no scope for collusion because at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy.

To establish that $(1,1)$ is the (weakly) harshest punishment, it remains to show that $U_{11} \leq U_{00}$. Suppose, by contradiction, $U_{11} > U_{00}$. If the wage scheme $\mathbf{w} \equiv \{w^x_{mn}\}$, $x = \{0,1\}$, $m,n = \{L,H\}$, creates strategic payoff substitutability, i.e., $U_{10} - U_{00} > U_{11} - U_{01}$, (Static NE) again implies that $(0,0)$ is not a stage-game equilibrium and thus cannot be used as a punishment strategy in the first place. If instead $\mathbf{w}$ creates (weak) strategic payoff complementarity, i.e., $U_{11} - U_{01} \geq U_{10} - U_{00}$, we have $U_{10} + U_{01} \leq U_{11} + U_{00} < 2U_{11}$, where the last inequality is due to the assumption $U_{11} > U_{00}$. But $U_{00} < U_{11}$ and $U_{10} + U_{01} < 2U_{11}$ together mean that at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy, meaning there is no scope for any collusion.

■

Let

$$V_t^i(\sigma) \equiv \sum_{\tau=1}^{\infty} \frac{U_{t+\tau}^i(a_\tau^i, a_\tau^j)}{(1+r)^\tau}$$

be agent i's continuation payoff from $t+1$ and forwards, discounted to period t, from playing $\{a_{t+\tau}^A, a_{t+\tau}^B\}_{\tau=1}^{\infty}$ specified in an action profile $\sigma = \{a_t^A, a_t^B\}_{t=0}^{\infty}$, for $a_t^A, a_t^B \in \{0,1\}$. We allow any σ satisfying the following condition to be a potential collusive strategy:

$$\sum_{t=0}^{\infty} \frac{U_t^i(a_t^i, a_t^j)}{(1+r)^t} \geq \frac{1+r}{r} U_{1,1}, \ \forall i \in \{A, B\}, \tag{5}$$

where $U_t^i(a_t^i, a_t^j)$ is Agent i's stage-game payoff at t given the action pair (a_t^i, a_t^j) specified in σ, and $\frac{1+r}{r}U_{1,1}$ is the agent's payoff from always working.

The outline of the sufficiency part of the Lemma is as follows:

Step 1: Any collusive strategy that contains only $(1,1), (1,0)$ and $(0,1)$ (i.e., without $(0,0)$ in any period) is easier for the principal to upset than *Cycling*.

Step 2: Any collusive strategy that ever contains $(0,0)$ at some period t would be easier for the principal to upset than either *Joint Shirking* or *Cycling*.

Step 1: The basic idea here is that, compared with *Cycling*, any reshuffling of $(0,1)$ and $(1,0)$ effort pairs across periods and/or introducing $(1,1)$, can only leave some shirking agent better off in some period if it also leaves another shirking agent worse off in another period, in terms of their respective continuation payoffs.

In order for the agents to be better-off under the collusive strategy σ that contains only $(1,1)$, $(1,0)$ and $(0,1)$ than under jointly work $(1,1)^\infty$, condition (5) requires $U_{1,0}+U_{0,1} > 2U_{1,1}$.

Therefore, we know

$$\overline{V}^{CYC} + \underline{V}^{CYC} \geq V_t^i(\sigma) + V_t^j(\sigma),\ \forall t, \tag{6}$$

where $\overline{V}^{CYC} = \sum_{t=1,3,5,...} \frac{U_{1,0}}{(1+r)^t} + \sum_{t=2,4,6,...} \frac{U_{0,1}}{(1+r)^t}$ and $\underline{V}^{CYC} = \sum_{t=1,3,5,...} \frac{U_{0,1}}{(1+r)^t} + \sum_{t=2,4,6,...} \frac{U_{1,0}}{(1+r)^t}$ are the continuation payoffs (under *Cycling*) of the shirking agent and the working agent, respectively. We drop the time index in $\overline{V}^{CYC}$ and $\underline{V}^{CYC}$ because they're time independent. Since (Static NE) and $U_{1,0} + U_{0,1} > 2U_{1,1}$ together imply $U_{1,0} > U_{1,1} \geq U_{0,1}$, simple algebra shows

$$\overline{V}^{CYC} > \max\{\underline{V}^{CYC}, V^*\}, \tag{7}$$

where $V^* \doteq \frac{U_{1,1}}{r}$ is the continuation payoff from playing $(1,1)^\infty$.

To prove the claim that the collusive strategy σ is easier for the principal to upset than *Cycling*, it is sufficient to show the following:

$$\exists\, t | \{(a_t^i = 0, a_t^j = 1) \wedge V_t^i(\sigma) \leq \overline{V}^{CYC}\}. \tag{8}$$

That is, there will be some period when the agents i is supposed to be the *only* "shirker" in that period faces a weakly lower continuation payoff (hence stronger incentives to deviate from shirking) under the collusive strategy σ than under *Cycling*. Suppose by contradiction that (8) fails. That is,

$$V_t^i(\sigma) > \overline{V}^{CYC},\ \forall\, t | \{(a_t^i = 0, a_t^j = 1)\}. \tag{9}$$

We know from (6) that (9) implies the following for the other agent j:

$$V_t^j(\sigma) < \underline{V}^{CYC},\ \forall\, t | \{(a_t^i = 0, a_t^j = 1)\}. \tag{10}$$

Since one agent always playing 0 is not a sub-game perfect equilibrium, there must be a switch from $(0,1)$ to $(1,0)$, possibly sandwiched by one or more $(1,1)$, in any strategy σ. We pick any such block in σ, and denote τ and $\tau + 1 + n$ ($n \in \{0,1,2,3,...\}$) as the time $(0,1)$

and $(1,0)$ are sandwiched by n period(s) of $(1,1)$.

We show below that, for all n, (9) leads to a contradiction, which then verifies (8) and proves the claim. We name the first agent as Agent A throughout the analysis.

- If $n = 0$, i.e., $(0,1)$ is followed immediately by a $(1,0)$ at $\tau + 1$. One can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,0} + V_{\tau+1}^A(\sigma)}{1+r} \\ &< \frac{U_{1,0} + \underline{V}^{CYC}}{1+r} \\ &= \overline{V}^{CYC}, \end{aligned}$$

  which contradicts (9). The inequality above applies (10) to $t = \tau + 1$.

- If n is an even number ($n = 2, 4, 6, ...$), i.e., there are even numbers of $(1,1)$ sandwiched between $(0,1)$ and $(1,0)$. We prove the case for $n = 2$ and same argument applies for all even n.

	...	$t = \tau$	$t = \tau + 1$	$t = \tau + 2$	$t = \tau + 3$	...
σ	...	$(0,1)$	$(1,1)$	$(1,1)$	$(1,0)$	...

  We can show the following for Agent A's continuation payoff at τ:

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + V_{\tau+3}^A(\sigma)}{(1+r)^3} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &< \frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^3} \\ &= \overline{V}^{CYC}, \end{aligned}$$

  which again contradicts (9). The first inequality applies (10) to $t = \tau + 3$. The second inequality applies (7) and the fact that $\overline{V}^{CYC} > V^*$ if and only if $\frac{U_{1,0}}{1+r} + \frac{U_{0,1}}{(1+r)^2} > \frac{U_{1,1}}{1+r} + \frac{U_{1,1}}{(1+r)^2}$.

- If n is an odd number ($n = 1, 3, 5, ...$). Consider, without loss of generality, the following case for $n = 1$, i.e., there is one $(1,1)$ sandwiched between $(0,1)$ and $(1,0)$.

	...	$t = \tau$	$t = \tau + 1$	$t = \tau + 2$	...
σ	...	$(0,1)$	$(1,1)$	$(1,0)$	...

We can show the following for Agent A's continuation payoff at τ

$$\begin{aligned} V_\tau^A(\sigma) &= \frac{U_{1,1}}{1+r} + \frac{U_{1,0} + V_{\tau+2}^A(\sigma)}{(1+r)^2} \\ &< \frac{U_{1,1}}{1+r} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{U_{1,0} + \underline{V}^{CYC}}{(1+r)^2} \\ &= \frac{rV^*}{1+r} + \frac{\overline{V}^{CYC}}{1+r} \\ &< \overline{V}^{CYC}, \end{aligned}$$

which contradicts (9). The first inequality applies (10) to $t = \tau + 2$, the second equalities is from the definition of $V^* = \frac{U_{1,1}}{r}$, and the last equality is by the definition of $\underline{V}^{CYC}$ and $\overline{V}^{CYC}$ so that $\overline{V}^{CYC} = \frac{U_{1,0} + \underline{V}^{CYC}}{1+r}$.

Step 2: Given any contract offered by the principal, one of the following must be true:

$$\max\{2U_{1,1}, 2U_{0,0}, U_{1,0} + U_{0,1}\} = \begin{cases} 2U_{1,1} & \text{case 1} \\ U_{1,0} + U_{0,1} & \text{case 2} \\ 2U_{0,0} & \text{case 3.} \end{cases}$$

Case 1 is trivially collusion proof as no other action profile Pareto-dominates the equilibrium strategy $\{1,1\}_{t=0}^{\infty}$.

Case 2 has two sub-cases: sub-case 2.1 where $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{1,1} \geq U_{0,0}$ and sub-case 2.2 in which $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$. In sub-case 2.1, first note that we can ignore collusive strategy σ that contains $(0,0)$ at some period without loss of generality. The reason is that we can construct a new strategy σ' by replacing $(0,0)$ in σ by $(1,1)$, and σ' is more difficult to upset than σ because (a) both agents' continuation payoffs are higher under σ' and (b) σ' does not have the possibility of upsetting the collusive strategy at $(0,0)$. After ruling out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

In sub-case 2.2 (i.e., $\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$), we argue that any collusive strategy σ that contains $(0,0)$ at some point is easier to be upset than (thus implied by) the *Cycling* strategy. The reason is clear by comparing the action profiles between σ and *Cycling* from any $\tilde{t}$ such that $a_{\tilde{t}}(\sigma) = (0,0)$:

	$t = \tilde{t}$	$t = \tilde{t}+1, \tilde{t}+2, \cdots$
σ	$(0,0)$	$\{a^A(\sigma), a^B(\sigma)\}_{\tilde{t}+1}^{\infty}$
Cycling	$(0,1)$	$\{Cycle\}_{\tilde{t}+1}^{\infty}$

$\frac{U_{1,0}+U_{0,1}}{2} \geq U_{0,0} > U_{1,1}$ implies $U_{1,0} - U_{0,0} > U_{1,1} - U_{0,1}$. That is, the benefit for either Agent A or B to unilaterally deviate from $(0,0)$ at $\tilde{t}$ is higher than the benefit for A to deviate from $(0,1)$ to $(1,1)$ at $\tilde{t}$ under the *Cycling* strategy. In addition, we know $V_{\tilde{t}}^{A}(CYC) + V_{\tilde{t}}^{B}(CYC) \geq V_{\tilde{t}}^{A}(\sigma) + V_{\tilde{t}}^{B}(\sigma)$ holds under Case 2, and therefore either $V_{\tilde{t}}^{A}(\sigma) \leq V_{\tilde{t}}^{A}(CYC)$ or $V_{\tilde{t}}^{B}(\sigma) \leq V_{\tilde{t}}^{B}(CYC)$. If $V_{\tilde{t}}^{A}(\sigma) \leq V_{\tilde{t}}^{A}(CYC)$ holds, then A has stronger incentive to deviate at $\tilde{t}$ under σ than he would have under *Cycling*. If it is $V_{\tilde{t}}^{B}(\sigma) \leq V_{\tilde{t}}^{B}(CYC)$, we make use of the observation that $V_{\tilde{t}}^{B}(CYC) < V_{\tilde{t}}^{A}(CYC)$ to conclude $V_{\tilde{t}}^{B}(\sigma) < V_{\tilde{t}}^{A}(CYC)$, which means that B has stronger incentive to deviate at $\tilde{t}$ under σ than A would have under *Cycling* at $\tilde{t}$. Again, once we rule out collusive strategies containing $(0,0)$, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

Case 3 implies $V_t^A(SHK) + V_t^B(SHK) = \max_\sigma V_t^A(\sigma) + V_t^B(\sigma)$, $\forall t$. If a collusive strategy σ contains $a_{t'}^A(\sigma) = a_{t'}^B(\sigma) = 0$ at some period t', then either $V_{t'}^A(\sigma) \leq V_{t'}^A(SHK)$ or $V_{t'}^B(\sigma) \leq V_{t'}^B(SHK)$, which means at least one of the agents who is supposed to (jointly) shirk at t' will have a weakly stronger incentive to deviate than he would have under *Joint Shirking* strategy. If the collusive strategy does not contain $a_t^A = a_t^B = 0$ in any period, we can refer to Step 1 and show that any such collusive strategy is deterred by the (No Cycling) constraint.

Having shown the sufficiency of the lemma, it remains to prove necessity. (No Joint Shirking) is necessary for a contract to be collusion-proof unless playing $(1, 1)$ indefinitely Pareto-dominates *Shirking* in the sense of (Pareto Dominance) defined in the text (i.e., $U_{11} \geq U_{00}$). Likewise, (No Cycling) is necessary unless $(1, 1)^\infty$ Pareto-dominates *Cycling*, which, according to (Pareto Dominance) in the text, requires $\frac{(1+r)U_{11}}{r} \geq \sum\limits_{t=0,2,4,\ldots} \frac{U_{01}}{(1+r)^t} + \sum\limits_{t=1,3,5,\ldots} \frac{U_{10}}{(1+r)^t}$. Note that this condition is identical to (No Cycling). ■

Proof of Lemma 2. Given any wage scheme $\mathbf{w}$, denote $M = \min_{m,n \in H,L}\{w_{m,n} + w_{n,m}\}$. We first show $\underline{w} \leq M$. Suppose by contradiction that $\underline{w} > M$, which means $\underline{w} > w_{m,n} + w_{n,m}$ for some outcome pair (m, n). Since the court will enforce $\underline{w}$, it will allocate the difference $\Delta = \underline{w} - (w_{m,n} + w_{n,m})$ between the two agents according to a certain allocation rule. The principal can directly give the agents the same payment as what they would have received through the court's enforcement by increasing the payments so that $(w'_{m,n} + w'_{n,m}) = \underline{w}$ and allocating it appropriately. Label the new wage scheme as $\mathbf{w}'$. It is easy to see that (i) $\mathbf{w}'$ costs the same as $\mathbf{w}$, and (ii) $\mathbf{w}'$ is feasible as long as the original $\mathbf{w}$ is. Since the principal can do better by optimizing over all possible $(w'_{m,n} + w'_{n,m}) = \underline{w}$ (as opposed to replicating the court's allocation rule), $\mathbf{w}'$ is at least weakly better than $\mathbf{w}$.

To further prove $\underline{w} = M$, suppose instead $\underline{w} < M$ in a wage scheme $\mathbf{w}$. We can construct a new wage scheme $\mathbf{w}'$ by increasing $\underline{w}$ (while keep the payment $w_{m,n}$ unchanged). It is easy to see that (i) $\mathbf{w}'$ and $\mathbf{w}$ yield the same objective value, and (ii) $\mathbf{w}'$ is feasible as long as $\mathbf{w}$ is. Since Principal's IC is more relaxed in $\mathbf{w}'$ than in $\mathbf{w}$, it is optimal to set $\underline{w} \geq M$. We know $\underline{w} = M$ as we already show $\underline{w} \leq M$ above. ■

The following parameters will be useful in the remainder of the appendix.

$$\begin{aligned}
r^A &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2 q_1 H - 1 - q_1^2 + \\ \sqrt{((q_0-q_1)^2 q_1 H - 1 - q_1^2)^2 + 4(q_0-q_1)^2(q_0+q_1)H - 4q_1^2} \end{array}\right], \\
r^B &= (q_0-q_1)^2 H - q_1, \\
r^C &= (q_0-q_1)^2(q_0+q_1)H - q_0 - q_1^2, \\
r^D &= \frac{1}{2}\left[\begin{array}{c} (q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2 + \\ \sqrt{((q_0-q_1)^2(2-q_1)H - 1 - 2q_1 + q_1^2)^2 + 4(q_1-2)q_1 - 8(q_0-q_1)^2(q_1-1)H} \end{array}\right], \\
r^L &= \frac{q_1+q_0-1}{(1-q_1)^2}, \\
r^H &= \frac{2q_1-1}{(1-q_1)^2}.
\end{aligned}$$

Note that r^A,r^B,r^C,and r^D increase in H, and we assume throughout the paper that H is larger enough to assure that (i) $r^C > r^B$ whenever $q_0 + q_1 > 1$, and (ii) $r^A < r^B < r^D$ and $r^A < r^C$. In addition, the agent's effort is assumed to be valuable enough ($q_1 - q_0$ is not too small) such that $r^A > \sqrt{2}$ and $(q_0 - q_1)^2 H \geq \max\{1 + \frac{1}{q_1-q_0}, \frac{q_1^2}{2q_1-1}, \frac{q_1(2-q_1)}{2(1-q_1)}, \frac{q_0-(1-q_1)q_1}{q_0+q_1-1}\}$.

Proof of Proposition 1. Given Lemma 2, we can simplify the program as follows:

$\min(1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$

$s.t$

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max_{m,n,m',n'}\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn \succ m'n'}$)

$(2-q_0-q_1+r(1-q_1))w_{LL} + (q_0+q_1+q_1 r-1)w_{LH} + (q_0+(q_1-1)(1+r))w_{HL} - (q_0+q_1+q_1 r)w_{HH} \leq \frac{-(1+r)}{q_1-q_0}$ (Mutual Monitoring, $\lambda_{Monitor}$)

$(2-q_0-q_1)w_{LL} + (q_0+q_1-1)w_{LH} + (q_0+q_1-1)w_{HL} - (q_0+q_1)w_{HH} \leq \frac{-1}{q_1-q_0}$ (Pareto Dominance, λ_{Pareto})

$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \leq \frac{1}{q_1-q_0}$ (Self-Enforcing Shirk)

$-w_{LL} \leq 0\,(\mu_{LL})$; $\quad -w_{HL} \leq 0(\lambda_{11})$; $\quad -w_{HH} \leq 0(\mu_{HH})$; $\quad -w_{LH} \leq 0(\mu_{LH})$;

Sketch of the proof: we first solve a *relaxed* program without the (Self-Enforcing Shirk) constraint and then verify that solutions of the relaxed program satisfy (Self-Enforcing Shirk).

Claim: Setting $w_{LH} = 0$ is optimal in the *relaxed program* (without Self-Enforcing Shirk constraint.)

Proof of the Claim. Suppose the optimal solution is $\mathbf{w} = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Construct a new solution $\mathbf{w}' = \left\{w'_{HH}, w'_{HL}, w'_{LH}, w'_{LL}\right\}$, with $w'_{LH} = 0$, $w'_{HL} = w_{HL} + w_{LH}$, $w'_{LL} = w_{LL}$ and $w'_{HH} = w_{HH}$. It is easy to see that $\mathbf{w}$ and $\mathbf{w}'$ generate the same objective function value, and that $\mathbf{w}'$ satisfies all the (Principal's IC) constraints $ICP_{mn \succ m'n'}$ as long as $\mathbf{w}$ does. Finally, we claim that, compared to $\mathbf{w}$, $\mathbf{w}'$ relaxes all the other constraints in the relaxed program. We illustrate the argument for (Pareto Dominance) only since the same argument applies to (Mutual Monitoring) and (No Cycling). Denote by C_{LH} (and C_{HL}) the coefficient of the payment w_{LH} (and w_{HL}, respectively). It is easy to verify that $C_{LH} - C_{HL} \geq 0$, and, therefore, $\mathbf{w}'$ relaxex (Pareto Dominance) relative to $\mathbf{w}$. ∎

The Lagrangian for the relaxed problem is

$$L(\mathbf{w}, \lambda, \mu) = f_0(\mathbf{w}) - \sum_i \lambda_i f_i(\mathbf{w}) - \sum_s \mu_s w_{mn}.$$

A contract $\mathbf{w} = \{w_{mn}\}$ for $m, n \in \{H, L\}$ is optimal if and only if one can find a pair $(\mathbf{w}, \lambda, \mu)$ that satisfies the following four conditions: (i) Primal feasible, i.e., constraints $f_i(w) \geq 0$ and $w_{mn} \geq 0$, (ii) Dual feasible, i.e., Lagrangian multiplier vectors $\lambda \geq 0$ and $\mu \geq 0$, (iii) Stationary condition, i.e., $\nabla_{\mathbf{w}} L(\mathbf{w}, \lambda, \mu) = 0$, and (iv) Complementary slackness, i.e. $\lambda_i f_i(\mathbf{w}) = \mu_s w_{mn} = 0$. The proof lists the optimal contract, in particular the pair $(\mathbf{w}, \lambda, \mu)$, as a function of the discount rate r.

For $r < r^A$, the solution, denoted as as *JPE1*, is:

$w_{LL} = 0, w_{HL} = 0, w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$;

$\lambda_{Pareto} = q_1, \lambda_{Monitor} = \frac{q_1^2}{q_0+q_1+q_1 r}, \lambda_{CYC} = 0, \lambda_{HH \succ LL} = 0,$

$\lambda_{HL \succ LL} = 0, \lambda_{HH \succ HL} = 0, \lambda_{HH \succ HH} = 0, \lambda_{LL \succ HH} = 0,$

$\lambda_{LL\succ HL}=0, \mu_{LL}=\frac{q_0-2q_0q_1+q_1(1+r-q_1r)}{q_0+q_1+q_1r}, \mu_{HL}=\frac{q_0q_1}{q_0+q_1+q_1r}, \mu_{HH}=0.$

The $ICP_{HH\succ LL}$ constraint yields the upper bound on r under *JPE1*.

For $r^A<r\leq \min(r^L,r^C)$, the solution, denoted as *BPC1*, is:

$w_{LL}=\frac{(q_1-q_0)^2(q_0+q_1+q_1r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)}, w_{HL}=2*w_{LL},$

$w_{HH}=\frac{(q_1-q_0)^2(q_0+q_1+(-1+q_1)r)H-(1+r)(-1+q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)};$

$\lambda_{Pareto}=0, \lambda_{Monitor}=\frac{-r}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_{CYC}=0, \lambda_{HH\succ LL}=\frac{q_0-(q_1-2)((q_1-q)r-1)}{q_0+q_1+q_1r-q_1^2r-r^2},$

$\lambda_{HL\succ LL}=0, \lambda_{HH\succ HL}=-\frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_{HH\succ HH}=0, \lambda_{LL\succ HH}=0,$

$\lambda_{LL\succ HL}=0, \mu_{LL}=0, \mu_{HL}=0, \mu_{HH}=0.$

Under *BPC1* , the non-negativity of w_{LL} and w_{HL} requires $r>r^A$, while (Pareto Dominance) and $\lambda_{HH\succ HL}\geq 0$ impose the upper bounds r^C and r^L, respectively.

For $\min\{r^L,r^A\}<r\leq r^B$, the solution, denoted as *JPE2*, is:

$w_{LL}=0, w_{HL}=\frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_0+q_1+q_1r)H}{(1-q_1)r(1+r)-q_0(q_1+r)},$

$w_{HH}=\frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0+(-1+q_1)(q+r))H}{(q_1-q_0)((-1+q_1)r(1+r)+q_0(q_1+r))};$

$\lambda_{Pareto}=0, \lambda_{Monitor}=\frac{(q_1-1)q_1r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{CYC}=0, \lambda_{HH\succ LL}=\frac{-q_0q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$

$\lambda_{HL\succ LL}=0, \lambda_{HH\succ HL}=0, \lambda_{HH\succ HH}=0, \lambda_{LL\succ HH}=0,$

$\lambda_{LL\succ HL}=0, \mu_{LL}=\frac{r(q_0-(q_1-1)((q_1-q)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \mu_{HL}=0, \mu_{HH}=0.$

Under *JPE2*, the non-negativity of w_{HL} requires $r>r^A$ and $\mu_{LL}\geq 0$ yields another lower bound r^L on r. The non-negativity of w_{HH} and w_{HL} also requires $r>s'\equiv\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}$. In addition, (Pareto Dominance) requires $r<r^B$. We claim $(\max\{s',r^A,r^L\},r^B]=(\max\{r^A,r^L\},r^B]$. The claim is trivial if $s'\leq r^A$ and therefore consider the case where $s'>r^A$. Since r^A increases in H while s' is independent of H, one can show $s'>r^A$ is equivalent to $H<H'$ for a unique positive H'. Meanwhile, algebra shows that $r^B<r^A$ for $H<H'$. Therefore, $s'>r^A$ implies $r^B<r^A$, in which case $(\max\{s',r^A,r^L\},r^B]=(\max\{r^A,r^L\},r^B]=\emptyset$.

For $\max\{r^L,r^B\}<r\leq r^C$ and $q_1+q_0\geq 1$, the optimal solution, denoted as *BPC2*, is:

$w_{LL}=\frac{q_0(q_1-q_0)^2H-q_0(q_1+r)}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}, w_{HL}=\frac{(q_1-q_0)^2+r-2q_0r-(q_1-q_0)^3H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))},$

$w_{HH}=\frac{(q_0+q_1)(q_1-1)+q_0r+(q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))};$

$\lambda_{Pareto} = \frac{-q_0+(-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_{Monitor} = \frac{q_0+q_1-1}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_{CYC} = 0, \lambda_{HH \succ LL} = \frac{q_0(1-q_1)}{(q_1-1)r+q_0(-1+q_1+r)},$

$\lambda_{HL \succ LL} = 0, \lambda_{HH \succ HL} = 0, \lambda_{HH \succ HH} = 0, \lambda_{LL \succ HH} = 0,$

$\lambda_{LL \succ HL} = 0, \mu_{LL} = 0, \mu_{HL} = 0, \mu_{HH} = 0.$

Under *BPC2* , the non-negativity of λ_{Pareto} requires $q_1 + q_0 \geq 1$. Given $q_1 + q_0 \geq 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > r^B$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0+q_1-1}$. The other lower bound r^L on r is generated by intersecting requirements for $\lambda_{Pareto} \geq 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leq r^C$. We claim $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$. To see this, note that a necessary condition for $(\max\{s'', r^B, r^L\}, r^C]$ to be non-empty is that $r^B < r^C$, which can be rewritten as $\frac{q_0-q_1(1-q_1)}{q_0+q_1-1} < (q_1 - q_0)^2 H$. Subtracting q_1 from both sides of the inequality and collecting terms, one can derive $s'' < r^B$. That is, $r^B < r^C$ implies $s'' < r^B$, and, therefore, $(\max\{s'', r^B, r^L\}, r^C] = (\max\{r^L, r^B\}, r^C]$ is verified.

As r becomes even larger, the problem $T = 1$ becomes infeasible because the intersection of the (Mutual Monitoring) and (Principal's IC) is an empty set. Finally, tedious algebra verifies that the solutions characterized above satisfy the (Self-Enforcing Shirk) constraint that we left out in solving the *relaxed* program. Therefore adding this constraint back does not affect the optimal contract. ■

Proof of Corollary 1. The corollary follows directly from Proposition 1. ■

Proof of Proposition 2. Similar argument as in the Proof of Proposition 1 shows that setting $w_{LH} = 0$ is optimal. Given $w_{LH} = 0$, one can rewrite the program as follows.

$$\min(1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

s.t

$\frac{2[q_1 H - \pi(1,1;\mathbf{w})] - 2q_0 H}{r} \geq \max\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$, (Principal's IC, $\lambda_{mn \succ m'n'}$)

$(1-q_1)w_{LL} - (1-q_1)w_{HL} - q_1 w_{HH} \leq \frac{-1}{q_1-q_0}$ (*Static NE*, λ_{SNE})

$((2-q_1-q_0)+(1-q_0)r)w_{LL} + ((q_0-1)(1+r)+q_1)w_{HL} - (rq_0+q_0+q_1)w_{HH} \leq \frac{-(1+r)}{q_1-q_0}$ (*No Joint Shirking*, λ_{SHK})

$(1-q_1)(2+r)w_{LL} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \leq -\frac{1+r}{q_1-q_0}$ (*No Cycling*, λ_{CYC})

$-w_{LL} \leq 0(\mu_{00})$; $\quad -w_{HL} \leq 0(\mu_{10})$; $\quad -w_{HH} \leq 0(\mu_{11})$.

For $r \leq r^B$, the solution, denoted as *IPE*, is:

$w_{LL} = 0$, $w_{HL} = w_{HH} = \frac{1}{q_1-q_0}$

$\lambda_{SNE} = q_1$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = 0$, $\lambda_{HH \succ LL} = 0$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = 1-q_1$, $\mu_{HL} = 0$, $\lambda_{12} = 0$.

Under IPE, the $ICP_{HH \succ LL}$ constraint imposes the upper bound r^B on r.

For $r^B < r < r^H$, the solution, denoted as *BPI*, is:

$w_{LL} = \frac{(q_1-q_0)^2(1+r)H-(1+r)(q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))}$, $w_{HL} = w_{HH} + w_{LL}$,

$w_{HH} = \frac{(q_1-q_0)^2H-(1+r)(-1+q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))}$;

$\lambda_{SNE} = 0$, $\lambda_{SHK} = \frac{r(1+r+q_1^2r-2q_1(q+r))}{(q_1-q_0)(1+r)(-1+(-1+q_1)r+r^2)}$, $\lambda_{CYC} = \frac{r(-1+q_0+q_1-r+q_0r-q_1^2r)}{(q_0-q_1)(1+r)(-q+(-1+q_1)r+r^2)}$, $\lambda_{HH \succ LL} = \frac{1+r-q_1r}{-1-(1-q_1)r+r^2}$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = 0$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *BPI*, both the non-negativity of w_{LL} and the (Static NE) constraints require $r > r^B$, and $\lambda_{SHK} \geq 0$ requires $r < r^H$.

For $\max\{r^H, r^B\} < r \leq r^D$, the optimal solution, denoted as *RPE*, is:

$w_{LL} = 0$, $w_{HL} = \frac{(q_1-q_0)q_1(2+r)H - \frac{(1+r)(q_1^2+r)}{q_1-q_0}}{q_1^2-r(1+r)+q_1r(2+r)}$,

$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(-1-r+q_1r(2+r))H}{(q_1-q_0)(q_1^2-r(1+r)+q_1r(2+r))}$,

$\lambda_{SNE} = 0$, $\lambda_{SHK} = 0$, $\lambda_{CYC} = \frac{(1-q_1)q_1r}{r(1+r)-q_1^2-q_1r(2+r)}$, $\lambda_{HH \succ LL} = \frac{q_1^2}{r(1+r)-q_1^2-q_1r(2+r)}$,

$\lambda_{HL \succ LL} = 0$, $\lambda_{HH \succ HL} = 0$, $\lambda_{HL \succ HH} = 0$, $\lambda_{LL \succ HH} = 0$,

$\lambda_{LL \succ HL} = 0$, $\mu_{LL} = \frac{r(1+r+q_1^2r-2q_1(1+r))}{r(1+r)-q_1^2-q_1r(2+r)}$, $\mu_{HL} = 0$, $\mu_{HH} = 0$.

Under *RPE*, the (Static NE) constraint and $\mu_{LL} \geq 0$ yields two lower bounds r^B and r^H on r. $ICP_{HL \succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leq r^D$. $w_{HH} \geq 0$ also requires $r > s \equiv \frac{2q_1-1+\sqrt{(2q_1-1)^2+4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, r^B, r^H\}, r^D] = (\max\{r^B, r^H\}, r^D]$.

Consider the case where $s > r^B$ (as the claim is trivial if instead $s \leq r^B$). Since r^B increases in H while s is independent of H, one can show that $s > r^B$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $r^D < r^B$ for $H < H^*$. Therefore, in the case of $s > r^B$, we know $r^D < r^B$ and hence $(\max\{s, r^B, r^H\}, r^D] = (\max\{r^B, r^H\}, r^D] = \emptyset$.

For $r > \max\{r^D, r^H\}$, the optimal solution, denoted as *BPS*, is:

$w_{LL} = \frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}, w_{HL} = 2w_{HH}$,

$w_{HH} = \frac{(1+r)(-(1-q_1)^2+r)-(q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}$,

$\lambda_{SNE} = q_1, \lambda_{SHK} = 0, \lambda_{CYC} = \frac{r}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ LL} = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2r-r^2-q_1(2+3r)}$,

$\lambda_{HL\succ LL} = \frac{2(1+r+q_1^2r-2q_1(1+r))}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_{HH\succ HL} = 0, \lambda_{HL\succ HH} = 0, \lambda_8 = 0$,

$\lambda_{LL\succ HL} = 0, \mu_{LL} = 1 - q_1, \mu_{HL} = 0, \mu_{HH} = 0$.

The two lower bounds r^D and r^H are derived from the non-negativity constraint of w_{LL} and $\lambda_{HL\succ LL}$, respectively. Collecting conditions verifies the proposition. ■

Proof of Corollary 2. The corollary follows directly from Proposition 2. ■

Proof of Proposition 3. The proposition is proved by showing the following sequence of claims.

Claim 1: If *JPE1* is optimal given $T = 1$, it is the overall optimal contract.

Claim 2: If *JPE2* is optimal given $T = 1$, it is the overall optimal contract.

Claim 3: If *BPC1* is optimal given $T = 1$, it is the overall optimal contract.

Claim 4: *BPC2* of $T = 1$ is never the overall optimal contract.

Proof of Claim 1: We know from Proposition 1 that *JPE1* is the optimal solution of $T = 1$ for $r \in (0, r^A]$, over which the optimal solution of $T = 0$ is *IPE* (Proposition 2). Substituting the two solutions into the principal's objective function, we obtain $obj_{JPE1} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1r)}$ and $obj_{IPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{IPE} - obj_{JPE1} = \frac{q_0q_1}{(q_1-q_0)(q_0+q_1+q_1r)} > 0$, which verifies the claim.

Proof of Claim 2: *JPE2* is the solution of $T = 1$ for $r \in (\max\{r^L, r^A\}, r^B]$, over which *IPE* is the corresponding solution of $T = 0$. Algebra shows that $obj_{JPE2} = \frac{(q_1-1)q_1r(1+r)+q_0(q_0-q_1)^2q_1H}{(q_0-q_1)((q_1-1)r(1+r)+q_0(q_1+r))}$,

$obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE2} - obj_{IPE} \leq 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)} \leq r \leq r^B$ (with strict inequality except at the boundary of the support). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}, r^L, r^A\} \leq r \leq r^B$, which is shown to be equivalent to $r \in (\max\{r^L, r^A\}, r^B]$ in the proof of Proposition 2. Therefore, $JPE2$ is the overall optimal contract whenever it is feasible.

Proof of Claim 3: We know that *BPC1* is the solution of $T = 1$ if $r \in (r^A, \min\{r^L, r^C\}]$. In this region, *IPE* and *BPI* are potential solutions in $T = 0$ because the other two solutions (*RPE* and *BPS*) require $r \geq r^H > r^L$. Let us compare first *BPC1* of $T = 1$ and *BPI* of $T = 0$. It is easy to show $obj_{BPC1} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1r-r^2)}$ and $obj_{BPI} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_0-q_1)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BPC1} < obj_{BPI}$ for $r^B < r \leq \min\{r^L, r^C\}$ where both solutions are feasible.

Comparing *BPC1* and *IPE* solution is more involved and we present the analysis in two steps. We first derive a sufficient condition for *BPC1* to outperform *IPE* (i.e., $obj_{BPC1} < obj_{IPE}$) and then show that the sufficient condition holds in the relevant region where the two solutions are optimal in their corresponding program, i.e., $r^A < r \leq \min\{r^L, r^B, r^C\}$. Given obj_{BPC1} and obj_{IPE} derived above, one can show that $obj_{BPC1} < obj_{IPE}$ if and only if $r < \delta$, where

$$\delta = \frac{1}{2(1-q_1)}[((q_1-q_0)^2H - q_1)q_1(1-q_1) - 1 + \sqrt{(((q_1-q_0)^2H-q_1)q_1(1-q_1)-1)^2 + 4(1-q_1)((q_1-q_0)^2H-q_1)(q_1+q_0)}].$$

We will show $r < \delta$ (hence $obj_{BPC1} < obj_{IPE}$) satisfies over the relevant region $r^A < r \leq \min\{r^L, r^B, r^C\}$ by considering two cases. First consider the case of $\delta \geq r^B$, in which $r < \delta$ satisfies trivially for $r^A < r \leq \min\{r^L, r^B, r^C\}$. Consider the second case in which $\delta < r^B$. The remainder shows that $\delta < r^B$ over the relevant region implies $r^L < \delta$, and, therefore, $r < \delta$ again satisfies given $r^A < r \leq \min\{r^L, r^B, r^C\}$. One can show that $\delta < r^B$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. We need only consider

$r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$ as the latter condition $r < q_1 \leq 1$ is outside the relevant region. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2H}$, algebra shows $r^L < \delta$ for any $q_0 \in [0, q_1)$.

Proof of Claim 4: Recall that *BPC2* of $T = 1$ is obtained by solving the following three binding constraints: (Mutual Monitoring), (Pareto Dominance), and $ICP_{HH \succ LL}$. When both (Mutual Monitoring) and (Pareto Dominance) are biding, it is easy to prove $U_{11} = U_{00} = U_{01}$. Since *BPC2* also creates strategic payoff complementarity (i.e., $U_{11} - U_{01} > U_{10} - U_{00}$), we can further show that the contract satisfies $U_{11} = U_{00} = U_{01} > U_{10}$, which means that *BPC2* would have satisfied the (No Cycling) had the constraint been part of $T = 1$. In other words, *BPC2* satisfies all the potentially binding constraints in the $T = 0$ program when the parameters are such that either *RPE* or *BPS* is optimal given $T = 0$ (recall that (No Joint Shirking) is not binding in these two solutions). Therefore, the optimality of *RPE* and *BPS* means that $obj_{BPC2} \geq obj_{RPE}$ and $obj_{BPC2} \geq obj_{BPS}$ over the corresponding parameter spaces. To see that *BPI* is more cost efficient than *BPC2* (i.e., $obj_{BPC2} \geq obj_{BPI}$), note that *RPE* is feasible over the region where *BPI* is optimal, and we know by revealed preference that $obj_{RPE} \geq obj_{BPI}$, and $obj_{BPC2} \geq obj_{RPE} \geq obj_{BPI}$ by transitivity.

Finally, showing *BPS* is eventually optimal for sufficiently large r is trivial as $T = 1$ does not have feasible solution in the region. ■

Proof of Proposition 4. Given Proposition 1, it is easy to verify that, fixing $T = 1$ and $\underline{w} = 0$, the solution to Program P is: (i) *JPE1* for $r \in (0, r^A]$; (ii) *JPE2* for $r \in (r^A, r^B]$; and (iii) infeasible otherwise. Similarly, given Proposition 2, one can verify that fixing $T = 0$ and $\underline{w} = 0$, the solution to Program P is: (i) *IPE* for $r \in (0, r^B]$; (ii) *RPE* for $r \in (r^B, r^D]$; and (iii) infeasible otherwise. Endogenizing the choice of T follows directly from Proposition 3 and yields the overall optimal contract stated in the proposition. Comparing the optimal contract with that in Proposition 3 verifies the three differences caused by the ability to commit to a bonus floor. ■

Proof of Lemma 3. Following Propositions 1 and 2, we list positive payments and Lagrangian multipliers of the binding constraints (μ for non-negativity constraints and λ for other constraints).

For $r < r1 \doteq \frac{2P\left(2Q^2+Q-2\right)+\sqrt{(2(P(2Q^2+Q-2)+Q^2)+Q)^2+8P(2P+1)(P(1-2Q)^2+4Q)}+Q(2Q+1)}{8P}$, the solution is: $w_{11}^H = \frac{4H(r+1)}{P(4Q(Q(r+2)+r)+r+2)+2Q(2Qr+r+2)}$, $\lambda_{Monitor} = \frac{4\left(P+\frac{1}{2}\right)\left(Q+\frac{1}{2}\right)^2}{P(4Q(Q(r+2)+r)+r+2)+2Q(2Qr+r+2)}$,
$\mu_{H00} = -\frac{(2P+1)Q\left(4P\left(4Q^2+1\right)+4Q^2(r-2)-r-2\right)}{2P(4Q(Q(r+2)+r)+r+2)+4Q(2Qr+r+2)}$, $\mu_{H10} = \frac{\left(4P^2-1\right)Q\left(4Q^2-1\right)}{2P(4Q(Q(r+2)+r)+r+2)+4Q(2Qr+r+2)}$,
$\mu_{L00} = \frac{\left(4PQ^2+P+2Q\right)\left(16PQ-4Q^2(r-2)+r+2\right)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$, $\mu_{L10} = \frac{\left(4Q^2-1\right)\left(P\left(4Q^2-1\right)r-2(P+Q)(4PQ+1)\right)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$,
$\mu_{L11} = -\frac{P(2Q-1)(2Q+1)^2(2Q(r-2)+r+2)}{4P(4Q(Q(r+2)+r)+r+2)+8Q(2Qr+r+2)}$.
$ICP_{H11\succ H00}$ imposes the upper bound $r < r1$.

To prove Part (ii), note that for $r1 < r \leq \min\{r2, -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}\}$, the contract is
$w_{11}^H = \frac{2\left(-2P^2(1-2Q)^2-P(4Q(3Q+r-1)+2r+3)+2Q(2Qr+r+2)\right)}{P\left(P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)\right)}$,
$w_{11}^L = \frac{2\left(2P^2(1-2Q)^2+P\left(Q^2(8r+4)+4Q(r+1)-8r(r+1)+1\right)+2Q(2Qr+r+2)\right)}{P\left(P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)\right)}$.
$\lambda_{Monitor} = -\frac{2(2P-1)(2Q+1)^2r}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,
$\lambda_{H11\succ H00} = -\frac{P(2Q-1)(2Q+1)^2(2Q(r-2)+r+2)}{2P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+16Qr(2Qr+r+2)}$,
$\mu_{H00} = \frac{2r\left(\left(4Q^2-1\right)r\left(4PQ^2+2PQ+P-Q\right)+2\left(4Q^2+1\right)(P-Q)(4PQ-1)\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,
$\mu_{H10} = -\frac{\left(4Q^2-1\right)r\left(P\left(8PQ+4Q^2(r-2)-r-2\right)+2Q\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,
$\mu_{L00} = -\frac{2(2P-1)Qr\left(4P\left(4Q^2+1\right)-4Q^2(r-2)+r+2\right)}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$,
$\mu_{L10} = \frac{2\left(4P^2-1\right)Q\left(4Q^2-1\right)r}{P\left((4Q(4Q(Q^2+Q-2)-1)-9)r-2(1-4Q^2)^2-4(2Qr+r)^2\right)+8Qr(2Qr+r+2)}$.
$w_{L11} > 0$ requires $r > r1$, $ICP_{L11\succ L00}$ requires

$$r < r2 \doteq \frac{4PQ^2 - 5P + 4Q^2 + 2Q + \sqrt{\left(4(P+1)Q^2 - 5P + 2Q\right)^2 - 16P\left(P(1-2Q)^2 - 4Q\right)}}{8P},$$

and $\mu_{H10} > 0$ requires $r < -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$. One can show that the interval $r1 < r \leq \min\{r2, -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}\}$ is non-empty if and only if $-\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)} \geq r1$, which is equivalent to $P \leq P^*$ for an (implicitly) determined unique threshold P^*.

The previous solution does not exist if, in contrast, $P > P^*$ (hence $r1 > -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$). As r increases from $r1$, the solution is

$$w^H_{10} = \frac{\left(2P^2(1-2Q)^2+P\left(Q^2(8r+4)+4Q(r+1)-8r(r+1)+1\right)+2Q(2Qr+r+2)\right)}{P((4P^2-1)Q(4Q^2-1)+4P(4Q^2-1)r+2(2Q-1)r^2(2(P+1)Q+P))},$$
$$w^H_{11} = \frac{(2P+1)(2PQ+P+2Qr)}{P((4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r)}; \lambda_{Monitor} = \frac{(2P+1)(2Q+1)r}{(4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r},$$
$$\lambda_{H11\succ H00} = -\frac{\left(4P^2-1\right)Q(2Q+1)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}, \mu_{H00} = -\frac{(2P+1)Q(2Q-1)r^2}{(4P^2-1)Q(2Q+1)+2r^2(2(P+1)Q+P)+4P(2Q+1)r},$$
$$\mu_{L00} = \frac{r\left(2(2Q+1)(P+Q)(4PQ+1)-(2Q-1)r\left(4PQ^2+P+2Q\right)\right)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}, \mu_{L10} = \frac{P\left(4Q^2-1\right)r(2Q(r-2)-r-2)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r},$$
$$\mu_{L11} = -\frac{(2Q+1)r\left(P\left(8PQ+4Q^2(r-2)-r-2\right)+2Q\right)}{2(4P^2-1)Q(2Q+1)+4r^2(2(P+1)Q+P)+8P(2Q+1)r}.$$

The non-negativity of w^H_{10} and μ_{L11} requires $r > r_1$ and $r > -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$, respectively; while $ICP_{H11\succ H00}$ and *Self-Shirking* impose an upper bound on r.

For Part (iii), note that the principal can always ignore the individual subjective measures and set $w^H_{mn} = \frac{H(r+1)}{P(r+2)}$ and $w^L_{mn} = 0$ for all $m, n \in \{0, 1\}$, which is feasible for all r. ■

Proof of Lemma 4. For $r < rA \doteq \min\{\frac{(2P+1)Q^2}{P(2(P+1)Q-P)}, \frac{(2P+1)Q(3-2Q)}{4P}\}$, the optimal solution satisfies $w^H_{11} = \frac{\frac{4H}{2(P+1)Q+P}+(2Q-1)w^H_{10}}{2Q+1}$. The principal can transfer between w^H_{11} and w^H_{10}, subject to the constraints. For instance, setting $w^H_{10} = 0$ is optimal for $r \leq \frac{(2P+1)Q(2Q+1)}{4P}$, and setting $w^H_{10} = \frac{(2P+1)Q(2Q+1)H-4PHr}{P(2Q-1)r(2(P+1)Q+P)}$ is optimal for $\frac{(2P+1)Q(2Q+1)}{4P} < r < rA$. The two upper bounds of r is required by *No Cycling* and $ICP_{H10\succ H00}$, respectively.

To prove Part (ii), we will show that the statement holds for $\underline{r} \doteq \frac{(2(P+1)Q-P)\left(4PQ^2+P-2Q\right)}{4P(P-Q)} < r < \bar{r} \doteq -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$. The optimal contract is

$$w^H_{00} = \frac{\left(P^2\left(2-8Q^2\right)-P\left(4Q^2(r+2)-4Q(r+1)+r(4r+5)\right)+4Q(r+1)\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w^H_{10} = -\frac{2\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w^H_{11} = -\frac{\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))},$$
$$w^L_{11} = -\frac{2(2P+1)(P+Qr)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}.$$

Substituting the payments verifies $\Delta U = 0$. Given the maintaining assumption $P < \frac{Q}{2(1+Q)}$, one can verify that $\bar{r} > \underline{r}$ is equivalent to $P > \underline{\Delta}$ for a uniquely determined $\underline{\Delta}$.

To prove Part (iii), note that for $r > -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$, the solution is

$$w^H_{00} = \frac{P\left(4\left((Q-2)Qr+Q(3Q-1)+r^2\right)+7r+3\right)+2Q(2Q(r+2)-3r-2)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w^H_{10} = \frac{2\left(4P^2+P\left(4Q^2(r+3)-4Q(r+1)+r(4r+7)+5\right)+4Q(Q(r+2)-r-1)\right)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w^H_{11} = \frac{4P^2+P\left(4Q^2(r+3)-4Q(r+1)+r(4r+7)+5\right)+4Q(Q(r+2)-r-1)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$

$$w^L_{10} = \frac{4(2P+1)(P+Qr)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$w^L_{11} = \frac{2(2P+1)(P+Qr)}{P(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))}.$$
$$\lambda_{CYC} = \frac{2(r+2)(2Pr+r)}{P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4)},$$
$$\lambda_{H11\succ H00} = -\frac{(2P+1)Q(2Q+1)((2Q-1)r-4)}{4P(4Q^2(r+2)-8Qr+(r+2)(4r+3))-4Q(4Q^2r-8Q(r+1)+r(3-4r)+4)},$$
$$\lambda_{L11\succ L00} = \frac{\left(4Q^2-1\right)r(2P(Q+1)-Q)+4(2Q+1)(Q-P)}{4P(4Q^2(r+2)-8Qr+(r+2)(4r+3))-4Q(4Q^2r-8Q(r+1)+r(3-4r)+4)},$$
$$\lambda_{H10\succ H00} = \frac{(2P+1)Q(4Q((Q-1)r-2)+r)}{2(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$\lambda_{L10\succ L00} = -\frac{(1-2Q)^2r(2P(Q+1)-Q)+4P+8Q^2}{2(P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4))},$$
$$\mu_{L00} = \frac{4Pr(-2Qr+r+2)}{P(4Q^2(r+2)-8Qr+(r+2)(4r+3))+Q(-4Q^2r+8Q(r+1)+r(4r-3)-4)}.$$

Taking the limit shows that, as $r \to \infty$, $w^L_{mn} = 0, w^H_{11} = w^H_{00} = \frac{1}{2}w^H_{10} = \frac{1}{P+Q}$. ■

Proof of Proposition 5. We list the complete optimal solutions, taking as given team incentives ($T = 1$) or individual incentives ($T = 0$), separately. Endogenizing $T = \{0, 1\}$ then follows from comparing the expected payments across the two programs.

Solutions given $T = 0$:

SolA: the optimal contract for $r \le \frac{(2P+1)Q^2}{P(2(P+1)Q-P)}$ is shown in Part (i) of Lemma 4. The maintaining assumption $P < \frac{Q}{2(1+Q)}$ helps simplify the upper bound on r.

SolB: for $\frac{(2P+1)Q^2}{P(2(P+1)Q-P)} < r \le \frac{P^2\left(2Q\left(-4Q^2+6Q+1\right)-3\right)+8PQ-8Q^2}{4P(2(P-1)Q+P)}$, the solution is $w^H_{10} = \frac{1}{Q}$, $w^H_{11} = \frac{P^2(2Q+1)(1-2Q)^2-4PQ+4Q^2}{PQ(P(2Q+1)(4Q^2-4r-1)+8Qr)}, w^L_{11} = -\frac{4\left(P^2r+(2P+1)Q^2-2(P+1)PQr\right)}{PQ(P(2Q+1)(4Q^2-4r-1)+8Qr)}$. The binding constraints are $w^H_{00} = w^L_{00} = w^L_{10} = 0$, *Static NE*, and $ICP_{H11\succ H00}$. The non-negativity $w^L_{11} > 0$ and the constraint $ICP_{H10\succ H00}$ impose the lower and upper bound on r, respectively. Note that we identify the binding constrains (rather than listing each Lagrangian multipliers) in this proposition for space considerations.

SolC: for $\frac{P^2\left(2Q\left(-4Q^2+6Q+1\right)-3\right)+8PQ-8Q^2}{4P(2(P-1)Q+P)} < r \le \frac{-P^2(1-2Q)^2(2Q-3)+4P(2Q-1)Q-8Q^2}{8(P-1)PQ-4P^2}$, the solution is $w^H_{10} = \frac{8(P-Q)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}, w^H_{11} = \frac{4(P-Q)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}$, and $w^L_{10} = \left[\frac{4((2P+1)Q(2Q-3)+4Pr)}{P(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)} + 2Qw^L_{11} + w^L_{11}\right]/(2Q-1)$. The solution is not unique: the principal can transfer between w^L_{11} and w^L_{10} subject to the constraints. For instance, setting $w^L_{10} = 0$ is optimal for $r \le Q\left(\frac{1}{P} - Q + 1\right) - \frac{1}{4}$; while, for higher r, setting $w^L_{10} = \frac{4\left(P\left((1-2Q)^2+4r\right)-4Q\right)}{P(2Q-1)(P(2Q+1)(4(Q-2)Q+4r+3)-8Qr)}$ is optimal. The binding constraints are $w^H_{00} = w^L_{00} = 0$, *Static NE*, $ICP_{H11\succ H00}$, and

$ICP_{H10\succ H00}$. The upper bound of r is imposed by *No Cycling* constraint.

SolD: for $\frac{-P^2(1-2Q)^2(2Q-3)+4P(2Q-1)Q-8Q^2}{8(P-1)PQ-4P^2} < r \leq \tilde{r} \doteq \frac{(2(P+1)Q-P)\left(4PQ^2+P-2Q\right)}{4P(P-Q)}$, the solution is $w_{00}^H = \frac{(2Q+1)\left(P^2(2Q-1)(4(Q-2)Q+4r+3)-4PQ(2Q+2r-1)+8Q^2\right)}{2PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$,

$w_{10}^H = \frac{\left(P^2(2Q-1)\left(2Q\left(4Q^2-2Q+4r+1\right)+4r-1\right)-2PQ(4Q(3Q+2r-1)+4r+3)+8Q^2\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$,

$w_{11}^H = \frac{\left(P^2(2Q-1)\left(2Q\left(4Q^2-2Q+4r+1\right)+4r-1\right)-2PQ(4Q(3Q+2r-1)+4r+3)+8Q^2\right)}{2PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$,

$w_{10}^L = \frac{2\left(P^2\left(8Q^3-4Q^2+2Q-4r-1\right)+4PQ(Q(2Q-1)+r+1)-4Q^2\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$, $w_{11}^L = \frac{\left(P^2\left(8Q^3-6Q+2\right)+P(-4(Q-1)Q-5)Q-8Q^3\right)}{PQ(P(16Q^2(Q^2+r)-12r-1)-2Q(4Q(Q+2r)-1))}$.

The binding constraints are $w_{00}^L = 0$, *Static NE*, *No Cycling*, $ICP_{H11\succ H00}$, $ICP_{H10\succ H00}$, and $ICP_{L11\succ L00}$. The non-negativity constraints $w_{00}^H > 0$ and $w_{10}^L > 0$ imposes the lower and upper bound of r, respectively.

SolE: for $\tilde{r} < r \leq r3 \doteq -\frac{4\left(P+2Q^2\right)}{(1-2Q)^2(2P(Q+1)-Q)}$, the solution is the one shown in Lemma 4 - Part (ii).

SolF: for $r > r3$, the solution is the one shown in Lemma 4 - Part (iii).

Solutions given $T = 1$:

Sol1: The optimal contract for $r \leq r1$ is *Pure JPE* shown in Part (i) of Lemma 3.

Sol2: Part (ii) of Lemma 3 (the one with $w_{11}^L > 0$) specifies the contract for $r1 < r \leq r2 \doteq \frac{4PQ^2+\sqrt{(4(P+1)Q^2-5P+2Q)^2-16P(P(1-2Q)^2-4Q)}-5P+4Q^2+2Q}{8P}$. The maintaining assumption $P < \frac{Q}{2(1+Q)}$ assures the existence of the interval $(r1, r2]$.

Sol3: for $r2 < r \leq -\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)}$, the solution is

$w_{10}^H = \frac{4\left(P\left(-4Q^2(r-1)-4Q+(r+1)(4r+1)\right)-2Q(2Qr+r+2)\right)}{P(P(4Q^2-1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-1)(2Q-2r^2+1))}$,

$w_{11}^H = -\frac{2(2P+1)(2PQ+P+2Qr)}{P(P(2Q+1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-2r^2+1))}$,

$w_{11}^L = -\frac{2(2P+1)(2PQ+P+2Qr)}{P(P(2Q+1)(4Q^2(r-2)-(r+2)(4r+1))+4Q(2Q-2r^2+1))}$. The binding constraints are $w_{00}^H = w_{00}^L = w_{10}^L = 0$, *Mutual Monitoring*, $ICP_{H11\succ H00}$, and $ICP_{L11\succ L00}$. The non-negativity $w_{10}^H > 0$ and $ICP_{H10\succ H00}$ impose the lower and upper bound of r, respectively.

Sol4: for $-\frac{2(P-Q)(4PQ-1)}{P(4Q^2-1)} < r \leq \tilde{r}$, a feasible solution is

$w_{00}^H = \frac{P^2\left(2-8Q^2\right)-P\left(4Q^2(r+2)-4Q(r+1)+r(4r+5)\right)+4Q(r+1)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,

$w_{10}^H = -\frac{2\left(r\left(P\left(4Q^2+5\right)-2Q\right)+2(P+1)\left(4PQ^2+P-2Q\right)+4Pr^2\right)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,

$w_{11}^H = \frac{r\left(2Q-P\left(4Q^2+5\right)\right)-2(P+1)\left(4PQ^2+P-2Q\right)-4Pr^2}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$,

$w_{11}^L = -\frac{2(2P+1)(P+Qr)}{P(P(8Q^3r-4Q^2(r+2)+2Qr-(r+2)(4r+1))+2Q(-2Qr-2r^2+r+2))}$.

$w_{00}^H > 0$ and *Pareto* require $r > r3$ and $r < \tilde{r}$, respectively. The solution is optimal if the parameters satisfy $\tilde{r} \leq \frac{4P(2Q+1)}{(2Q-1)(2P(Q+1)-Q)}$ (the binding constraints are $w_{00}^L = w_{10}^L = 0$, *Mutual Monitoring*, $ICP_{H11\succ H00}$, $ICP_{L11\succ L00}$, and $ICP_{H10\succ H00}$.) Setting $w_{10}^L = 0$ is no longer optimal if the condition is violated (i.e., $\tilde{r} > \frac{4P(2Q+1)}{(2Q-1)(2P(Q+1)-Q)}$), in which case, depending the value of r, either w_{00}^H or *Self-Shirking* replaces $w_{10}^L = 0$ as the binding constraint in the optimal solution. We verify that Part (iii) of the proposition holds for both cases as well.

Sol 5: for $\tilde{r} < r \leq -\frac{P^2-4PQ^3+4(P(3P+2)+1)Q^2-3PQ}{4P(P-Q)}$, the optimal solution is
$w_{00}^H = \frac{P^2(2Q-1)\left(12Q^2+4r+1\right)+4PQ\left((1-2Q)^2+2r\right)-8Q^2}{2PQ(2Q+1)(4PQ^2+4Pr+P-2Q)}$, $w_{11}^H = \frac{2P+1}{4PQ^2+4Pr+P-2Q}+w_{00}^H$, $w_{11}^L = \frac{2P+1}{4PQ^2+4Pr+P-2Q}$, $w_{10}^H = \frac{P^2\left(4Q^2+1\right)\left(12Q^2+4r+1\right)+2PQ(4Q(Q(4Q-3)+2r+1)-1)-16Q^3}{PQ(2Q+1)^2(4PQ^2+4Pr+P-2Q)}$. The binding constraints are $w_{00}^L = w_{10}^L = 0$, *Self-Shirking*, $ICP_{H11\succ H00}$, $ICP_{L11\succ L00}$, and *Pareto*. $ICP_{H11\succ H10}$ and *Mutual Monitoring* require the upper and lower bound on r, respectively.

Sol 6: for $r > -\frac{P^2-4PQ^3+4(P(3P+2)+1)Q^2-3PQ}{4P(P-Q)}$, the solution is $w_{00}^H = \frac{1-\frac{2(2P+1)Q}{4PQ^2+4Pr+P-2Q}}{2P}$, $w_{01}^H = \frac{\frac{\left(8P^2-2\right)Q}{4PQ^2+4Pr+P-2Q}-\frac{P}{Q}+1}{2P}$, $w_{10}^H = \frac{-\frac{2(2PQ+Q)^2}{4PQ^2+4Pr+P-2Q}+P+Q}{2PQ}$, $w_{11}^H = \frac{P(4P+4(Q-1)Q+4r+3)-4Q}{2P(4PQ^2+4Pr+P-2Q)}$, $w_{11}^L = \frac{2P+1}{4PQ^2+4Pr+P-2Q}$. The binding constraints are $w_{00}^L = w_{10}^L = w_{01}^L = 0$, *Mutual Monitoring*, $ICP_{H11\succ H00}$, $ICP_{L11\succ L00}$, $ICP_{H11\succ H10}$, and *Pareto*. The non-negativity of w_{01}^H imposes the lower bound on r.

Endogenizing $T = \{0, 1\}$: Having solved the optimal solution given $T = \{0, 1\}$, endogenizing the choice of T is conceptually straightforward: comparing the expected payments between the two program. We sketch the three main steps of the comparison and skip the tedious algebra. First, the relevant solutions for $r \leq \tilde{r}$ are SolA to SolD (given $T = 0$) and Sol1 to Sol4 (given $T = 1$). Second, tedious but straightforward algebra shows that Sol1 to Sol4 are overall optimal as long as they are optimal given $T = 1$. Finally, Sol5 and Sol6 of $T = 1$ are never the overall optimal contract. The argument is similar to that in Claim 4 of the proof of Proposition 3: for parameters over which the two solutions are optimal given $T = 1$, they satisfy all the potentially binding constraints of program with $T = 0$. ∎

Proof of Corollary 3. For $T = 1$, recall from Proposition 1 that the (old) *No Cycling* constraint does not bind and that all solutions in Proposition 1 satisfy strategic complements, i.e., $U(1,1) - U(0,1) > U(1,0) - U(0,0)$. It is easy to show that strategic complements and the *Pareto Dominance* constraint together imply $2U(1,1) > U(0,1) + U(1,0)$. Therefore, if $T = 1$, agents will not collude on playing any strategies that involve only *work* and *shirk*. This proves the part (i).

For $T = 0$, it is straightforward to plug in the new cycling constraint, investigate the Lagrangian, and verify the first part of Corollary 4 - (ii). The second part of (ii) follows from the observation that the (old) *No Cycling* constraint is more restrictive than the new cycling constraint. Note that the two agents are willing to collude on playing the off-diagonal *work* and *shirk* only if their *joint* payoff satisfies $U(1,0) + U(0,1) > 2U(1,1)$. Given $T = 0$, $U(1,0) + U(0,1) > 2U(1,1)$ implies $U(1,0) - U(1,1) > U(1,1) - U(0,1) \geq 0$, where the last inequality follows from the *Static NE* constraint that must hold if $T = 0$. The observation $U(1,0) - U(1,1) > U(1,1) - U(0,1) \geq 0$ further implies $U(1,0) > U(1,1) > U(0,1)$, which together with the time value argument, suggests that our *No Cycling* constraint provides strictly higher continuation payoff for the shirking agent in (0,1) (hence more costly for the principal to break) than any collusive strategy having agents randomizes between *work* and *shirk*.

Part (iii) follows directly form parts (i) and (ii) of the Corollary. ■

Appendix B: Stationary Collusive Strategies

In addition to allowing for a joint bonus floor, another difference between our paper and Kvaløy and Olsen's (2006) is that they allow the agents to play correlated strategies, while we do not. If we allow for such correlated strategies (while keeping the ability to commit to a bonus floor), we would rewrite the (No Cycling) constraint as follows:

$$\frac{1+r}{r}\left[\pi(1,1;w)-1\right] \geq \pi(0,1;w) + \frac{[\pi(1,0;w)-1+\pi(0,1;w)]/2}{r}.$$

The second term of the right hand side of the equation is the continuation payoff for the shirking agent from indefinitely playing either $(1,0)$ or $(0,1)$ with equal probability in each period. Similar argument as Lemma 1 shows that, if we restrict the agents collusive strategy to be stationary as in Kvaløy and Olsen (2006),[27] the contract is collision proof if it satisfies both the (No Joint Shirking) and the modified cycling constraint above. The corollary below shows that the modified cycling constraint does not qualitatively change the nature of the optimal contract studied in the main text.

Corollary 3 *If we confine attention to stationary collusive strategies,*

(i) Given $T = 1$, the wage contract and the cutoffs are same as in Proposition 1.

(ii) Given $T = 0$, the wage contract and the cutoffs are characterized by the same binding constraints as those in Proposition 2. The expected wage payment, $\pi(1,1)$, is weakly lower than that in Proposition 2.

(iii) The overall optimal contract is same as Proposition 3 (the closed-form of cutoffs r^D and r^H are different).

Part (i) shows that the way we formulate cycling constraint is immaterial if the principal choses to explore mutual monitoring (given $T = 1$). The reason is that exploring mutually

[27] Stationary (symmetric) collusive strategies are characterized in Kvaløy and Olsen (2006) as probabilities $(a, b, b, 1-a-2b)$ on effort combinations $(1,1), (1,0), (0,1)$, and $(0,0)$, respectively.

monitoring endogenously requires the wage contract to satisfy strategic complements in payoffs, and the endogenous complementarity further makes the two agents' total payoff is the highest on the equilibrium path and therefore collusion-proof by definition.

Part (ii) of Corollary 4 follows from observation that the *No Cycling* constraint formulated in the main text is more restrictive than the (correlative) cycling strategy above. Intuitively, because of the time value, the *Cycle* strategy formatted in the text gives the shirking agent a higher continuation payoff (hence more difficult for the principal to upset) than what he would receive under the correlated collusive strategy. As a result, the agents earns more rent by colluding on the *Cycle* strategy than by colluding on the correlated collusive strategy.

Given the alternative cycling constraint, it is not surprising that some of the cutoffs of the optimal contract are different. However, as shown in Corollary 4 - Part (iii) shows, the way that the optimal contract depends on the discount rate is same as that in the main text. The main insight is that that having a positive bonus floor ($w_{LL} > 0$) is still useful for the same reason as in the main text: either encourages mutual monitoring or mitigates agents' collusion problem. Finally, it is worth noting that, unlike in Kvaløy and Olsen's (2006), relational contracts can be sustained for all discount rates r because of the commitment of the bonus floor.

Appendix C: Private Communications From Agents to The Principal

Following the multi-agent relational contract literature, we assume away the communication between the agents and the principal in the main model. We now discuss a setting in which each agent privately sends a message to the principal about his opponent's effort. Continuing with the setting in the Extension in which two agents perfectly observe each other's effort choice $e^i = \{0, 1\}$, and e^i, e^j together affects the realization of the objective team measure $y \in \{H, L\}$ so that $p_1 = \Pr(y = H|e^i = e^j = 1) > p = \Pr(y = H|e^i \neq e^j) > p_0 = \Pr(y = H|e^i = e^j = 0)$. We allow agent i sends a private message $m_j^i = \hat{a}^j \in \{0, 1\}$ to the principal about j's effort choice and the report m_j^i does not have to be truthful. Agent i's pure strategy in a given stage game is characterized by a triple $< e^i, \{m_j^i(e^i, e^j = 1), m_j^i(e^i, e^j = 1)\} >$ that prescribes his effort choice e^i, and his report m_j^i after observing $e^j = 1$ and $e^j = 0$, respectively. Given the binary support of each of the three elements of agent i's strategy, the normal form of the stage game is summarized by a 8×8 payoff matrix (in contrast to a 2×2 payoff matrix of the main model). That is, agents' strategy spaces become larger because of the sequential nature of the stage game.

Denote by $\mathbf{w} = \{w_{\hat{e}^i, \hat{e}^j}^y\}$ the symmetric wage contract that Agent i receives if he reports $\hat{e}^j$ about j's effort and receives a report $\hat{e}^i$ from Agent j, and the team output is y. We assume the principal can credibly commit honoring the contract $\mathbf{w}$ given the privately communicated messages $(\hat{a}^i, \hat{a}^j)$. This assumption is necessary because we would otherwise run into technical difficulties of private monitoring in repeated games (e.g., Mailath and Samuelson, 2006). In particular, since the agents' reports are private, relational contract is hard to sustain when one agent who is unfairly paid cannot distinguish whether it is because the other agent has deviated in terms of mis-reporting or because the principal deviated from honoring the contract. Such private monitoring breaks the link between agents' current period mis-reporting and future play, and in general, deviations cannot be unambiguously detected. Any equilibrium will thus involve history dependent (i.e., non stationary) strategies and punishments occurs on the equilibrium path. We are unaware of technologies that are able to incorporate such non-stationary, private monitoring into relational contracting. Assuming that principal can credibly honor the contract $\mathbf{w}$ restores the agent's ability to detect and punish any deviations by the other agent, which then will occur only off-the-equilibrium. In particular, the following condition assures that agent i truthfully reports the other agent j's effort choice in the relational contract.

$$\mathbf{E}_y[w_{10}^y - w_{11}^y | e^i = e^j = 1] \leq \frac{\mathbf{E}_y[w_{11}^y | e^i = e^j = 1] - 1}{r} - \frac{\mathbf{E}_y[w_{00}^y | e^i = e^j = 0]}{r},$$

(Report Monitoring)

where the expectation $\mathbf{E}$ is taken over team measure y, the left-hand side (LHS) is the current period benefit from unilaterally misreport, and the RHS is the present value of future punishment. We know from the last term of the condition that, a deviation by agent i from truthful reporting, triggers agent j's punishment of reporting $\hat{e}^i = 0$ thereafter, and, therefore, reverting future play to the *punishment phase* in which $e^i = e^j = \hat{e}^i = \hat{e}^j = 0$ indefinitely.

In addition to mutually monitoring each other to assure truthful reporting, the agents can also mutually monitor each others' effort choice as in the main model. The following constraint parallels the mutual monitoring constraint in the main model.

$$\frac{1+r}{r}(\mathbf{E}_y[w_{11}^y | e^i = e^j = 1] - 1) \geq \mathbf{E}_y[\max\{w_{01}^y, w_{00}^y\} | e^i = 0, e^j = 1] + \frac{1}{r}\mathbf{E}_y[w_{00}^y | e^i = e^j = 0].$$

(Effort Monitoring)

The first term on the RHS is the difference between the constraint above and the (Mutual Monitoring) in the main model. The maximum operator $\max\{w_{01}^y, w_{00}^y\}$ takes into account the double deviation by the same agent as well as the sequential nature of the stage game. In particular, agent i who unilaterally deviated from *work* can also deviate in the reporting state to maximize his current period payoff.

Introducing agents' private reporting also makes the agents' potential collusive strategy space larger, for the agent can collude both on the effort choice dimension as well as the subsequent reporting stage. The principal needs to prevent collusive strategies that involves such *joint deviation*. The potential binding (pure strategy) collusive strategies can be as follows.

Collusion 1 in which both agents from shirking, after which they both report 1 for each other. The following constraint is required to upset this collusive strategy.

$$\mathbf{E}_y[w_{10}^y|e^i = e^j = 0] + \frac{1}{r}\mathbf{E}_y[w_{00}^y|e^i = e^j = 0] \geq \frac{1+r}{r}\mathbf{E}_y[w_{11}^y|e^i = e^j = 0], \tag{No Shirk and over report}$$

where the LHS is the payoff agent i receives from unilaterally deviates by reporting 0 for the other agent, which then triggers the continuation game into a punishment phase in which both agents *shirk* and report 0 indefinitely, i.e., $e^i = e^j = \hat{e}^i = \hat{e}^j = 0$. Note that the punishment supporting collusive strategy is different from playing (*work, work*) as we have seen in the main model where private communication is blocked. This is because playing (*work, work*) is no longer a stage game equilibrium when each agent correctly anticipates that, in the punishment phase, his effort choice will be under-reported subsequently by the other agent.

Collusion 2 in which both agents shirk and then take turn to report 1 for each other. That is, the reported effort pair $(\hat{a}^i, \hat{a}^j)$ is $(1, 0)$ for even periods and $(0, 1)$ for odd periods. The following condition upsets this strategy.

$$\mathbf{E}_y[w_{00}^y|e^i = e^j = 0] + \frac{1}{r}\mathbf{E}_y[w_{00}^y|e^i = e^j = 0] \geq \frac{(1+r)^2}{r(2+r)}\mathbf{E}_y[w_{01}^y|e^i = e^j = 0] + \frac{(1+r)}{r(2+r)}\mathbf{E}_y[w_{10}^y|e^i = e^j = 0]. \tag{No Shirking but Cycling Report}$$

The LHS is the payoff agent i receives from unilaterally deviates by reporting 0 for the other agent when he is supposed to report 1 instead.

Collusion 3 in which both agents shirk and then report each others' shirking truthfully. The contract needs to satisfy the following Pareto dominant condition to upset the shirking strategy.

$$\mathbf{E}_y[w_{11}^y|e^i = e^j = 1] - 1 \geq \mathbf{E}_y[w_{00}^y|e^i = e^j = 0]. \tag{Pareto Dominant}$$

Unlike (No Joint Shirking) in the main model in which an agent who *unilaterally* deviates from shirking is punished only in the future plays, any deviation of effort choice is immediately punished by the other agent in the current period by mis-reporting the observed high effort to a low effort. Because agents' incentive to deviate unilaterally in the effort choice is weakened, it is more cost efficient for the principal to implement the Pareto-dominant condition above.

Collusion 4 in which agents take turns to work (i.e., cycling effort choices), after which they both report 1 for each other. The collusion proof constraint is:

$$\mathbf{E}_y[w_{10}^y|e^i = 0, e^j = 1] + \frac{\mathbf{E}_y[w_{00}^y|e^i = e^j = 0]}{r} \geq \frac{(1+r)^2\mathbf{E}_y[w_{11}^y|e^i \neq e^j]}{r(2+r)} + \frac{(1+r)\mathbf{E}_y[w_{11}^y - 1|e^i \neq e^j]}{r(2+r)}. \tag{No Cycling effort and over report}$$

The LHS is the payoff the shirking agent i receives from unilaterally deviating by reporting $e^j = 0$ when he is supposed to report $e^j = 1$ (while agent j still reports $\hat{a}^i = 1$ according to the collusive strategy) and triggering the punishment stage game in all future periods.[28]

Collusion 5 in which agents take turns to work and then truthfully report each other's effort. The collusion-proof constraint is:

$$\mathbf{E}_y[w_{00}^y|e^i = 0, e^j = 1] + \frac{\mathbf{E}_y[w_{00}^y|e^i = e^j = 0]}{r} \geq \frac{(1+r)^2}{r(2+r)}\mathbf{E}_y[w_{01}^y|e^i \neq e^j] + \frac{(1+r)}{r(2+r)}\mathbf{E}_y[w_{10}^y - 1|e^i \neq e^j], \tag{No Cycling effort}$$

where the LHS is the payoff the shirking agent i receives from unilaterally deviates by reporting $e^j = 0$ when he is supposed to report $e^j = 1$ and is then punished indefinitely in future plays.

[28] Although the principal can also induce the working agent j to deviate in the reporting stage, she optimally targets the shirking agent to save costs. The reason is the shirking agent's continuation payoff under the collusive strategy, which is the cost of deviation, is lower (than the working agent's) because of the time-value consideration. Meanwhile, the current period benefit of deviation is same across the two agent because the cost of effort is sunk by the time of reporting stage.

The following program formalizes the principal's optimization problem.

Program P': $\min_{\mathbf{w}} \pi(1,1)$

$s.t.$ (Report Monitoring), (Effort Monitoring),

(No Shirk and over report), (No Shirk and cycling report), (Pareto Dominant),

(No Cycling effort and over report), (No Cycling effort),

$w^{y}_{\hat{e}^i,\hat{e}^j} \geq 0.$

It is clear that the focus of the problem has been shifted from the principal's credibility consideration to assuring that (i) each agent truthfully report his opponent's effort, and (ii) agents will not collude, either in the effort choice stage or in the subsequent reporting stage (or both). We therefore view the problem is qualitatively different from what we have studied in the main model.