

Inequality Led Instability

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Abstract

During the last three decades, income inequality has risen considerably across many developed and developing countries. This rise has stimulated a debate among academic scholars and policymakers on whether widening income inequality may cause economic instability and how to cope with such instability. A commonly accepted policy prescription for breaking the link between income inequality and economic instability is to tighten the credit market and prevent debt accumulation to unsustainable levels, which is a pre-condition for instability. This paper offers some new insights into this debate, first, by proposing an internally consistent mechanism through which income inequality may lead to economic instability and second, by arguing that a policy proposal to tighten the credit market for breaking the link between inequality and instability may not always succeed and sometimes can even make things worst.

Keywords: Credit policy intervention, endogenous fluctuations, income inequality, income instability, resource misallocation;

JEL Classification: D31; E32; E44; O16; O41

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“As long as poverty, injustice, and gross inequality exist in our world, none of us can truly rest” – Nelson Mandela.

1 Introduction

Over the past few decades, income inequality has risen in most major economies, reaching levels not seen since the period immediately preceding the Great Depression. According to The World Social Report (2020), since 1980, income inequality has increased in all developed and many middle-income countries, touching the lives of more than 70% of the global population. Income inequality has been rising in all nine advanced economies members of the G-20.¹ The sharpest increase was in the US, where the Gini index of income inequality increased from 0.34 in 1985 to 0.42 in 2021. Among major emerging economies that are members of the G-20, the largest increase in income inequality occurred in China, Russia, and South Africa. Brazil is the only major emerging economy that experienced a moderate decline in income inequality. According to the OECD (2022) report, out of the 22 member countries of OECD, the Gini index of income inequality increased in 20 countries by varying amounts.² The average annual growth rate of the real per capita income across OECD countries between 1980 and 2021 was 1.6% per year, however, the income of the top 10% grew much faster than those of the poorest 10%, resulting in an increase of the Gini index of income inequality from 0.29 in the mid-1980s to 0.32 in 2021. Today, the average per capita income of the richest 10% of the population across OECD countries is ten times higher than the average per capita income of the poorest 10%. The same ratio was seven in the 1980s.³ Rising level of income inequality and its social, economic, and political implications are a widespread concern among researchers and policymakers. One such concern discussed in the literature is the inequality-led instability hypothesis.

The idea that an elevated level of income inequality may cause income instability is not new. In his writing on the causes of the Great Depression of 1929–33, Marriner Eccles⁴ wrote

“...a giant suction pump had by 1929–1930 drawn into a few hands an increasing portion of currently produced wealth. This served them as capital accumulations. But by

¹The G20 is the group of the world’s largest economies, including both industrialized and developing nations. G20 accounts for 80% of the gross world product, 65% of international trade, two-thirds of the global population, and roughly half the world’s land area.

²OECD stands for Organization for Economic Cooperation and Development and is made up world’s developed economies. As of 2017, the OECD member countries collectively comprised 62.2% of global GDP, and 80% of world trade and investment, with a collective population of 1.38 billion.

³This ratio varies widely across OECD countries. It is much lower than the OECD average in the Nordic and many Continental European countries but reaches around 10 in Italy, Japan, Korea, Portugal, and the United Kingdom, between 13 and 16 in Greece, Israel, Turkey, and the United States, and about 27 and 30 in Mexico and Chile.

⁴Marriner Eccles served as 7th Chairman of the Federal Reserve Board between 11/1934 and 01/1948.

taking purchasing power out of the hands of mass consumers, the savers denied themselves the kind of effective demand for their products that would justify a reinvestment of their capital accumulations in new plants. In consequence, as in a poker game where the chips were concentrated in fewer and fewer hands, the other fellows could stay in the game only by borrowing. When their credit ran out, the game stopped.”⁵.

Keynes (1936) also identified income inequality as one of the major destabilizing features of the capitalist system. In a similar vein, Galbraith (1954) highlights the “bad distribution of income” as being the first of “five weaknesses ... ensuing an economic disaster”. In more recent literature, Rajan (2010), Tridico (2012), Wisman (2013), Kumhof et al. (2015), Perugini et al. (2016), and Gu et al. (2019) among others argue that in the presence of high level of income inequality, low and middle-income households borrow excessively for maintaining the existing consumption levels. At some point, household indebtedness becomes unsustainable, eventually leading to economic instability, accompanied by a contraction in the real economy. Kumhof et al. (2015) reports that in the U.S., between 1920 and 1928, the top 5% income share increased from 27.5% to 34.8% leading to the increase in household debt-to-GDP ratio from 16.9%, to 37.1% and eventually causing the Great-Depression of 1929-33. Between 1983 and 2007, the top 5% income share increased from 21.8% to 33.8% leading to the increase of the household debt-to-GDP ratio from 49.1% to 98.0% and eventually causing the Great Recession of 2007-08. Central to this argument is the debt leverage channel through which the widening income inequality contributes to the excessive accumulation of debt, which is widely recognized as the ultimate cause of economic instability. Kumhof et al. (2015) formalized this line of argument by building a tractable theoretical model in which the economic crisis is the result of an endogenous and rational default decision on the part of bottom earners, who first accumulate the debt to unsustainable levels and then default on it dragging the economy into a crisis which is characterized by partial household debt defaults and an abrupt output contraction.⁶

This paper contributes to the inequality-led instability debate, first by proposing a new and internally consistent mechanism through which income inequality may lead to income instability and second by discussing how to cope with such instability. To describe how income inequality may lead to income instability, we consider and analyze a dynamic general equilibrium model with overlapping generations of agents. The proposed model is similar to one considered in Matsuyama (2004), Kikuchi and Vachadze (2015, 2018), and Vachadze (2018). The main modification we make in this paper is the introduction of heterogeneity among young and old agents with respect to their labor-endowed and entrepreneurial productivity. This way, we expose the impacts of income inequality on credit (miss)allocation among agents with different net worth and entrepreneurial productivity.

⁵This citation can be found on p. 76 in Reich (2010)

⁶Perugini et al. (2016) produced empirical evidence based on panel data of 18 OECD countries for 1970–2007 confirming a positive relationship between income inequality and private sector indebtedness.

Young agents inelastically supply the labor endowment and earn the labor income. They do not consume during the first period but have two options to transfer the current net worth into the second-period consumption. First, young agents may become financiers by lending their entire net worth in the competitive credit market. Second, young agents may become entrepreneurs by borrowing funds in the competitive credit market and setting up an investment project which produces capital.⁷ Produced capital can be later rented to the final commodity-producing firm in exchange for the rental rate paid in terms of the final commodity. Setting up an investment project is subject to a minimum investment requirement, and young agents cannot borrow any amount because of the limited pledgeability of profit (generated from an investment project) for debt repayment.⁸

Due to the heterogeneity of agents' labor endowment and entrepreneurial productivity, the pool of young agents is divided into four types: agents of type-11 are ones who have high net worth and are productive entrepreneurs; agents of type-10 have high net worth and are unproductive entrepreneurs, agents of type-01 have low net-worth and are productive entrepreneurs, and agents of type-00 have low net-worth and are unproductive entrepreneurs. Agents of type-11 always have a competitive advantage for attracting external funding for their entrepreneurial activity because they rely less on external financing and the investment project are highly profitable. The exact opposite is true about agents of type-00 who depend more on external funding because of low net worth, and the investment project they set up is not highly profitable. However, the relative competitiveness of agents of type-01 and type-10 depends on income and income inequality. In particular, we find that credit will flow to unproductive entrepreneurs when income and income inequality are at the intermediate level. Such flow negatively affects overall entrepreneurial productivity as it changes the composition of the pool of productive and unproductive agents who become entrepreneurs. As a result, capital formation slows down, leading to an economic downturn. At the same time, an economic slowdown creates a "cleansing effect" on aggregate entrepreneurial productivity by preventing high-net-worth but unproductive agents from accessing the credit market again and improving overall entrepreneurial productivity. As a result, there is an acceleration in capital accumulation which leads to an economic boom. In other words, we describe a mechanism through which economic success may breed failure, and the failure can become a precondition for success so that the boom-bust cycles in per capita income may become endogenous. This argument is consistent on the one hand with the empirical findings of Bresnahan and Raff (1991), and Davis and Haltiwanger (1992), who found a counter-cyclical productivity growth using the

⁷We interpret capital broadly to include physical capital, human capital, or any other reproducible asset used as a factor in production.

⁸Since young agents do not consume while young, it follows that the first-period labor income also represents the entrepreneur's net worth. This, combined with the assumption of the Cobb–Douglas production function, implies that the agent's net worth, wage rate, labor income, and per capita income are always proportional to each other. That's why we use these terms interchangeably unless it causes any confusion.

firm-level data, and on the other hand with the empirical findings of Cingano (2014) who argue that a sharp increase in income inequality across OECD countries slowed down economic growth and the economic recovery from recessions by slowing human and physical capital accumulation.

The rest of the paper is organized as follows. In Section 2, we propose a basic model and analyze agents' optimal behavior. In Section 3, we consider a benchmark case with no income inequality and identify conditions (i.e., parameter restrictions) under which economic stability is guaranteed in the absence of income inequality. In the rest of the paper, we keep the same parameter restrictions and discuss the case with the presence of income inequality. In Section 4, we consider the case when only productive entrepreneurs compete for credit. In Section 5, we allow the participation of all four types of agents in the credit market. In the latter case, we derive conditions under which unproductive agents will borrow and set up an investment project in equilibrium. In Section 6, we discuss how credit allocation affects capital formation and the equilibrium income dynamics. In particular, we present scenarios under which per capital income converges to regular or chaotic cycles instead of displaying monotonic convergence to a unique steady state as in the benchmark case. In Section 7, we discuss the model's main empirical and policy implications. Finally, in Section 8, we summarize the paper and conclude. All remaining proofs are located in the Appendix.

2 Model set up

In this section, we set up and then analyze a simple model that will allow us to demonstrate how and why income inequality may lead to resource misallocation, income instability, and the cyclical behavior of the economy. Central to the hypothesis is: (1) the heterogeneity of agents with respect to their labor endowment and entrepreneurial productivity, (2) the presence of the borrowing constraint, and (3) a minimum investment requirement for running an investment project that produces capital.

We consider a discrete-time economy populated by an infinite sequence of two-period lived overlapping generations. Agents during the first and second periods of life are referred to as *young agents* and *old agents*, respectively. There is a single competitive firm that combines labor and capital inputs to produce a single final commodity, the *numeraire*, according to a Cobb-Douglas production technology. Produced output at time t is $y_t = Ak_t^\alpha$, where k_t is the capital per capita, while parameters $A > 0$ and $\alpha \in (0, 1)$ represent the total factor productivity and the capital share in production respectively. Labor and capital inputs, which are supplied by young and old agents respectively, are paid according to the marginal product rule, i.e., $w_t = A(1 - \alpha)k_t^\alpha$ is the wage rate and $f'(k_t) = A\alpha k_t^{\alpha-1}$ is the rental rate of capital.

Heterogeneity in labor endowment: In each period, a young generation of measure one is born. Half of these agents are endowed with $e_0 = 1 - e$ units of labor,

Agent Type	Labor Endowment	Entrepreneurial Productivity	Type Size
Type-11	$e_1 = 1 + e$	p_1	0.25
Type-10	$e_1 = 1 + e$	p_0	0.25
Type-01	$e_0 = 1 - e$	p_1	0.25
Type-00	$e_0 = 1 - e$	p_0	0.25

Table 1: Labor endowment and entrepreneurial productivity of each of four types of agents.

and the other half are endowed with $e_1 = 1 + e$ units of labor. These young agents supply the labor endowment inelastically and receive the wage incomes, $e_0 w_t$ and $e_1 w_t$, respectively. Under this specification of the distribution of labor endowment, the aggregate labor supply is always normalized to unity, and the Gini index of labor income inequality is always $\frac{e}{2}$. This implies that by varying the parameter $e \in [0, 1)$, we can trace out different degrees of income inequality without affecting the aggregate labor supply.⁹

Young agents have two options to transfer the current labor income, $e_m w_t$ for $m = 0, 1$, into the second-period consumption.¹⁰ First, a young agent may become a financier by lending the entire labor income to the competitive credit market at the gross rate of return r_{t+1} and consume

$$c_{t+1} = e_m w_t r_{t+1} \quad (1)$$

units of the final commodity during the second period. Second, a young agent may become an entrepreneur by running an investment project which requires a minimum of one unit of the final commodity for investment at time t and which transforms the invested $i_t \geq 1$ units of the final commodity into $p_n i_t$, for $n = 0, 1$, units of physical capital at time $t + 1$.¹¹

Heterogeneity in entrepreneurial productivity: Young agents are heterogeneous with respect to their entrepreneurial productivity, i.e., the ability to produce capital from an investment project. Half of the young agents are productive entrepreneurs with an entrepreneurial productivity p_1 , and the other half of young agents are unproductive entrepreneurs with entrepreneurial productivity, p_0 . Without loss of generality, we assume that the relative productivity, which represents the heterogeneity of entrepreneurial productivity, satisfies $p = \frac{p_0}{p_1} \in [0, 1]$. Young agents' labor endowment and entrepreneurial productivity are perfectly observable and independent from each other. I.e., all market participants know the labor endowment and entrepreneurial

⁹Alternatively, e_0 and e_1 can be interpreted as efficiency units of labor, and w_t can be interpreted as wage per efficiency unit of labor.

¹⁰Since young agents do not consume during the first period, the current labor income also represents the young agent's net worth.

¹¹As in Kikuchi and Vachadze (2015) and Vachadze (2018) young agents are subject to a minimum investment requirement, the value of which we normalize to unity.

productivity of a given agent, and the proportions of young agents with high and low entrepreneurial productivity remains the same no matter what the agent's labor endowment is. This means that there are four different types of agents with equal sizes at any moment. Agents of type-11 and type-10 have high net worth but are productive and unproductive entrepreneurs, respectively. Agents of type-01 and type-00 have low net worth but are productive and unproductive entrepreneurs, respectively. The Labor and entrepreneurial productivity of each of the four types of agents is summarized by Table 1, with the first index indicating the net worth and the second telling the entrepreneurial productively.

Young agent's incentive to becoming an entrepreneur: Suppose a young agent of type- mn , for $m, n = 0, 1$, becomes an entrepreneur by borrowing $i_t - e_m w_t$ units of final commodity at the rate r_{t+1} and by investing $i_t \geq 1$ units of final commodity in an investment project which produces $p_n i_t$ units of capital. The entrepreneur will rent the produced capital to the final commodity-producing firm for an exchange of $p_n i_t f'(k_{t+1})$ units of the final commodity to be received at time $t+1$. Rented capital fully depreciates within a period implying the following consumption level for an entrepreneur.

$$c_{t+1} = p_n i_t f'(k_{t+1}) - (i_t - e_m w_t) r_{t+1} = (p_n - \xi_{t+1}) i_t f'(k_{t+1}) + e_m w_t r_{t+1}, \quad (2)$$

where $\xi_{t+1} = \frac{r_{t+1}}{f'(k_{t+1})}$ denotes the relative borrowing rate as it represents the ratio of borrowing rate of one unit of the final commodity to the rental rate of one unit of capital. Direct comparison between (1) and (2) implies that a young agent strictly prefers to become an entrepreneur when $\xi_{t+1} < p_n$ and is indifferent between becoming an entrepreneur or a financier when $\xi_{t+1} = p_n$. If $\xi_{t+1} > p_n$, a young agent prefers to become a financier.

Young agent's ability to become an entrepreneur: If $i_t \in (0, 1)$ then the young agent cannot set up an investment project, cannot produce capital, and thus cannot consume during the second period.¹² Young agents decide how much to borrow and how much to invest by maximizing second-period consumption. While making the optimal decision, young agents face the borrowing constraint because of a limited pledgeability of the entrepreneur's future income for debt repayment. A young agent can borrow and set up an investment project when

$$i_t \geq 1 \quad \text{and} \quad (i_t - e_m w_t) r_{t+1} \leq (1 - \lambda) p_n i_t f'(k_{t+1}) \Leftrightarrow (i_t - e_m w_t) \xi_{t+1} \leq (1 - \lambda) p_n i_t. \quad (3)$$

The first inequality represents the minimum investment requirement, while the second represents the borrowing constraint. As indicated by the second inequality, the maximum loan size a young agent may get while becoming an entrepreneur is $(i_t - e_m w_t) r_{t+1}$ which is limited by $1 - \lambda \in (0, 1)$ fraction of entrepreneur's second-period income. Justifications for the limited pledgeability of entrepreneurs' future income can be numerous,

¹²Beyond the fixed investment cost, justifications for a minimum investment requirement include a fixed cost for entrepreneurs to acquire know-how in management, marketing and other areas of the organization.

including the asymmetry of information between borrowers and lenders, limitations of legal and financial institutions, and implementation of credit policy (such as liquidity support, liability guarantees, minimum down payment standard, etc.). Under such formulation, at most $1 - \lambda$ fraction of the next period's income can be externally financed or the down payment on debt used to fund the investment project has to be at least λ fraction of the future profit. I.e., by varying parameter $\lambda \in [0, 1]$, we can trace out all degrees of tightness of the credit market and access the impact of credit policy change on the income inequality and income instability nexus.

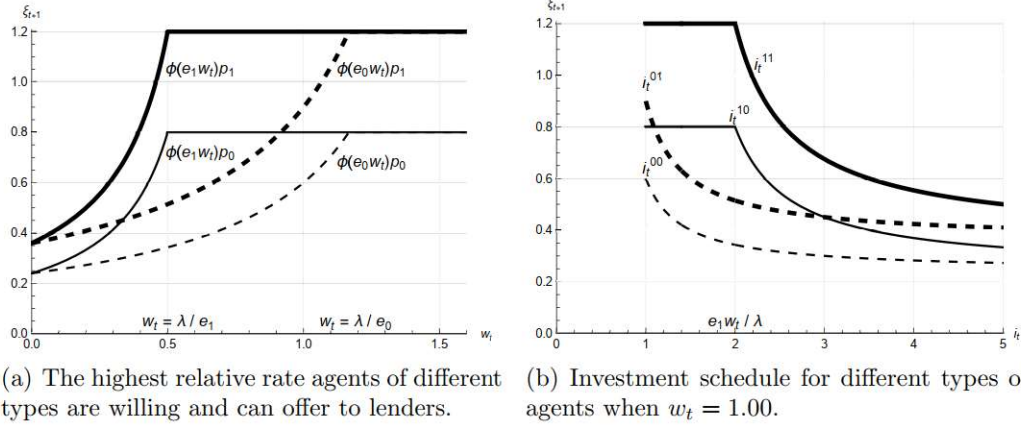


Figure 1: Both figures are constructed when $\lambda = 0.70$, $e = 0.40$, $p_1 = 1.20$, and $p_0 = 0.80$. All figures presented in this paper are plotted using Mathematica, version 11.3. The code is available upon request.

Young agent's optimal decision: As discussed above, an agent of type- mn

- (a) strictly prefers to become a financier when $\xi_{t+1} > p_n$.
- (b) is indifferent between becoming an entrepreneur or a financier when $\xi_{t+1} = p_n$,
- (c) strictly prefers to become an entrepreneur when $\xi_{t+1} < p_n$,

When an agent becomes a financier, she lends the entire wage income to the competitive credit market. If $\xi_{t+1} \leq p_n$, then it follows from (2) and (3) that entrepreneur's optimization problem can be written as

$$\max i_t \quad \text{subject to} \quad i_t \geq 1 \quad \text{and} \quad (i_t - e_m w_t) \xi_{t+1} \leq (1 - \lambda) p_n i_t. \quad (4)$$

One can easily verify that the solution to the above optimization problem is

$$i_t = \begin{cases} \infty & \text{if} \quad \xi_{t+1} \leq (1 - \lambda) p_n \\ \frac{e_m w_t \xi_{t+1}}{\xi_{t+1} - (1 - \lambda) p_n} & \text{if} \quad (1 - \lambda) p_n < \xi_{t+1} \leq \phi(e_m w_t) p_n, \end{cases} \quad (5)$$

where

$$\phi(x) = \begin{cases} \frac{1-\lambda}{1-x} < 1 & \text{if } x < \lambda \\ 1 & \text{if } x \geq \lambda. \end{cases} \quad (6)$$

Two important observations should be highlighted here. First, if $\xi_{t+1} \leq (1-\lambda)p_n$, a young will maximize the second-period consumption by borrowing and investing an unlimited amount because of the lack of a borrowing constraint. Second, $\xi_{t+1} = \phi(e_m w_t)p_n$ is the highest relative rate at which a young agent of type- mn is willing and can borrow enough to satisfy the minimum investment requirement. This implies that for $e_m w_t \in (0, \lambda) \Leftrightarrow \phi(e_m w_t) < 1$, a young agent will strictly prefer to become an entrepreneur, but she will be able to do so only for $(1-\lambda)p_n < \xi_{t+1} \leq \phi(e_m w_t)p_n$. At the same time, the young agent's borrowing constraint will not bind for $e_m w_t \geq \lambda \Leftrightarrow \phi(e_m w_t) = 1$. As a result, a young agent will be willing and able to become an entrepreneur for any $(1-\lambda)p_n < \xi_{t+1} \leq p_n$.

Figure 1(a) visualizes the relationship $w_t \mapsto \phi(e_m w_t)p_n$ for all agents. As the figure indicates, the high net wealth and highly productive entrepreneurs can offer the highest relative rate to lenders, and the low net worth and unproductive entrepreneurs may offer the lowest relative rate to lenders. However, the wage rate and income distribution will determine who among agents of type-01 and type-10 will have a comparative advantage in becoming an entrepreneur. In particular, an increase in income inequality will increase the competitiveness of agents of type-10 compared to the agent of type-01. This happens because an increase in income inequality (i.e., an increase of parameter e) causes an increase in the net worth of agents of type-10 and a simultaneous reduction of the net worth of agents of type-01. However, the relationship $w_t \mapsto \phi(e_m w_t)p_n$ is non-monotonic for a given level of income inequality. As indicated by Figure 1(a), agents of type-10 have a comparative advantage over agents of type-01 in obtaining credit when $\phi(e_1 w_t)p_0 > \phi(e_0 w_t)p_1$ and this can happen only for intermediate values of income. Figure 1(b) visualizes the investment schedule for different types of agents when $w_t = 1.00$. As the figure indicates, increase of i_t reduces agent's ability to compete for credit because $i_t \mapsto \phi\left(\frac{e_m w_t}{i_t}\right)p_n$ is a decreasing function for $i_t > \frac{e_m w_t}{\lambda}$.

3 Benchmark case with no income inequality

In this section, we discuss the case when there is no income inequality, $e = 0$, so that everybody receives the same income, w_t , and the pool of young agents is divided into productive and unproductive agents with equal sizes. For notational convenience, productive and unproductive agents will be referred to as agents of type-1 and type-0, respectively.

Suppose productive entrepreneurs are the only ones who run investment projects. Then the relative borrowing rate is $\xi_{t+1} = \phi(w_t)p_1$. At such a rate, the investment made by

	ξ_{t+1}	i_t^1	ϵ_t^1	i_t^0	ϵ_t^0
$w_t \in (0, \frac{1}{2})$	$\frac{(1-\lambda)p_1}{1-w_t}$	1	w_t	0	0
$w_t \in [\frac{1}{2}, 1 - \frac{p}{2}]$	$2(1-\lambda)p_1$	$2w_t$	$\frac{1}{2}$	0	0
$w_t \in (1 - \frac{p}{2}, \lambda)$	$\frac{(1-\lambda)p_0}{1-w_t}$	$\frac{pw_t}{w_t+p-1}$	$\frac{1}{2}$	1	$\left(1 - \frac{1}{2} \frac{p}{w_t+p-1}\right) w_t$
$w_t \in [\lambda, \infty)$	p_0	$\frac{pw_t}{\lambda+p-1}$	$\frac{1}{2}$	$\frac{w_t}{\lambda}$	$\left(1 - \frac{1}{2} \frac{p}{\lambda+p-1}\right) \lambda$

Table 2: Relative borrowing rate and the investment made by productive and unproductive entrepreneurs when $\lambda \in (\frac{1}{2}, 1)$ and $p \in (\bar{p}, 1)$. \bar{p} is defined in Equation (10).

productive entrepreneurs and the size of productive agents who set up an investment project are

$$i_t^1 = \frac{w_t \xi_{t+1}}{\xi_{t+1} - (1-\lambda)p_1} = \begin{cases} 1 & \text{if } w_t \in (0, \lambda) \\ \frac{w_t}{\lambda} & \text{if } w_t \in [\lambda, \infty). \end{cases} \quad \text{and} \quad \epsilon_t^1 = \begin{cases} w_t & \text{if } w_t \in (0, \lambda) \\ \lambda & \text{if } w_t \in [\lambda, \infty). \end{cases} \quad (7)$$

This is only possible if $\lambda \leq \frac{1}{2}$ because the size of productive agents is $\frac{1}{2}$ and thus ϵ_t^1 must belong to the interval $(0, \frac{1}{2}]$. If $\lambda \in (\frac{1}{2}, 1]$, then the relative rate productive entrepreneurs will offer to lenders is

$$\xi_{t+1} = \begin{cases} \frac{(1-\lambda)p_1}{1-w_t} & \text{if } w_t < \frac{1}{2} \\ 2(1-\lambda)p_1 & \text{if } w_t \geq \frac{1}{2} \end{cases} \quad (8)$$

so that

$$i_t^1 = \begin{cases} 1 & \text{if } w_t \in (0, \frac{1}{2}) \\ 2w_t & \text{if } w_t \in [\frac{1}{2}, \infty). \end{cases} \quad \text{and} \quad \epsilon_t^1 = \begin{cases} w_t & \text{if } w_t \in (0, \frac{1}{2}) \\ \frac{1}{2} & \text{if } w_t \in [\frac{1}{2}, \infty). \end{cases} \quad (9)$$

In this case, the value of parameter p determines whether unproductive entrepreneurs can compete for credit. If $p_0 \leq \min\{1, 2(1-\lambda)\}p_1 \Leftrightarrow$

$$p \in (0, \bar{p}] \quad \text{where} \quad \bar{p} = \begin{cases} 1 & \text{if } \lambda \in (0, \frac{1}{2}) \\ 2(1-\lambda) & \text{if } \lambda \in [\frac{1}{2}, 1), \end{cases} \quad (10)$$

then only productive agents will set up an investment project and produce capital. As a result, the following period capital stock and the wage rate will be $k_{t+1} = p_1 w_t$ and $w_{t+1} = A(1-\alpha)(p_1 w_t)^\alpha = w_\infty^{1-\alpha} w_t^\alpha$ respectively, where $w_\infty = [A(1-\alpha)p_1^\alpha]^{\frac{1}{1-\alpha}}$ represents the steady state wage rate. It follows from $w_{t+1} = w_\infty^{1-\alpha} w_t^\alpha$ that w_∞ is a unique and globally stable steady state towards which the economy will converge monotonically for any initial value, w_0 .

If $p \in (\bar{p}, 1)$, then the equilibrium relative rate and the investments made by productive and unproductive agents while setting up an investment project are summarized by

Table 2. As the table indicates, the absence of income inequality, $e = 0$, does not guarantee the credit flow only to productive agents.¹³ Since the purpose of this paper is to describe a mechanism through the resource misallocation and income instability that occurs due to the presence of income inequality, for the rest of the paper, we keep the parameter restriction (10) so that the monotonic convergence to a unique and globally stable steady state is guaranteed in the absence of income inequality.

4 Credit flow to productive entrepreneurs

In this section, we discuss the case when there is income inequality between young agents, $e > 0$, and p is sufficiently low, so unproductive entrepreneurs cannot compete for credit.¹⁴ In the presence of income inequality, young agents of type-11 have higher net worth than young agents of type-01. This means a relatively tighter credit constraint for agents of type-01 compared to agents of type-11.

Credit flow to only agents of type-11: Suppose young agents of type-11 are the only ones who become entrepreneurs by offering the following relative rate to lenders, $\xi_{t+1} = \phi(e_1 w_t) p_1$. At such rate, the investment made by agents of type-11 and the size of agents of type-11 who become entrepreneurs should satisfy

$$i_t^{11} = \frac{e_1 w_t \xi_{t+1}}{\xi_{t+1} - (1 - \lambda) p_1} = \begin{cases} 1 & \text{if } w_t \in (0, \frac{\lambda}{e_1}) \\ \frac{e_1 w_t}{\lambda} & \text{if } w_t \in [\frac{\lambda}{e_1}, \infty). \end{cases} \quad \text{and} \quad \epsilon_t^{11} = \begin{cases} w_t & \text{if } w_t \in (0, \frac{\lambda}{e_1}) \\ \frac{\lambda}{e_1} & \text{if } w_t \in [\frac{\lambda}{e_1}, \infty), \end{cases} \quad (11)$$

so that $i_t^{11} \epsilon_t^{11} = w_t$ holds for any w_t . This is only possible if $\frac{\lambda}{e_1} \leq \frac{1}{4}$ so that $\epsilon_t^{11} \in (0, \frac{1}{4}]$. In the rest of the paper, we consider the parameter restriction $\frac{\lambda}{e_1} > \frac{1}{4} \Leftrightarrow e \in (0, \hat{e})$, where

$$\hat{e} = \begin{cases} 0 & \text{if } \lambda \in [0, \frac{1}{4}] \\ 4\lambda - 1 & \text{if } \lambda \in (\frac{1}{4}, \frac{1}{2}) \\ 1 & \text{if } \lambda \in [\frac{1}{2}, 1), \end{cases} \quad (12)$$

so that agents of type-11 are not able to absorb all available credit.¹⁵ By combining parameter restrictions, (10) and (12), for the rest of the paper, we focus our attention on the case when

$$(\lambda, p, e) \in \left(\frac{1}{4}, 1\right) \times (0, \bar{p}) \times (0, \hat{e}), \quad (13)$$

so that agents of type-11 are not the only ones who set up an investment project.

¹³This case with its full generality is considered and analyzed in Vachadze (2022).

¹⁴As we see later, when $e > 0$, then the credit will only flow to unproductive entrepreneurs if p is sufficiently high.

¹⁵If income inequality is sufficiently high, $e \in [\hat{e}, 1]$, then agents of type type-10 could not compete with agents of type-11. To guarantee the credit flow to agents of type-10, we restrict the income inequality as follows $e \in (0, \hat{e})$.

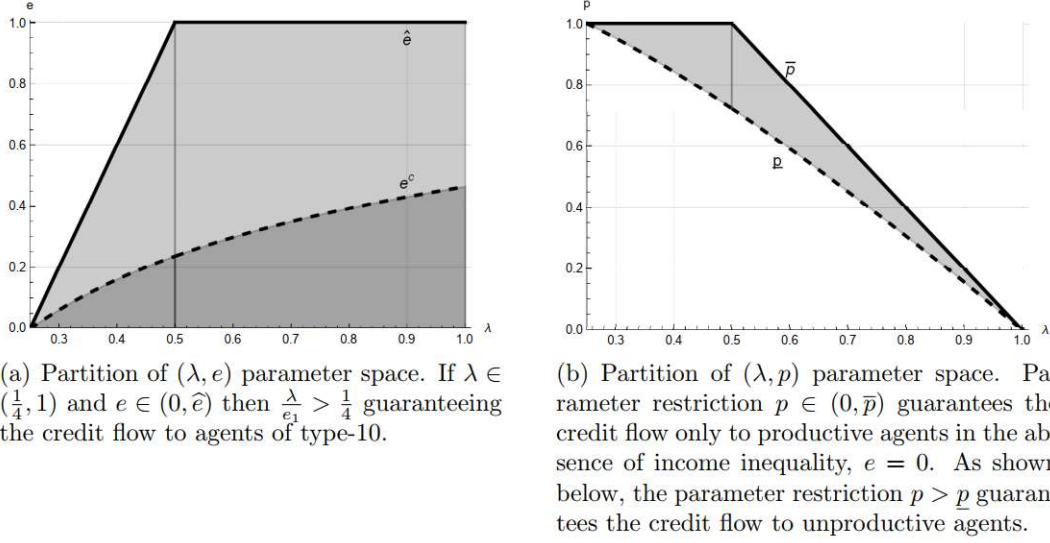


Figure 2: Partition of the parameter space.

	$\xi_{t+1} = \xi(w_t, e)p_1$	i_t^{11}	ϵ_t^{11}	i_t^{01}	ϵ_t^{01}
$w_t \in (0, \frac{1}{4})$	$\frac{1-\lambda}{1-e_1 w_t} p_1$	1	w_t	0	0
$w_t \in [\frac{1}{4}, \frac{e_1}{4e_0}]$	$\frac{4(1-\lambda)}{4-e_1} p_1$	$4w_t$	$\frac{1}{4}$	0	0
$w_t \in (\frac{e_1}{4e_0}, \frac{\lambda}{e_0})$	$\frac{1-\lambda}{1-e_0 w_t} p_1$	$\frac{e_1}{e_0}$	$\frac{1}{4}$	1	$w_t - \frac{e_1}{4e_0}$
$w_t \in [\frac{\lambda}{e_0}, \infty)$	p_1	$\frac{e_1 w_t}{\lambda}$	$\frac{1}{4}$	$\frac{e_0 w_t}{\lambda}$	$\frac{\lambda}{e_0} - \frac{e_1}{4e_0}$

Table 3: Relative borrowing rate and investments made by productive agents when $(\lambda, e) \in (\frac{1}{4}, \frac{1}{2}) \times (0, 4\lambda - 1) \Leftrightarrow \frac{1}{4} < \frac{e_1}{4e_0} < \frac{\lambda}{e_0} < \frac{1}{2e_0}$.

Credit flow to agents of type-10: How credit is allocated among agents of type-11 and type-10 when $\frac{\lambda}{e_1} > \frac{1}{4}$? We consider two cases separately: case with $(\lambda, e) \in (\frac{1}{4}, \frac{1}{2}) \times (0, 4\lambda - 1)$ and case with $(\lambda, e) \in [\frac{1}{2}, 1) \times (0, 1]$. If $(\lambda, e) \in (\frac{1}{4}, \frac{1}{2}) \times (0, 4\lambda - 1)$ then $0 < 4\lambda - e_1 \leq e_0 \Leftrightarrow 0 < \frac{\lambda}{e_0} - \frac{e_1}{4e_0} \leq \frac{1}{4}$ and thus $\epsilon_t^{01} \in (0, \frac{1}{4}]$. The relative borrowing rate and investments made by productive entrepreneurs are summarized on Table 3. If $(\lambda, e) \in [\frac{1}{2}, 1) \times (0, 1]$ then $\epsilon_t^{01} = \frac{1}{4}$ and the relative borrowing rate and investments made by productive entrepreneurs are summarized on Table 4.

It is worthwhile to mention here that under both configurations of parameter values, (a) $\epsilon_t^{11} \in (0, \frac{1}{4}]$, $\epsilon_t^{01} \in [0, \frac{1}{4}]$,

$$i_t^{11} = \frac{e_1 w_t \xi_{t+1}}{\xi_{t+1} - (1 - \lambda)p_1} \geq 1, \quad \text{and} \quad i_t^{01} = \frac{e_0 w_t \xi_{t+1}}{\xi_{t+1} - (1 - \lambda)p_1} \geq 1, \quad (14)$$

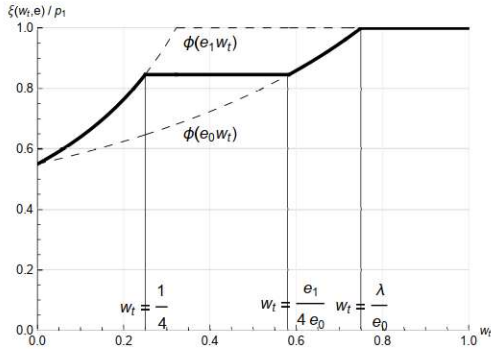
are continuous and piecewise smooth functions, and (b) aggregate investment is equal

	$\xi_{t+1} = \xi(w_t, e)p_1$	i_t^{11}	ϵ_t^{11}	i_t^{01}	ϵ_t^{01}
$w_t \in (0, \frac{1}{4})$	$\frac{1-\lambda}{1-e_1 w_t} p_1$	1	w_t	0	0
$w_t \in [\frac{1}{4}, \frac{e_1}{4e_0}]$	$\frac{4(1-\lambda)}{4-e_1} p_1$	$4w_t$	$\frac{1}{4}$	0	0
$w_t \in (\frac{e_1}{4e_0}, \frac{1}{2e_0})$	$\frac{1-\lambda}{1-e_0 w_t} p_1$	$\frac{e_1}{e_0}$	$\frac{1}{4}$	1	$w_t - \frac{e_1}{4e_0}$
$w_t \in [\frac{1}{2e_0}, \infty)$	$2(1-\lambda)p_1$	$2e_1 w_t$	$\frac{1}{4}$	$2e_0 w_t$	$\frac{1}{4}$

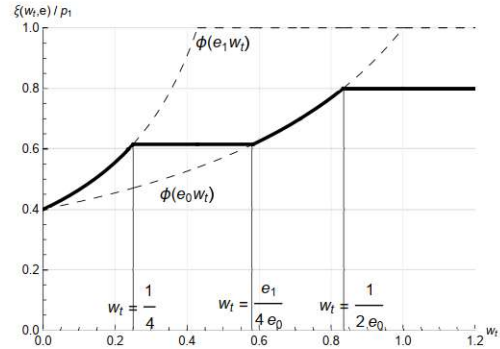
Table 4: Relative borrowing rate and investments made by productive agents when $(\lambda, e) \in [\frac{1}{2}, 1) \times (0, 1) \Leftrightarrow \frac{1}{4} < \frac{e_1}{4e_0} < \frac{1}{2e_0} < \frac{\lambda}{e_0}$.

to the aggregate saving, $\epsilon_t^{11} i_t^{11} + \epsilon_t^{01} i_t^{01} = w_t$, for any value of w_t . The equilibrium relative rates in these two cases are visualized in Figures 3(a) and 3(b), respectively.

These two figures deserve some interpretation. When $w_t \in (0, \frac{1}{4})$, then the competition for credit occurs only among agents of type-11 who only satisfy the minimum investment requirement while becoming entrepreneurs. An increase of w_t beyond the level $w_t = \frac{1}{4}$ allows all agents of type-11 to borrow and become entrepreneurs. These agents can increase their second-period consumption by increasing the investment above the minimum investment requirement and offering the same lending rate to financiers. An increase of the aggregate saving above the level $w_t = \frac{e_1}{4e_0}$ allows agents with low net worth to compete for credit and become entrepreneurs. This competition drives the equilibrium rate up again, which eventually stabilizes for $w_t \geq \frac{1}{e_0} \min\{\lambda, \frac{1}{2}\}$ at $\xi_{t+1} = \min\{1, 2(1-\lambda)\}p_1$.



(a) Case with $(\lambda, e) \in (\frac{1}{4}, \frac{1}{2}) \times (0, 4\lambda - 1)$. This figure is contracted when $(\lambda, e) = (0.45, 0.40)$.



(b) Case with $(\lambda, e) \in (\frac{1}{2}, 1) \times (0, 1)$. This figure is contracted when $(\lambda, e) = (0.60, 0.40)$.

Figure 3: Relative borrowing rate when only productive agents borrow and set up an investment project to produce capital.

5 Credit flow to unproductive entrepreneurs

In the previous section, we considered the case when only productive agents were allowed to participate in the credit market. What happens when unproductive agents become able to compete for credit? The highest relative rate agents of type-10 may offer to lenders is $\xi_{t+1} = \phi(e_1 w_t) p_0$. This implies that the unproductive agents will become able to obtain credit and become an entrepreneur when

$$\phi(e_1 w_t) p_0 > \xi(w_t, e) p_1 \Leftrightarrow p > P(w_t, e) = \frac{\xi(w_t, e)}{\phi((1+e)w_t)} \quad (15)$$

holds for some levels of income and income inequality, (w_t, e) . Let, for a given $\lambda \in (\frac{1}{4}, 1)$, define the following derived parameters

$$e^c = -(2\lambda + 1) + \sqrt{4\lambda(2 + \lambda)} \in (0, \widehat{e}) \quad \text{and} \quad \underline{p} = (1 - \lambda) \left(1 + \sqrt{\frac{\lambda}{2 + \lambda}}\right) \in (0, \bar{p}). \quad (16)$$

Configurations of (λ, e^c) and (λ, \underline{p}) are visualized on Figures 2(a) and 2(b) respectively. One can easily verify that for a given $\lambda \in (\frac{1}{4}, 1)$, $e = e^c$ is the positive solution of the quadratic equation, $\frac{\lambda}{e_1} = \frac{e_1}{4e_0}$. The parameter pair, (λ, e^c) , divides the considered parameter region into two sub-regions. When $e \in (0, e^c)$ then $\frac{\lambda}{e_1} \in (\frac{e_1}{4e_0}, \frac{\lambda}{e_0})$ and when $e \in (e^c, \widehat{e})$ then $\frac{\lambda}{e_1} \in (\frac{1}{4}, \frac{e_1}{4e_0}]$. Likewise, the parameter pair, (λ, \underline{p}) , divides the considered parameter region into two sub-regions. When $p \in (0, \underline{p}]$ then $p \leq P(w_t, e)$ holds for any pair (w_t, e) and thus unproductive agents are not able to obtain external funding and set up an investment project. In contrast, if $p \in (\underline{p}, \bar{p})$ then $p > P(w_t, e)$ will hold for some pairs, (w_t, e) . This is proved using the following technical lemma.

Lemma 1 (a) *for any $\lambda \in (\frac{1}{4}, 1)$ and $e \in (0, \widehat{e})$,*

$$P^c(e) = \min_{w_t \geq 0} P(w_t, e) = P\left(\frac{\lambda}{e_1}, e\right) = \begin{cases} \frac{(1-\lambda)(1+e)}{1-\lambda+(1+\lambda)e} & \text{if } e \in (0, e^c] \\ \frac{4(1-\lambda)}{3-e} & \text{if } e \in (e^c, 1] \end{cases} \quad (17)$$

(b) *for any $\lambda \in (\frac{1}{4}, 1)$,*

$$\underline{p} = \min_{e \in (0, \widehat{e})} P^c(e) = P^c(e^c) = (1 - \lambda) \left(1 + \sqrt{\frac{\lambda}{2 + \lambda}}\right). \quad (18)$$

Proof of Lemma 1 can be found in the Appendix. Two important observations can be made based on Lemma 1. First, for a given level of income inequality, e , $w_t \mapsto P(w_t, e)$ is a “U” shaped curve achieving a local minimum $P^c(e)$ at $w_t = \frac{\lambda}{e_1}$; and second $e \mapsto P^c(e)$ is a “U” shaped curve achieving a local minimum at $e = e^c$. These two properties imply that if $p \in (0, \underline{p})$ then $p < P^c(e^c) < P^c(e) < P^c(w_t, e)$ for any pair

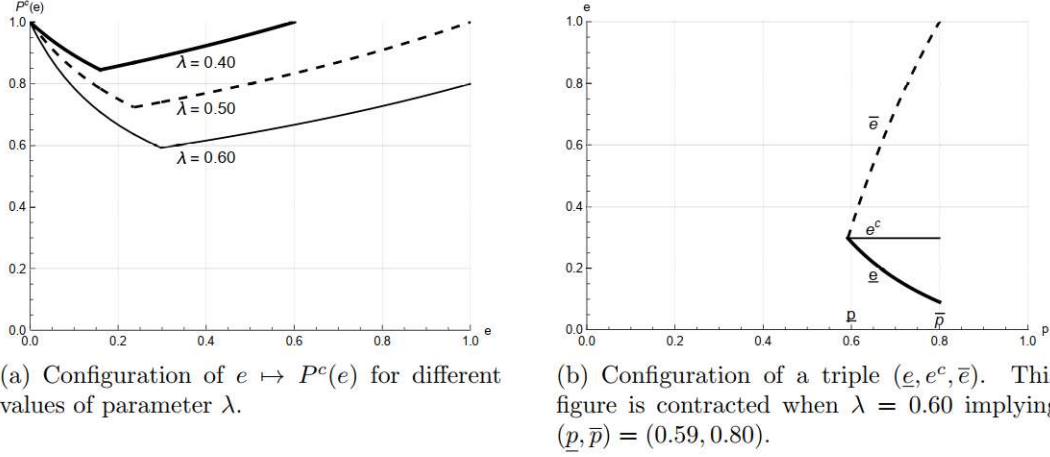


Figure 4: Relative borrowing rate when only productive agents borrow and set up an investment project to produce capital.

(w_t, e) . I.e., unproductive agents are not able to compete for credit with productive ones if $p \in (0, \underline{p})$.¹⁶ In contrast, if $p \in (\underline{p}, \bar{p})$ then $p = P^c(e)$ has two solutions

$$\underline{e} = \frac{(1-\lambda)(1-p)}{(1+\lambda)p - (1-\lambda)} \in (0, e^c) \quad \text{and} \quad \bar{e} = 3 - \frac{4(1-\lambda)}{p} \in (e^c, \widehat{e}) \quad (19)$$

such that $p > P^c(e)$ for intermediate values of income inequality, $e \in (\underline{e}, \bar{e})$, and $p < P^c(e)$ for either a sufficiently low or a sufficiently high level of income inequality, $e \in (0, \underline{e}] \cup [\bar{e}, 1)$. Figure 4(a) visualizes the configuration of $e \mapsto P^c(e)$ for different values of parameter λ . As the figure indicates, $e \mapsto P^c(e)$ is a piecewise continuous and “U” shaped curve achieving its minimum at $e = e^c$. This implies that $p > P^c(e)$ holds for an intermediate values of productivity and income inequality $(p, e) \in (\underline{p}, \bar{p}) \times (\underline{e}, \bar{e})$. Figure 4(b) visualizes the configuration of triple $(\underline{e}, e^c, \bar{e})$ for a given value of parameter $\lambda \in (\frac{1}{4}, 1)$ and for $p \in (\underline{p}, \bar{p})$.

At this point, we can highlight that if $(\lambda, p, e) \notin (\frac{1}{4}, 1) \times (\underline{p}, \bar{p}) \times (\underline{e}, \bar{e})$ then only productive agents set up investment projects and produce capital. As a result, the capital accumulation is described by $k_{t+1} = p_1 w_t$ and the evolution of income is given by $w_{t+1} = A(1-\alpha)k_{t+1}^\alpha = w_\infty^{1-\alpha} w_t^\alpha$, where $w_\infty = [A(1-\alpha)p_1^\alpha]^\frac{1}{1-\alpha}$ is the unique and interior steady-state income level towards which the economy converges monotonically for any initial value of w_0 .

Proposition 1 *Suppose the parameter triple satisfies,*

$$(\lambda, p, e) \in \left(\frac{1}{4}, 1\right) \times (\underline{p}, \bar{p}) \times (\underline{e}, \bar{e}) \quad (20)$$

¹⁶This case was analyzed in Section 4.

	ξ_{t+1}	i_t^{11}	ϵ_t^{11}	i_t^{10}	ϵ_t^{10}
$w_t \in \left(\underline{w}, \frac{\lambda}{e_1}\right)$	$\frac{(1-\lambda)p_0}{1-e_1 w_t}$	$\frac{e_1 p w_t}{e_1 w_t + p - 1}$	$\frac{1}{4}$	1	$\left(1 - \frac{1}{4} \frac{e_1 p}{e_1 w_t + p - 1}\right) w_t$
$w_t \in \left[\frac{\lambda}{e_1}, \bar{w}\right)$	p_0	$\frac{e_1 p w_t}{\lambda + p - 1}$	$\frac{1}{4}$	$\frac{e_1 w_t}{\lambda}$	$\left(1 - \frac{1}{4} \frac{e_1 p}{\lambda + p - 1}\right) \frac{\lambda}{e_1}$

Table 5: Relative borrowing rate and investments made by agents with high net-worth when $(\lambda, p, e) \in (\frac{1}{4}, 1) \times (\underline{p}, \bar{p}) \times (\underline{e}, \bar{e})$ and $w_t \in (\underline{w}, \bar{w})$.

then $\phi(e_1 w_t)p > \xi(w_t, e)$ for $w_t \in (\underline{w}, \bar{w})$, where

$$\underline{w} = \max \left\{ \frac{1-p}{1-p+(1+p)e}, \frac{4-(3-e)p}{4(1+e)} \right\} \in \left(\frac{1}{4}, \frac{\lambda}{e_1}\right) \quad \text{and} \quad \bar{w} = \frac{p-(1-\lambda)}{p(1-e)} \in \left(\frac{\lambda}{e_1}, \frac{\lambda}{e_0}\right). \quad (21)$$

implying the credit flow to agents of type-10 when $w_t \in (\underline{w}, \bar{w})$.

Proof of Proposition 1 can be found in the appendix. As the figure indicates, $\underline{w} \in (0, \frac{\lambda}{e_1})$

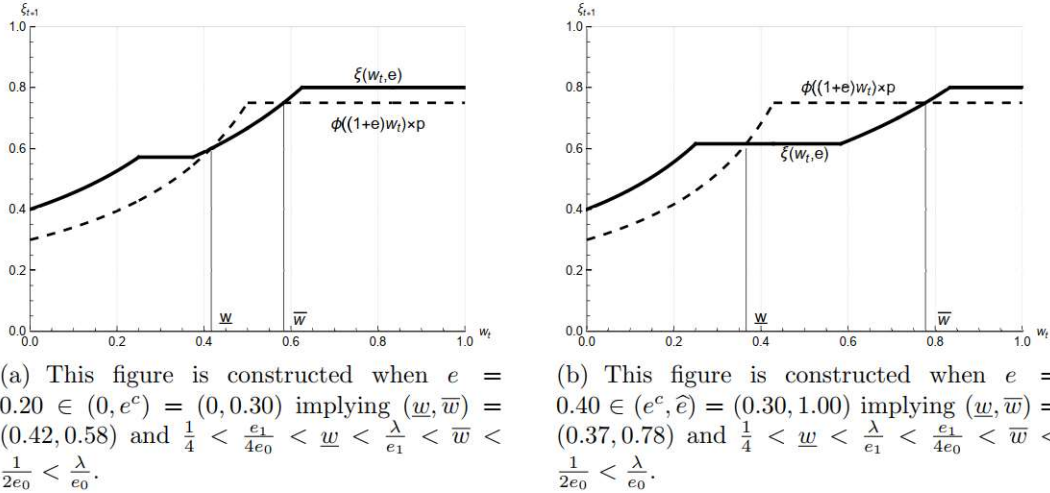


Figure 5: Configurations of $w_t \mapsto \xi(w_t, e)$ and $w_t \mapsto \phi((1+e)w_t)p$. This figure is constructed when $(\lambda, p) = (0.60, 0.75)$ implying $(e^c, \bar{e}) = (0.30, 1)$ and $(\underline{p}, \bar{p}) = (0.59, 0.80)$.

solves $\max \left\{ \frac{4(1-\lambda)}{4-e_1}, \frac{1-\lambda}{1-e_0 w_t} \right\} = \frac{(1-\lambda)p}{1-e_1 w_t}$, while $\bar{w} \in (\frac{\lambda}{e_1}, \frac{\lambda}{e_0})$ solves $p = \frac{1-\lambda}{1-e_0 w_t}$. Figure 5 visualizes two cases when unproductive agents can borrow and set up an investment project.

6 Equilibrium Income Dynamics

Capital Accumulation: When parameter restriction (26) is satisfied, then credit will flow only to productive entrepreneurs if $w_t \in (0, \underline{w}) \cup (\bar{w}, \infty)$. As a result, the

next period relative rate and the capital stock will be $\xi_{t+1} = \xi(w_t, e)$ and $k_{t+1} = p_1 w_t$, respectively. If $w_t \in (\underline{w}, \bar{w})$, then credit will flow to productive and unproductive entrepreneurs. The next period's relative lending rate will be $\xi_{t+1} = \phi(e_1 w_t) p_0$, and only agents with a high net worth will be able to set up an investment project. In such a case, the investment made by agents of high net worth is summarized in Table 5. Resulting the next period capital is $k_{t+1} = p_1 i_t^{11} \epsilon_t^{11} + p_0 i_t^{10} \epsilon_t^{10} = p_1 w_t m(w_t)$ where

$$m(w_t) = \begin{cases} 1 & \text{if } w_t \in (0, \underline{w}] \\ m_c(w_t) = \left(1 + \frac{1}{4} \frac{e_1(1-p)}{e_1 w_t + p - 1}\right) p & \text{if } w_t \in (\underline{w}, \frac{\lambda}{e_1}) \\ m_c(\frac{\lambda}{e_1}) = \left(1 + \frac{1}{4} \frac{e_1(1-p)}{\lambda + p - 1}\right) p & \text{if } w_t \in [\frac{\lambda}{e_1}, \bar{w}) \\ 1 & \text{if } w_t \in [\bar{w}, \infty) \end{cases} \quad (22)$$

measures the resource misallocation.¹⁷

Since $k_{t+1} = p_1 w_t m(w_t)$, it follows from $w_{t+1} = A(1-\alpha)k_{t+1}^\alpha$ that the following equation describes the dynamics of income.

$$w_{t+1} = A(1-\alpha)k_{t+1}^\alpha = A(1-\alpha)p_1^\alpha [w_t m(w_t)]^\alpha = w_\infty^{1-\alpha} [w_t m(w_t)]^\alpha = W(w_t). \quad (23)$$

Classification of Steady States: It follows from (23) that the steady state wage rate solves

$$\Phi(w) = w_\infty \quad \text{where} \quad \Phi(w) = \left(\frac{w}{[w m(w)]^\alpha}\right)^{\frac{1}{1-\alpha}} = \frac{w}{[m(w)]^{\frac{\alpha}{1-\alpha}}}. \quad (24)$$

Two important properties of Φ should be highlighted here. First, $w \mapsto \Phi(w)$ is a non-decreasing function with one or two discontinuities at $w = \underline{w}$ and $w = \bar{w}$. Second, $\Phi(w) = w$ for $w \in (0, \underline{w}] \cup [\bar{w}, \infty)$. Let

$$\underline{\Phi} = \lim_{w \downarrow \underline{w}} \Phi(w) \quad \text{and} \quad \bar{\Phi} = \lim_{w \uparrow \bar{w}} \Phi(w) \quad (25)$$

denotes the right and left limits of Φ at $w = \underline{w}$ and $w = \bar{w}$ respectively. Monotonicity property of $w \mapsto m(w)$ implies that $\underline{w} \leq \min\{\underline{\Phi}, \bar{w}\} < \max\{\underline{\Phi}, \bar{w}\} < \bar{\Phi}$. Figure 6 visualizes two possible configurations of $w \mapsto \Phi(w)$. By this visualization, we can conclude that

(a) there exists no steady state if $w_\infty \in (\underline{w}, \min\{\underline{\Phi}, \bar{w}\})$;

¹⁷To be more precise, $1 - m(w_t)$ measures the resource misallocation as it represents the percentage deviation of actual capital stock from the potential capital stock, $k_{t+1} = p_1 w_t$. At the same time, $1 - [m(w_t)]^\alpha$ measures the percentage deviation of output from the potential output, $y_{t+1} = A k_{t+1}^\alpha = A [p_1 w_t]^\alpha$.

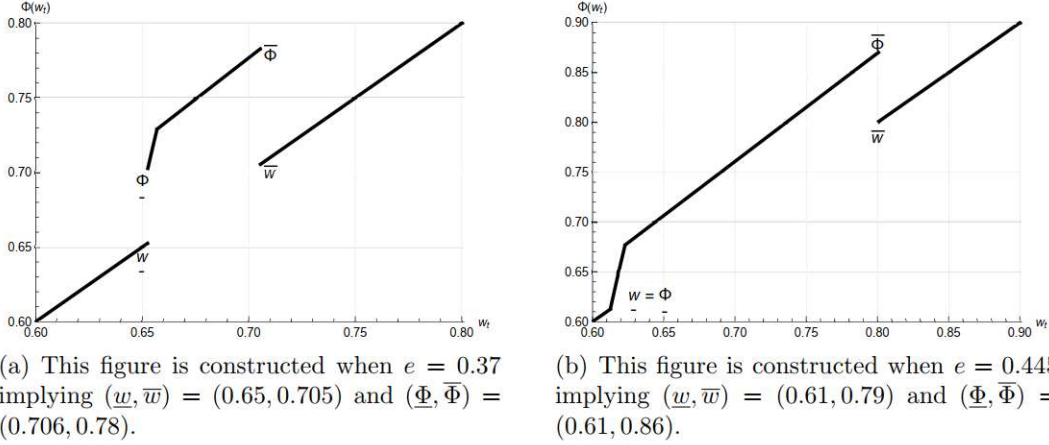
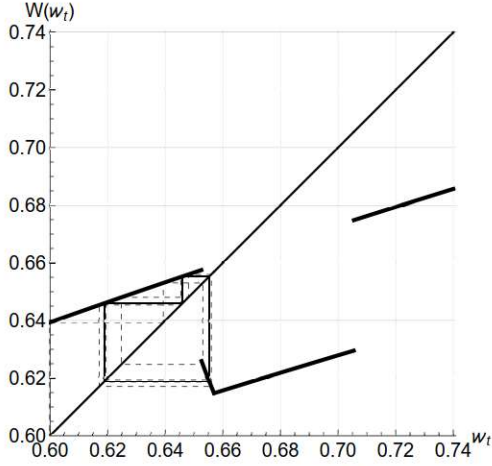


Figure 6: Two alternative configurations of $w_t \mapsto \Phi(w_t)$. This figure is constructed when $(\lambda, p) = (0.90, 0.18)$.

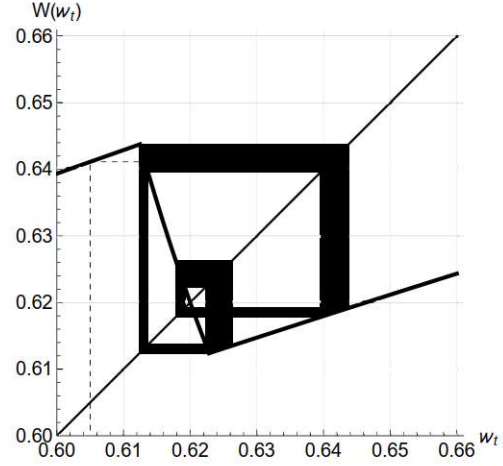
- (b) there exists a unique steady state if $w_\infty \in (0, \underline{w}) \cup [\min\{\underline{\Phi}, \bar{w}\}, \max\{\underline{\Phi}, \bar{w}\}) \cup (\bar{\Phi}, \infty)$;
- (c) there exists two steady states if $w_\infty \in [\max\{\underline{\Phi}, \bar{w}\}, \bar{\Phi}]$.

Different Scenarios of Income Dynamics: Figure 7 visualizes a cobweb plot displaying a qualitative behavior of wage rate when $(\lambda, p) = (0.90, 0.18)$. As shown by figure 6(a), when $e = 0.37$ and $w_\infty = 0.66$, a steady state does not exist. As indicated by figure 7(a), under such parameter configuration, the wage rate converges to a stable cycle-3. As shown by figure 6(b), when $e = 0.445$ and $w_\infty = 0.66$, a unique steady state exists. As indicated by figure 7(b), under such parameter configuration, the wage rate converges to a stable chaotic cycle-4. Thus both figures demonstrate that the economy might display a cyclical behavior in the presence of income inequality. Figure 8 displays two bifurcation diagrams of w_t with respect to income inequality e . It shows that a system's fixed points, periodic orbits, or chaotic attractors as inequality (represented by the parameter e) change. Figure 8(a) is constructed when $(\lambda, p) = (0.90, 0.18)$. As the figure demonstrates, the economy converges to a unique and globally stable steady state w_∞ when income inequality is sufficiently small, $e \in (0.34, 0.355)$. When $e = 0.37$, cycle-3 is a unique and globally stable periodic orbit. When $e = 0.445$, then the wage rate converges to a chaotic cycle-4.¹⁸ The instability of steady state continues until $e = 0.475$ and stabilises for $e \in (0.475, 0.78)$. Figure 8(b) displays yet another bifurcation diagram when $(\lambda, p) = (0.92, 0.15)$. Again, periodic orbits and chaotic attractors arise for intermediate values of income inequality. For sufficiently small and sufficiently large values of income inequality, $e \in (0.34, 0.38) \cup (0.52, 0.78)$, the unique steady state is a stable fixed point which loses its stability for an intermediate level of

¹⁸Respective Cobweb plot are visualized on Figure 7.



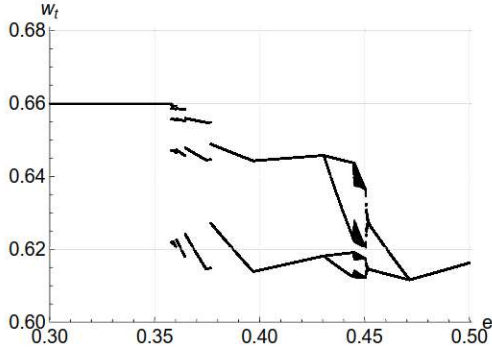
(a) This figure is constructed when $e = 0.37$ implying $(\underline{w}, \bar{w}) = (0.56, 0.61)$ and $(\underline{\Phi}, \bar{\Phi}) = (0.60, 0.66)$.



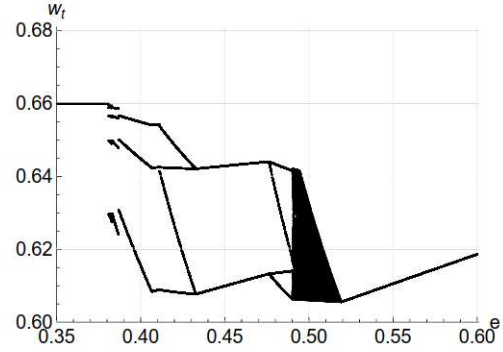
(b) This figure is constructed when $e = 0.445$ implying $(\underline{w}, \bar{w}) = (0.61, 0.79)$ and $(\underline{\Phi}, \bar{\Phi}) = (0.61, 0.86)$.

Figure 7: Cobweb plot displaying a qualitative behaviour of wage rate. This figure is constructed when when $(\lambda, p) = (0.90, 0.18)$.

income inequality, $e \in (0.38, 0.52)$. For $e \in (0.38, 0.49)$, the economy converges to a periodic orbit, while $e \in (0.49, 0.52)$ convergence occurs to a chaotic attractor.



(a) This figure is constructed when $(\lambda, p) = (0.90, 0.18)$ implying $(\underline{p}, \bar{p}) = (0.16, 0.20)$ and $(\underline{e}, \bar{e}) = (0.34, 0.78)$.

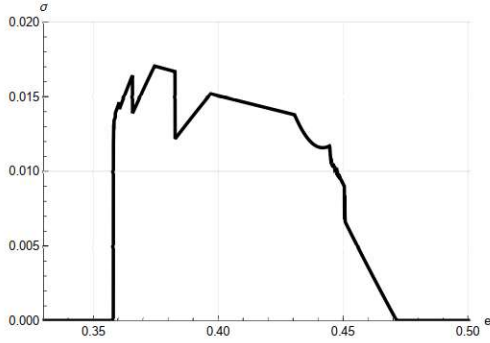


(b) This figure is constructed when $(\lambda, p) = (0.92, 0.15)$ implying $(\underline{p}, \bar{p}) = (0.13, 0.16)$ and $(\underline{e}, \bar{e}) = (0.33, 0.87)$.

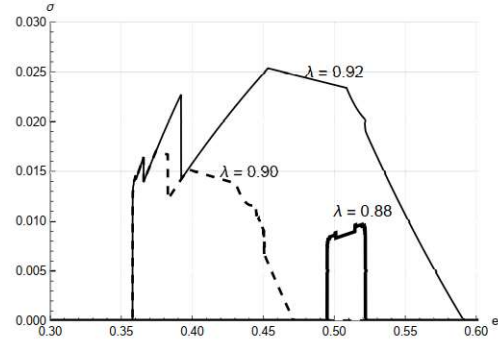
Figure 8: Bifurcation diagrams of w_t with respect to $e \in (\underline{e}, \bar{e})$. Both figures are constructed when $w_\infty = 0.66$.

7 Main Implications

We considered a deterministic model and demonstrated the possibility of endogenous fluctuations due to income inequality. Endogenous fluctuations arise either due to the disappearance of a steady state or a loss of dynamic stability of a unique steady state. In both cases, the economy converges into a regular or chaotic cycle. Figure 9(a) visualizes the dependence of income volatility (i.e., the standard deviation of w_t) on income inequality. As the figure indicates, income volatility displays the inverted “U” shaped relationship with respect to income inequality.



(a) This figure is constructed when $\lambda = 0.90$.

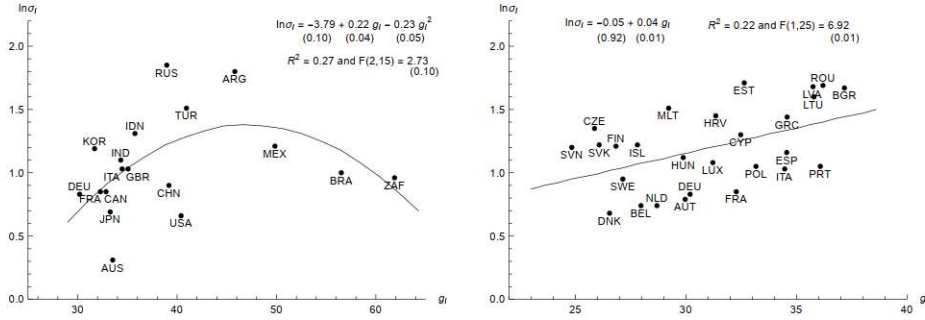


(b) This figure is constructed for different values of parameter λ .

Figure 9: Relationship between income inequality and income volatility.

Empirical Implications: Figures 10(a) and 10(b) displaying the regression results with dependent and independent variables being the natural logarithm of the annual income growth standard deviation and the Gini index of income inequality, respectively, across G20 and European Union (EU) countries. Figures 1(a) display the quadratic fit when the sample consists of G20 countries. As results indicate, both coefficients of the linear and quadratic terms are significant, and the coefficient of the quadratic term is negative, indicating an initial increase and then a decrease of the standard deviation of the annual income growth with respect to the increase of income inequality. The regression is significant at 90% confidence, and 27% of the cross-country variation of the annual income growth standard deviation is explained by income inequality. Figure 1(b) displays the linear regression fit between the same quantities when the sample consists of EU countries. In this case, the quadratic term of the regression is insignificant, that's why we have not included it in the regression equation. The coefficient of the linear terms is positive and significant. The regression is significant at 99% confidence, and 22% of the cross-country variation of the dependent variable is explained by income inequality. Of course, the presented regression results do not show the causality between income inequality and income volatility, but the observed

data is consistent with model prediction.¹⁹



(a) G20 countries include: (1) Argentina, (2) Australia, (3) Brazil, (4) Canada, (5) China, (6) France, (7) Germany, (8) India, (9) Indonesia, (10) Italy, (11) South Korea, (12) Japan, (13) Mexico, (14) Russia, (15) Saudi Arabia, (16) South Africa, (17) Turkey, (18) United Kingdom, (19) United States, (20) European Union.

(b) The EU countries are: Austria, Belgium, Bulgaria, Croatia, Republic of Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain and Sweden.

Figure 10: Income Inequality vs. Income Volatility. P-values are reported in parentheses.

Policy Implications: Figure 9(b) visualizes the dependence of income volatility (i.e., the standard deviation of w_t) on income inequality for different values of parameter λ . As the figure indicates, an increase in the value of parameter λ (which stands as a credit policy parameter) causes the shift of the relationship between two variables from right to left and the widening of the domain of income inequality for which endogenous fluctuations occur. This result suggests that a policy proposal to weaken a link between income inequality and income volatility by tightening the credit market may not always succeed and sometimes may create the opposite effect. It is important to recognize that apart from the benefits of making it hard for low-income families to accumulate debt, tight credit policy may sometimes impede efficient credit allocation.

8 Summary and Conclusions

In this paper, we link two strands of the literature that have been mainly evolving separately: the literature on income and income distribution and the literature on the efficient functioning of credit markets. In doing so, we propose and analyze a dynamic general equilibrium model with heterogeneous agents. We find that credit

¹⁹Data to obtain these plots are obtained from World Bank development indicators. To calculate the income growth volatility, we relied on the indicator - GDP per capita growth (annual percentage). For measuring income inequality, we relied on the indicator - Gini index.

misallocation takes place for intermediate levels of income and income inequality. The strength of such misallocation determines whether income inequality leads to income instability. The model is kept as simple as possible to allow for a clear understanding of the mechanisms at work.

The proposed model allows us to put forward several research hypotheses about the predictive and explanatory potential of income inequality on income instability. Just to highlight a few: (1) income inequality negatively affects capital accumulation. This hypothesis is consistent with the empirical findings of Cingano (2014), who argue that a sharp increase in income inequality across OECD countries slowed down economic growth and recovery from recessions by slowing human capital accumulation. (2) Productively growth is counter-cyclical. This hypothesis is consistent with empirical findings of Bresnahan and Raff (1991), and Davis and Haltiwanger (1992), who found the “cleansing effect” of a recession on productivity using firm-level data. (3) Credit tightening does not always imply income stability. It is important to recognize that by tightening credit constraints, we not only prevent debt accumulation but also hamper the flow of credit from less productive to more productive borrowers and prevent low-income agents from getting credit and realizing their human capital potential. This will slow down productivity growth and stagnate human and physical capital accumulation.²⁰

The analytical framework through which we demonstrate the main results is based on an overlapping generations model with capital accumulation modified only to include a minimum investment requirement for setting up a firm, heterogeneity of agents’ characteristics with respect to their labor and entrepreneurial productivity, and the existence of borrowing constraint. Within this framework, we demonstrate the existence and uniqueness of equilibrium and spelled out parameter restrictions under which resource misallocation and endogenous business cycles occur. As long as the assumption of a minimum investment requirement, partial pledgeability of an entrepreneur’s profit for debt repayment, and heterogeneity among agents’ entrepreneurial productivity are maintained, the alternative specifications of the model would not eliminate the key result. However, they may complicate the analysis. However, if we eliminate assumptions about a minimum investment requirement or partial pledgeability of an entrepreneur’s profit for debt repayment, the credit will always flow to the most productive agents. Under the current setting, the investment made by productive entrepreneurs is limited because of the credit constraint. Alternatively, if we eliminate the assumption about the heterogeneity of the agent’s entrepreneurial productivity in the current setting, then we could no longer discuss the resource misallocation because the productivity of every entrepreneur will be the same.

This paper address only one aspect of income inequality and income instability to reconcile the contradictory views of credit policy. We identify two-sided spillover effects

²⁰This is especially important in a world of rapidly evolving technological change and increasing skill premium.

between income and credit misallocation so that high income implies a high level of credit misallocation, leading to low income, and low income implies an improvement in the efficient functioning of the credit market, leading to increased income again. One should interpret the main results with care. We do not argue that income inequality is the only source to be blamed for income instability or that other sources, like technological change, demographics, fiscal and monetary policy, etc., are unimportant. Endogenous income volatility does NOT mean that exogenous sources for volatility, like wars, pandemics, energy price shocks, etc., are not important. Instead, we argue that an intermediate level of income inequality may magnify income volatility.

At this point, we would like to point out some limitations of the model presented in this paper. First, the model has only one type of investment project (which is to set up a firm producing the single final commodity). Second, the model does not allow for technology or labor force growth. Third, the economy is closed and thus does not interact with other economies. Fourth, agents save the entire labor income and do not consume during the first period. Due to these limitations, we can think of many ways to extend the model. The present paper represents a step toward research in this direction.

9 Appendix

Proof of Proposition 1: (a) $\frac{\lambda}{e_1} = \frac{e_1}{4e_0}$ is a quadratic equation with respect to e . One of its positive solution is $e^c = -(2\lambda + 1) + \sqrt{4\lambda(2 + \lambda)}$. One can easily verify that $\frac{\lambda}{e_1} \in (\frac{e_1}{4e_0}, \frac{\lambda}{e_0})$ for $e \in (0, e^c)$ and $\frac{\lambda}{e_1} \in (\frac{1}{4}, \frac{e_1}{4e_0}]$ for $e \in [e^c, \hat{e})$.

(b) Suppose $e \in (0, e^c] \Leftrightarrow \frac{\lambda}{e_1} \in (\frac{e_1}{4e_0}, \frac{\lambda}{e_0})$ then

$$w_t \mapsto P(w_t, e) = \frac{\xi(w_t, e)}{\phi(e_1 w_t)} = \begin{cases} 1 & \text{if } w_t \in (0, \frac{1}{4}) \\ \frac{4-4e_1 w_t}{4-e_1} & \text{if } w_t \in [\frac{1}{4}, \frac{e_1}{4e_0}] \\ \frac{1-e_1 w_t}{1-e_0 w_t} & \text{if } w_t \in [\frac{e_1}{4e_0}, \frac{\lambda}{e_1}) \\ \frac{1-\lambda}{1-e_0 w_t} & \text{if } w_t \in [\frac{\lambda}{e_1}, \frac{\min\{\lambda, 1/2\}}{e_0}) \\ \min\{1, 2(1-\lambda)\} & \text{if } w_t \in [\frac{\min\{\lambda, 1/2\}}{e_0}, \infty) \end{cases} \quad (26)$$

which is a strictly decreasing function for $w_t \in (\frac{1}{4}, \frac{\lambda}{e_1})$ and is a strictly increasing function for $w_t \in (\frac{\lambda}{e_1}, \frac{\min\{\lambda, 1/2\}}{e_0})$. This means that

$$P\left(\frac{\lambda}{e_1}, e\right) = \frac{(1-\lambda)e_1}{e_1 - \lambda e_0} = \frac{(1-\lambda)(1+e)}{1-\lambda + (1+\lambda)e} = P^c(e). \quad (27)$$

(c) Suppose $e \in (e^c, \hat{e}(\lambda)) \Leftrightarrow \frac{\lambda}{e_1} \in (\frac{1}{4}, \frac{e_1}{4e_0})$ then

$$w_t \mapsto P(w_t, e) = \frac{\xi(w_t, e)}{\phi(e_1 w_t)} = \begin{cases} 1 & \text{if } w_t \in (0, \frac{1}{4}) \\ \frac{4-4e_1 w_t}{4-e_1} & \text{if } w_t \in [\frac{1}{4}, \frac{\lambda}{e_1}) \\ \frac{4(1-\lambda)}{4-e_1} & \text{if } w_t \in [\frac{\lambda}{e_1}, \frac{e_1}{4e_0}] \\ \frac{1-\lambda}{1-e_0 w_t} & \text{if } w_t \in (\frac{e_1}{4e_0}, \frac{\min\{\lambda, 1/2\}}{e_0}) \\ 1 & \text{if } w_t \in [\frac{\min\{\lambda, 1/2\}}{e_0}, \infty) \end{cases} \quad (28)$$

is a strictly decreasing function for $w_t \in [\frac{1}{4}, \frac{\lambda}{e_1})$, is a strictly increasing function for $w_t \in (\frac{e_1}{4e_0}, \frac{\lambda}{e_0})$, and is constant for $w_t \in [\frac{\lambda}{e_1}, \frac{e_1}{4e_0}]$. This means that

$$P\left(\frac{\lambda}{e_1}, e\right) = \frac{4(1-\lambda)}{4-e_1} = \frac{4(1-\lambda)}{3-e} = P^c(e). \quad (29)$$

(d) One can easily verify that

$$P^c(e) = \begin{cases} \frac{(1-\lambda)(1+e)}{1-\lambda+(1+\lambda)e} & \text{if } z \in (0, e^c) \\ \frac{4(1-\lambda)}{3-e} & \text{if } e \in (e^c, \hat{e}) \end{cases} \quad (30)$$

is a continuous and "U" shaped curve achieving a local minimum at $e = e^c$. This means that for

$$p \in (\underline{p}, \bar{p}), \text{ where } \underline{p} = P^c(e^c) = (1-\lambda) \left(1 + \sqrt{\frac{\lambda}{2+\lambda}}\right), \quad (31)$$

QED.

Proof of Lemma 1: If $p \in (\underline{p}, \bar{p})$ then $p = P^c(e)$ admits two solutions

$$\underline{e} = \frac{(1-\lambda)(1-p)}{(1+\lambda)p-(1-\lambda)} \in (0, e^c) \text{ and } \bar{e} = 3 - \frac{4(1-\lambda)}{p} \in (e^c, \hat{e}) \quad (32)$$

such that $p > P^c(e)$ for $e \in (\underline{e}, \bar{e})$. If $p \in (\underline{p}, \bar{p})$ and $e \in (\underline{e}, \bar{e})$ then $p = P(w_t, e)$ admits two solutions

$$\underline{w} = \max \left\{ \frac{1-p}{1-p+(1+p)e}, \frac{4-(3-e)p}{4(1+e)} \right\} \text{ and } \bar{w} = \frac{p-(1-\lambda)}{p(1-e)} \quad (33)$$

such that $p > P(w_t, e)$ for $w_t \in (\underline{w}, \bar{w})$. This means that $\phi(e_1 w_t) p_0 > \xi(w_t, e) p_1$ holds for $w_t \in (\underline{w}, \bar{w})$.

QED.

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