Lie for the Sake of Peace:
Information Mediation in Repeated Hawk-Dove Game

Sungmin Kang

April 2023

Abstract

The paper considers a modified Hawk-Dove model in which one player's type is unknown, and the other wants to respond in accordance with the opponent's action. While the warfare outcome prevails with a high common prior belief toward a dominant Hawk (bad) type, a mediator, or a central information system, with commitment can induce an individual's optimistic belief by transmitting strategically manipulated signals. As a result, there is an incentive for players to refrain from constructing common knowledge about the true state, which leads them to stay in a doubtful but peaceful outcome. Private signaling makes the bad type uncertain about how the other player would believe about his type. If he believes that the other player has received a positive but false signal about his type, he is willing to choose the peace-making action. For the less-informed player, the signaling gives her the advantage to hide her private information and pursue her dynamic benefit by choosing the peace-making action as well, although she was initially aware of the truth of the bad type. Hence, the signaling raises a chance of a peaceful outcome, and given that all players expect a higher ex-ante payoff with the intervention of the information system, all players agree on allowing some degree of false signals.

1. Introduction

In the 1990s, concerns about North Korea's aim to develop a nuclear program raised military tensions and worsened the relationship not only between the two Koreas but also between the United States and North Korea. Given this circumstance, upon South Korea's newly elected President, Kim Dae-jung, took the office in 1998, Kim implemented an appeasement policy, known as "Sunshine Policy",[1] towards North Korea rather than giving diplomatic pressure. As a result, in June 2000, the leaders of both nations met in Pyongyang, and the first inter-Korea summit became a remarkable historic moment when the two Korean leaders shook hands and pledged to cooperate peacefully toward reunifying the Korean peninsula since the Korean War in the early 1950s. Despite the pledge, North Korea continued with nuclear development, and whenever political issues arose in this regard, President Kim stated that North Korea's intention was not to develop nuclear weapons. The US, under President Clinton's administration, accordingly also kept a similar policy for nuclear deterrence. For this notable contribution, President Kim was awarded the Nobel Peace Prize in 2000.

Today, North Korea is notorious for its ambition to progressively develop and test nuclear weapons. Were the former administrations of South Korea and the US unaware of North Korea's secret plan and progress back then? After the inauguration of former President George W. Bush in 2001, the peace policy got turned over, and later it was revealed that Kim's administration was aware of North Korea's nuclear program. The national intelligence agencies of both nations informed the administrations well about the project's progress at that time. Typically, telling the truth is considered to be virtuous. Nevertheless, we are also afraid of confronting even before crying over spilled milk. In this sense, when conflicts escalate, hiding the truth gives a chance to pretend

[1] The term originates from Aesop's Fables, "The North Wind and the Sun".

to be ignorant. It must be a sensitive matter to evaluate which policies would have been the wiser: to reveal or hide the bad truth. As we can learn from this case, however, hiding the truth may bring at least short-term peace.

Although there was no peaceful outcome, a similar misinforming strategy was used during World War II to obtain desired success. The British intelligence agency, MI5, operated the Double Cross system, a counter-espionage against German intelligence. In this warfare, the British arrested some German spies, which were clandestinely converted to become British spies, and made them deliver disinformation back to Germany, which successfully resulted in deceiving the German authorities with the wrong time and location of the Allies' D-day for Operation Overload, later known as Normandy landings. Similarly, the Soviet Union's spy obtained Japan's top secret – their reluctance to engage in the Eastern Front. Acquiring this valuable information led the Soviet Union to boldly decide to dispatch its military force against Germany, and, moreover, their secret police organization, NKVD, used double agents to carry out Operation Snow and to successfully provoke Japan against the United States. As such, disinformation system may create a chance to execute preferable actions to take for players.

Based on the previous two scenarios, this paper aims to explore the possibility of intended misleading information benefitting some parties. In other words, there could be a desirable, sometimes peaceful, outcome. To see this, we will construct a game theoretic model in which one player's type is unknown, and each can choose either a peace-making or a hostile action. Most of the time, although the players attempt to communicate to achieve a better result, it may not work when the belief is highly pessimistic. Thus, the best scenario requires the opponent to cooperate for peace; however, there always exists a bad type, as usual, that tries to derive benefit from deceiving the opponent. The Hawk-Dove game, therefore, fits well into this setting. Furthermore, a peacekeeping and well-informed third-party mediator needs to intervene to control the other two players' beliefs without the coercive power of policy-making. When a player builds an optimistic belief, even for the potentially bad guy, they could maintain peace for a while.

Precisely, this paper demonstrates one of many ways to resolve a conflict. Assuming that players' negotiation or communication does not work effectively in a conflict, the control and disclosure of the information are characterized to induce a peaceful outcome. Mostly, the "information design" problem solely structures strategic models' information/knowledge. As many prominent contexts of information design imply, when the interests of all players, including the third-party mediator, are not perfectly aligned, only partial information or perturbed signals are necessary at a certain level. This paper adopts the information controller, called a mediator, who is omniscient (with perfect information) and has a commitment power (does not reverse her initial plan when the game is proceeding), can design the signaling system without restriction and incurring any costs. We will see that the system releasing false signals with some degree about the bad type actually mitigates the conflict and attains more probability of peace outcome. The improvement resorts to commitment power and manipulations of the player's higher-order beliefs, even for the player who had perfect information about the game, which would not have been possible through their direct communications.

For example, in the case of using spies during World War II, the mediator's signaling corresponds to the information operation system of an intelligence agency. The core role here is not limited to obtaining the enemy's information but to modifying their belief in order to induce favorable action and outcome. They must be able to play with both sides' information asynchronously with covert operations. The espionage includes information acquisition and injection, which is the same as operating the central information system, but it nonetheless makes use of a discrepancy in the player's beliefs.

Of course, the equilibrium requires game theory criteria: players' beliefs must be consistent, and decisions must be rational. Players' interests are fundamentally not aligned in conflict models, so sending biased, advantageous information only to one side is untrustworthy to the others. Having this said, the key idea is how much to design the mix of both true and false information to persuade all sides. In equilibrium, the combination should be balanced between true and false probabilities. We will examine the equilibrium from the signaling system and track how much false information is allowable.

The sections proceed as follows: Section 2 reviews related literature, Section 3 sets up the model and checks equilibria without mediation, Section 4 introduces a mediator/central information system and finds the equilibrium, Section 5 extends the signaling to more variations, Section 6 discusses potential issues, and, lastly, Section 7 concludes.

2. Literature

Some papers are notable in that they build up the foundations of the information system. Crawford and Sobel (1982) pioneer a cheap talk model of the informed agent (sender)'s strategic signaling when agents' preferences are not perfectly aligned, and information from the sender is not verifiable by the receiver. They characterize the equilibrium that the sender delivers only partial information with intended noise. Rayo and Segal (2010) and Kamenica and Gentzkow (2011) analyze a similar model to show how signals should be pooled or separated to construct the mechanism of partial information disclosure. Aumann and Maschler (1995) and Kamenica and Gentzkow (2011) interpret the sender's problem to induce the optimal receiver's posterior beliefs as geometrically searching for the concave closure of his expected payoff that internalizes the receiver's posterior beliefs. This paper applies this signaling concept to a conflict model, which is necessarily partially informative as well.

Since the advent of "Correlated equilibrium (Aumann, 1987)", the equilibrium concept of the central signaling system has been extended to incomplete games, depending on how much information the system can identify. Myerson (1986) and Forges (1986, 1988) suggest "Communication equilibrium," an information transmission mechanism that requires both incentive-compatible conditions: an honest report of players' type (which the system can not identify) and obedience to the recommended action. Forges (1993) extends it to the "Bayesian solution" that introduces a mediator who can identify all types of players. Bergemann and Morris (2013) suggest "Bayes Correlated equilibrium" that the mediator is omniscient as she has comprehensive knowledge of the state beyond the collective information from all agents. It correlates not only with agents' actions but also between their actions and the states of the world. Forge (2020) surveys a comparison of cheap talk equilibrium, communication equilibrium, and Bayesian solution (or Bayes correlated equilibrium), implying that the Bayesian solution is the most relaxed of the informed agent's incentive conditions among the three so that the sender can achieve the highest ex-ante expected payoff.[2] The difference between communication and the Bayesian solution is based on the mediator's knowledge power, resulting in fewer restrictions of incentive-compatible conditions in the latter. As Taneva (2019) points out, the difference between the cheap talk and the Bayesian solution comes from the sender's commitment power. Mathevet, Perego, and Taneva (2020) raise an issue of discrepancy in higher-order beliefs among multiple receivers caused by private signals, and they impose a "belief type" into the equilibrium concept, following the sense of Harsayi (1967).[3] Not surprisingly, all the

[2] See Forge (2020)'s example 3 for more details.

[3] Epistemic structure in game theory is a clever way to describe players' mutual knowledge and higher-order belief

equilibrium concepts require players' consistent belief to choose the best reply unilaterally based on his/her first-order belief. This paper adopts an omniscient third party, a better-informed mediator than players, as her private signaling possibly generates uncertainty beyond their pre-given knowledge. So, it resorts to Bayes Correlated equilibrium in each subgame and concerns the higher-order belief of multiple receivers.

Furthermore, we must seek dynamic consistency to deal with a repeated game. Brocas and Carrillo (2007) and Au (2015) study a dynamic model to explore sequential information disclosure. They examine the possibility of the sender's dynamic information delivery with variations of feasible mechanisms or the sender's commitment power.[4] This paper allows the sender's discretionary right to construct a plentiful signaling setup and full commitment power over all periods. Che et al. (2022) add a signaling cost and show that equilibrium signaling is a dynamic spreading of the receiver's posterior belief, a combination of discrete transition and gradual evolutions of the receiver's belief. Compared to them, this paper does not assume the signaling cost but still shows a discrete or gradual flow of beliefs motivated by the signal sender. In addition, while most other models rely on an irreversible option to step out of the game, this paper is based on a repeated game so that all players must keep showing their position every period. With a chance of being stuck in a bad state forever, players' reluctance to reveal or to learn of true (bad) state could arise, walking on their eggshells but not breaking them by themselves for a long time.

Some others are about the conflict model and information problem. Baliga and Sjöström (2022) illustrate a good scenario how two players' conflictual bargaining problem is translated to a stage Hawk-Dove game. With the uncertainty of payoff or players' type, Carlsson and van Damme (1993) and Baliga and Sjöström (2004) show how even very a small fear of bad outcome keeps

[4] Au (2015) shows that the sender is likely to hurry to reveal information in earlier stages when her commitment power is restricted in the short-term, while the optimal one with a full commitment power turns out static.

encroaching throughout the entire game by players' iterative reasonings.[5] Baliga and Sjöström (2004) study a cheap talk, an exchange of partial information of player's type, to increase the chance of peace as it cuts the cascade of fear to confine player's beliefs on a particular range, pooling both extremes but separating the moderate types. Baliga and Sjöström (2012) show that a third party's signaling is ineffective for strategically complementary games as she is biased toward sending only peace signals, inducing them non-informative. This paper's model also shows strategic complementarity, but it adds the third party's commitment power, making it work to send only some significant signals with different probabilities. This paper's model closely relates to Hörner, Morelli, and Squintani's (2015) mediation mechanism that hinders common knowledge about the types works better than a communication protocol of players. However, it is more probable, for example, international diplomacy not relying on binding laws, that the mediator may not possess much bargaining advantage to control the participant's options. So, this paper resorts mainly to a signaling system, interpreted as action recommendations, that does not change the original options.

There are more topics and applications with dynamic information systems: Type uncertainty (Aumann and Maschler, 1995; Powell, 1999), Type variations (Powell, 1996), Screening (Fearon, 2013), and Misperception (Acemoglu and Wolitzky, 2014). This paper is based on the type uncertainty model, which does not change over time. The result implies that a perturbed information system rather than screening schemes could be agreeable. Furthermore, the paper approaches the characterization of the system rather than examining only feasible payoffs.

[5] It is called "Hobbesian trap" in Baliga and Sjöström (2010)'s conflict model.

3. Model

There are two players, 1 (he) and 2 (she). They are involved in war-imminent affairs, and each player i has two options to choose either a favorable gesture D or a hostile action H. When both players choose D, they stay in the status quo with a payoff 0. When both players choose H, they must be engaged in a battle with an equal expected payoff. The player i taking an offensive pose (H) when j shrinks back (D) can gain a benefit β, which is the best scenario for i. However, it is the worst one for j as he fights in an adversarial position, causing him damage γ. As the hostile action H requires one's hard determination, assume that it always incurs a war cost θ_i for player i. The basic game has the same structure as the Hawk-Dove game, described as following:[6]

		2	
		D	H
1	D	$0, 0$	$-\gamma, \beta - \theta_2$
	H	$\beta - \theta_1, -\gamma$	$-\theta_1, -\theta_2$

We will assume that θ_i is private information representing the player i's type. When θ_i is large (small), i is more likely to choose D (H). In specific, if $\theta_i < \beta$ or $\theta_i > \gamma$, then there is a unique dominant action for i. Let's call these types "dominant types." The mediator or a central information system can't turn over the outcome when both players are dominant types If θ_i is moderate $\beta < \theta_i < \gamma$, then i wants to respond coordinately to the opponent's action. Let's call it "coordination type."

The correlated equilibrium can represent the mediation based on perfect information of θ. In particular, we will focus on a peace meditation, the correlated equilibrium that maximizes the

[6] Baliga and Sjöström (2022) show how a two-period bargaining game for a territory contest can be translated to a static Hawk-Dove game as a valid conflict model. They introduce winning probability in war and a commitment cost of choosing actions to parameterize the game.

probability of peaceful outcome (D, D). When both players are coordination types, there is only a duty to match their actions: Correlates them perfectly on the desirable outcome with probability 1.[7]

To make the model more interesting, assume that players have different types: player 1 is either a Hawkish dominant type ($\theta_1 < \beta$) or a Dovish dominant one ($\theta_1 > \gamma$), and player 2 is a coordination one ($\beta < \theta_2 < \gamma$). Then, any further information can not overturn player 1's equilibrium strategy, but player 2's information about 1's type is crucial to determine her best response. For each θ_2, player 2 constructs a belief on Θ_1, which is correlated with player 1's choice: a probability p for 1's H. The equilibrium strategy can be characterized only by player 2's belief about θ_1. Furthermore, if the player's types are independent, then player 1 can guess a distribution of player 2's type and her belief in ex-ante stage, which is translated to 1's belief on player 2's choice, independently of θ_1. So, we can fix θ_2 to represent player 1's belief, for example, the expectation of distribution of θ_2, and set up player 2's belief about θ_1 as common knowledge.

We can consider a coordination type of player 1, too. Two scenarios of treating the coordination type are possible: fully revealed or pooled with Hawk type. The key idea of the system is about how much belief for Hawk type should be pooled with others. If the coordination type absorbs it fully, we can construct the signaling easily. Otherwise, the rest of the Hawk types must be pooled with the dominant Dove ones. The problem ultimately boils down to how to mix two dominant types. Once the mixture of two dominant types is solved, treating the coordination type becomes a trivial matter: fully revealing it and coordinating perfectly.[8] We will focus on how to pool only two dominant types.

[7] In particular, a correlated equilibrium exists with only one signal that correlates players' actions perfectly for every (θ_1, θ_2). So, any information of θ_j do not change i's best responses as he is sure about the j's action.

[8] Baliga and Sjöström (2004)'s cheap talk also shows that the continuous type set is divided into three ranges, and then the coordination type is revealed, but two dominant types are pooled.

As action outcomes are finite, we can reduce the type space into a simpler one. If signals s and s' when θ_1 result in the same outcome, we can incorporate them into one signal with a sum of their probabilities. The number of total signals needs not be larger than the number of possible outcomes. As each type of player 1 should choose his dominant action, we don't have to introduce different Hawk types. In addition, a value p $(1-p)$ can represent player 2's prior belief distribution over all dominant Hawk (Dove) types. So, it is sufficient to consider only two types of player 1. Summarizing, we can reduce the game with two dominant types of θ_1, one coordination type of θ_2, and a common prior p on θ_H.

Assume that

$$\Theta_1 = \{\theta_H, \theta_D\},\ \Pr(\theta_H) = p,\ \Pr(\theta_D) = 1 - p$$

$$\theta_H < \beta < \theta_2 < \gamma < \theta_D \tag{1}$$

$$\theta_2 < p\gamma + (1-p)\beta \tag{2}$$

The condition (1) implies that θ_H and θ_D are dominant types, and that θ_2 is a coordination one. Furthermore, player 2 will likely choose H because of a high prior on θ_H or severe damage from D. The game is like a bilateral relation of small and hegemonic countries. The small country considers weapon development to gain recognition for its international status. If the cost is quite affordable (Hawk type), he will launch the development project secretly. Until the project matures, He wants the project to be regarded as frustrated (Dove type) to gain from tricking the opponent later (β). The hegemonic country (coordination type) intends to respond to the small country's stance by using an eye for an eye. Based on her current information, she is so concerned about the weapon (a high p or a high γ), so pushing (H) the country, such as international sanctions, is the best security option.

Now, let's construct a dynamic setup. Two players must keep their relations for a while. In the meantime, each has to take his stand against the other. Assuming no change of types and payoffs, we will discuss repeated games of the basic model. For each time $\tau = 1, ..., T$, let h^τ be a true history until τ. Notice that player 1's type is determined at the beginning and that each player can observe all previously chosen actions (a_1^τ, a_2^τ) in τ. Thus, the true history can be expressed recursively with a^τ:

$$\begin{aligned} h^0 &= \theta_1 \in \{\theta_H, \theta_D\} \\ h^\tau &= (h^{\tau-1}, a^\tau) \ (\tau \geq 1) \\ &= (\theta_1, a^1, a^2, ..., a^\tau) \end{aligned}$$

Note that each player observes different histories: only player 1 can observe h^0. As we will introduce private signals later, it is convenient to distinguish players' observable histories. Define i's observable history h_i^τ as following:

$$\begin{aligned} h_1^\tau &= h^\tau \text{ for all } \tau \\ h_2^0 &= \phi \\ h_2^\tau &= (h_2^{\tau-1}, a^\tau) \ (\tau \geq 1) \\ &= (a^1, a^2, ..., a^\tau) \end{aligned}$$

, that is, player 1 knows the true history in every period (omniscient), while player 2 can observe only action histories. Notice that $h^\tau = h_1^\tau = (\theta_1, h_2^\tau)$ in every τ. Let b_i^τ denote player i's belief distribution over i's uncertain space X_i in τ. $margb_i \in \Delta(Y_i)$ denotes i's marginal belief over Y_i, a subspace of X_i. Player 2 constructs her belief on player 1's type space Θ_1 depending on her history in τ:

$$
\begin{aligned}
b_2^0 &= (p, 1-p) \\
b_2^\tau &: H_i^{\tau-1} \to \Delta(\Theta_1) \ (\tau \geq 1)
\end{aligned}
$$

Each player i should decide his/her action in τ depending on the current history and private belief, $a_i^\tau : H_i^{\tau-1} \times B_i^\tau \to \Delta(\{H, D\})$, which can be reduced, by the mapping $b_2^\tau : H_i^{\tau-1} \to \Delta(\Theta_1)$, to

$$
a_i^\tau : H_i^{\tau-1} \to \Delta(\{D, H\})
$$

i's strategy is a plan of all a_i^τ for every $h_i^{\tau-1}$ and τ: $a_i = (a_i^\tau(h_i^{\tau-1}))_{h_i^{\tau-1} \in H_i^{\tau-1}, \tau}$. At $h_i^{\tau-1}$, i has a plan of future actions $a_i^{\geq\tau}(h_i^{\tau-1}) \equiv (a_i^t(h_i^{t-1}))_{h_i^{t-1} \in H_i^{t-1}(h_i^{\tau-1}), t \geq \tau}$ where H_i^{t-1} is the set of histories reachable from h_i^{t-1}. Abusing notations, we will use a_i^τ as a stage action or an action plan.

We will adopt the Perfect Bayesian equilibrium (PBE) concept: A profile of strategy plans and belief system (a, b) satisfy that $a_i^{\geq\tau}(h_i^{\tau-1})$ is sequentially rational with b_i^τ and that i's belief is consistent with $(a^t, b^t)_{t \leq \tau-1}$ for all $h_i^{\tau-1}$ and τ.

Lemma 1 *In Perfect Bayesian equilibrium (PBE), the Dove type must choose D in every history and period.*

Hawk type can mimic whatever Dove's strategy that to pool his type. So, in equilibrium, Dove type can not reveal his type by his will. All he can do is choose his dominant action of the stage game in all cases. Naturally, player 2 interprets the bad action that must have come from Hawk type, but the good action is not informative to her. Some countries have been suspected of developing mass destruction weapons and have repeatedly denied it, but the international community is not likely to believe it.

In the repeated game, Hawk type must determine when is the best to launch a surprise attack at the cost of revealing his type. Player 2 also predicts D-day as long as all game parameters are known. Recall that the prior belief on Hawk type is not favorable for peace. Then, the Hawk type should take action one period ahead of her. Iteratively, the optimal timing approaches the first period, so the PBE outcome yields that both Hawk type and player 2 end up playing H in all periods. As the global game (Carlsson and van Damme, 1993) and Hobbesian trap model (Baliga and Sjöström, 2010) show, players' fear of extreme cases spreads quickly throughout the game. In this paper's model, the concern of the warlike outcome in the last period keeps encroaching throughout all periods.

However, players can be motivated endogenously to play D if we consider mixed strategies. The Hawk type can play D with some probability, not too high, to mimic the Dove type. Then, player 2's belief becomes more optimistic as long as the type is not revealed, or she expects the opponent's H with a small probability. She is willing to respond with the same action. There exists mixed strategy PBE in the model. We can sufficiently check strategies of two periods: One with a prior belief p and the other when Hawk type plays H for sure. After attaining the outcome of the first period from a strategy profile, we can track the static game of the next period with a posterior based on the history. Once the two-period model shows a possibility of (D, D) in the first period, we can extend the equilibrium to more periods by spreading the distributions of mixed strategies. So, this paper will focus on solving the two-period model.

Let $\delta > 0$ be the common intertemporal ratio of payoff evaluation. If $\delta < 1$, it is known as "discount factor". If $\delta > 1$, it means players appreciate future payoffs higher than present ones, such as the investment model. Using a brief notation, "$a_i^1.\hat{a}_i^2/\widetilde{a}_i^2$" denotes i's plan in which "." distinguishes periods and "/" does histories: a_i^1 for period 1, $\hat{a}_i^2/\widetilde{a}_i^2$ for some relevant histories of period 2. For example, $a_{1H} = D.H$ denotes player 1's strategy: $a_1^1(h_1^0 = \theta_H) = D$, $a_1^2(h_1^1 = (\theta_H, \cdot)) = H$. Since he must choose H in $\tau = 2$, we can reduce the notation with only one action for all histories in $\tau = 2$. Similarly, $a_2 = D.D/H$ denotes player 2's strategy: $a_2^1 = D$, $a_2^2(a_1^1 = D, \cdot) = D$, $a_2^2(a_1^1 = H, \cdot) = H$, omitting other histories that are not reachable from a_2^1. A behavior strategy mixed of actions will be expressed as "$m_i^\tau D + (1 - m_i^\tau)H$".

For given $a_2 = D.D/H$, Hawk type expects $0 + \delta[\beta - \theta_H]$ with $a_{1H} = D.H$ and $\beta - \theta_H + \delta[-\theta_H]$ with $a_{1H} = H.H$. We are interested in the case that Hawk type is willing to cooperate in the short-term peace agreement with $a_{1H} = D.H$, i.e.,

$$(1 - \delta)\beta < \theta_H \tag{3}$$

Proposition 1 *Assume the condition (3) in a two-period repeated game. In Perfect Bayesian equilibrium (PBE), the Dove type plays D in every history and period, and player 2 believes the Hawk type with a probability 1 for given any histories with 1's action H.*

The following describes all PBE strategies:

Case 1. $1 - \frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta} \leq \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$ *and* $\theta_H \leq \gamma - \delta\beta$

1.1. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H, a_2 = D.(m_2^2 D + (1 - m_2^2)H)/H$

where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}, m_2^2 = \frac{\beta - \theta_H}{\delta\beta}$,

1.2. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H, a_2 = m_2^1 D + (1 - m_2^1)H.D/H$

where $m_{1H}^1 = 1 - \frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta}, m_2^1 = \frac{\gamma - \theta_H - \delta\beta}{\gamma - \beta}$,

1.3. $a_{1H} = H.H, a_2 = H.D/H$.

Case 2. $1 - \frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta} \leq \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$ *and* $\theta_H > \gamma - \delta\beta$

$a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H, a_2 = D.(m_2^2 D + (1 - m_2^2)H)/H$

where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}, m_2^2 = \frac{\beta - \theta_H}{\delta\beta}$

Case 3. $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ *and* $\theta_H \leq \gamma - \delta\beta$

$a_{1H} = H.H, a_2 = H.D/H$

Case 4. $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ *and* $\theta_H > \gamma - \delta\beta$

$a_{1H} = (m_{1H}^1 D + (1-m_{1H}^1)H).H, a_2 = H.(m_2^2 D + (1-m_2^2)H)/H$

where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}, m_2^2 = \frac{\gamma-\theta_H}{\delta\beta}$.

The expression "$\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$" is notable in that player 2's maximum belief (a probability) for Hawk type mimicking Dove one such that she is willing to choose D in the stage game. The probability can be interpreted as 2's maximum tolerated probability of the false signal from Hawk type. Given the probability, she is willing to play $a_2^2 = D$ against $a_1^1 = D$, that is, an eye for an eye strategy ($a_2 = \cdot.D/H$). We will see that this probability matters later in the mediation problem. The expression "$1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$" is player 2's maximum belief that "He is the Hawk type but chooses $a_{1H}^1 = D$" such that $a_2 = D.D/H$ is better than $a_2 = H.D/H$. That is, if Hawk type plays $a_{1H}^1 = D$ with a probability higher than $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$, then her belief is still pessimistic toward Hawk type, so she will choose $a_2^2 = H$ even after observing $a_1^1 = D$. To play $a_2 = D.D/H$, she needs fairly accurate information from 1's action. Otherwise, she will suffer from being tricked ($-\gamma$) in $\tau = 2$. Thus, to make her play $a_2 = D.D/H$, it is necessary $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ (Case 1 and 2). The expression "$\frac{\beta-\theta_H}{\delta\beta}$" is another important value. It is Hawk type's minimum belief for $a_2^2 = D$, provided $a_2^1 = D$, such that he is willing to choose D in $\tau = 1$. He needs compensation with a chance to trick her (to get β) in $\tau = 2$ for his endurance in $\tau = 1$.

Lessons from Proposition 1 are that there is a possibility of a peaceful outcome if beliefs are optimistic enough for the opponent's peace-making action and that it is impossible to attain the outcome (D, D) in the first period with 100% in any cases. As p is high (Case 3 or 4), PBE with $a_{1H}^1 = H$ and $a_2^1 = H$ prevails then. Player 2 requires more accurate information to take peaceful action, but it is beyond players' incentives. Intuitively, the more the players believe that war is very impending, the more difficult to agree on a peace treaty because their information/action requirements are getting more demanding to each other.

Now, check if communication can improve the situation. They can seek a better outcome at a negotiation table before going to the game. When worrying about taking discordant actions among multiple equilibria, cheap talk can help them to focus on one of them. To check the possibility, recall that case 1 in Proposition 1 yields three PBEs.

Lemma 2 *Recall Proposition 1. In the case 1,* $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ *and* $\theta_H \leq \gamma - \delta\beta$*, there are three PBEs: (1.1), (1.2), and (1.3). In the stage when player 1 just realizes his type before choosing an action in period 1, players' preferences are as following:*

Hawk type:

$$(1.1) \succ (1.2) \succ (1.3)$$

Dove type:

$$\text{If } \delta < \frac{\gamma-\theta_H}{\beta} < \frac{2\beta-\theta_H}{\beta}, \text{ then } (1.1) \succ (1.2) \succ (1.3)$$

$$\text{If } \delta < \frac{2\beta-\theta_H}{\beta} < \frac{\gamma-\theta_H}{\beta}, \text{ then } (1.2) \succ (1.1) \succ (1.3)$$

$$\text{If } \frac{2\beta-\theta_H}{\beta} < \delta < \frac{\gamma-\theta_H}{\beta}, \text{ then } (1.2) \succ (1.3) \succ (1.1)$$

Player 2:

$$\text{If } \delta < \frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2}, \text{ then } (1.1) \succ (1.2) \succ (1.3)$$

$$\text{If } \frac{\beta}{\gamma-\theta_2} < \delta < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2}, \text{ then } (1.1) \succ (1.3) \succ (1.2)$$

$$\text{If } \frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \delta < \frac{\gamma}{\gamma-\theta_2}, \text{ then } (1.3) \succ (1.1) \succ (1.2)$$

$$\text{If } \frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2} < \delta, \text{ then } (1.3) \succ (1.2) \succ (1.1)$$

Any type of player 1 prefers (1.2) to (1.3) with a more chance of $a_2^1 = D$. (1.1) is the best scenario for Hawk type as it gives the highest chance of concealing his type in period 1. It is true for Dove type when δ and γ are not relatively high. He can put up with a loss in the future, provided that player 2's D is guaranteed in $\tau = 1$. As δ, γ, or p is large, she is likely to play (1.3) in which 1's true type is definitely revealed $\tau = 2$. In this case, she can avoid the damage $-\gamma$ in every period. This strategy $a_2 = H.D/H$ serves to test the opponent's type (See the section 6.3 for more details). In the other cases, she prefers (1.1) the most, that is, she seeks the peaceful outcome in $\tau = 1$. So, all players will agree to play (1.1) to get the highest probability of (D, D) in $\tau = 1$, and notice that a small p is necessary for this case.

Depending on parameter conditions, only some of PBEs may be attainable and negotiable. Let's assume that the negotiation process yields an agreeable equilibrium anyway, say such equilibrium as "the best PBE for all players." We also should check if the cheap talk changes some probabilities of the outcome of the unique equilibrium.

Proposition 2 *There does not exist cheap talk equilibrium such that all players are better off than the best PBE for all players.*

Obviously, any messages in $\tau = 2$ do not overturn player 1's plan, so they are not informative at all. One's strategic distribution over meaningful messages is necessarily the spreading of PBE strategy in $\tau = 1$. The consistency condition restricts the belief spreading critically. While player 1's informative message induces a higher belief on θ, the other one should yield a lower of it. As long as the incentive compatible condition for a peace-making message is slack, there is a chance to increase the peaceful outcome. However, the mixed strategy already eats up such a probability through the indifference condition between D and H. Thus, cheap talk can not make different probabilities compared to mixed strategy PBEs. In another possibility, the cheap talk can produce a convex combination among PBEs. As Lemma 2 shows, players' preference for multiple PBE is globally ordered in all cases. So, any combinations must yield a lower expected payoff than one's best scenario.

Intuitively, player 1's message must be informative to make a difference. The only way is to separate Hawk type by himself with some probability. However, the pooling messages with different probabilities of D is unstable because revealing the type should give him a lower expected payoff. Without commitment, any peace-making messages are not trustful.

In the sense of Forges (2020), the cheap talk without commitment is geometrically interpreted as "quasi-concavification" of the sender's expected payoff graph by picking some receiver's feasible belief distributions. Because the indifferent condition between messages in the support, a mixture of them, or a flatting of the graph could attain a higher payoff for the non-monotonic range. However, the conflict model in this paper shows a monotonic payoff: With a higher p, a lower expected payoff.

We can impose a commitment power "for each type," but the result is not improving.

Proposition 3 *There does not exist communication equilibrium such that all players are better off than the best PBE for all players.*

The proof is quite intuitive. Resorting to the Revelation principle, it is sufficient to check central signaling systems that gather type reports and then implement an outcome following a particular distribution. Two kinds of incentive compatibilities are raised for each player: a truthful report and obedience to the recommended action. If the system produces different distributions for each θ, then both types must report the one that induces a higher probability $a_2 = D.D$. The system can not separate two types beyond the prior, so it should result in the same outcome as PBE.

We assumed that the commitment power is endowed in the stage right after player 1 realizes his type. This situation is like an uninformed third party mediating a peace agreement based on two countries' pre-designed negotiation plans. Even if the mediator is empowered to implement the point of contracts, a new incentive constraint arises: the suspicious country says, "We are ready to freeze our nuclear program in exchange for political and economic concessions from the opponent," and then is this statement trustful?[9]

4. Mediator's Signaling

We will introduce a mediator, or a proactive information system, that can verify the true state and not violate each player's discretion on decision-making. She has the power to design information systems before the true state is realized. The system requires the commitment power to operate: Otherwise, it can not make a difference from other kinds of communications. This setup is based on the persuasion model.

The timeline is as following:

<1> The mediator designs and announces her signaling rule.

<2> Player 1 observes his own type.

<3> Following the rule, a signal profile s^1 is sent to players.

<4> Each player choose an action a_i^1.

<5> Players observe action profile (a_1^1, a_2^1) in period 1.

<6> Following the rule based on the history, a signal profile s^2 is sent to players.

[9] In 2004, North Korea allowed the US delegation to tour its nuclear facilities and showed a willingness to give up nuclear weapons.

<7> Each player choose an action a_i^2.

<8> Players attain payoffs.

Assume that the signaling does not incur costs and allows a set of plentiful signals. As the number of outcomes is finite, we need only finitely many signals. With private signaling, a higher-order belief issue arises. For example, while player 1 could observe the true history in every moment, i.e., $h_1^\tau = h^\tau$ for all τ without the signaling, now he may be uncertain about player 2's private signals. For given a private signal, player 2 faces a similar uncertainty, and so on. Controlling players' higher-order beliefs is the key to achieving a more peaceful outcome. To characterize the higher-order belief, we need to extend the space with "belief type" of players, following the sense of Harsayi (1967). A collection of player's belief types and each type's distribution over the space[10]: state, strategy, and other's belief type sufficiently describe the infinite belief hierarchy.[11] One signal yields a player's unique belief over the space that includes other players' belief types. So, constructing signaling extends the game space with more belief type spaces.

In specific, let S_i^τ be a set of i's private signals in τ. Each s_i^τ induces i's private belief in τ. We need to trim some notations. For $\theta_1 \in \Theta_1$ and $s_i^1 \in S_i^1$, let histories such that

$$\begin{aligned} h^0 &= (\theta_1, s_1^1, s_2^1),\ h^1 = (\theta_1, s_1^1, s_2^1, a_1^1, a_2^1, s_1^2, s_2^2) \\ h_1^0 &= (\theta_1, s_1^1),\ h_1^1 = (\theta_1, s_1^1, a_1^1, a_2^1, s_1^2) \\ h_2^0 &= (s_2^1),\ h_2^1 = (s_2^1, a_1^1, a_2^1, s_2^2) \end{aligned}$$

An action profile a^τ is the common history, but s_i^τ is private information. The mediator designs a

[10] The space must be coherent, that is, a belief should produce marginal distributions consistently with its lower order beliefs.

[11] The following papers are notable about the belief type structure: Harsanyi (1967), Mertens and Zamir (1985), Brandenburger and Dekel (1993), Penta (2012), Dekel and Siniscalchi (2015), Perea (2012, 2023).

signaling such that

$$\sigma^\tau : H^{a,\tau-1} \times H_1^{s,\tau-1} \times H_2^{s,\tau-1} \to \Delta(S_1^\tau \times S_2^\tau)$$

for each τ, where $H^{a,\tau}$ and $H_i^{s,\tau}$ denote the set of action histories and i's signal histories, respectively, until $\tau - 1$. Obviously, public signaling is possible in that $\sigma^\tau(s_j^\tau \mid s_i^\tau; h_i^{\tau-1})$ could be a singleton for all i and in every τ.[12]

Consider i's belief map

$$b_i : H^{a,\tau-1} \times H_i^{s,\tau-1} \times S_i^\tau \to \Delta(\Theta \times H_j^{s,\tau'-1} \times S_j^{\tau'} \times A_j^{\tau'}) \tag{4}$$

Notice that, at the true history $h^\tau \equiv (h^{a,\tau-1}, h^{s,\tau-1}, s^\tau)$, all players share the common action history $h^{a,\tau-1}$, but each has private information $(h_i^{s,\tau-1}, s_i^\tau)$ which can be interpreted as player i's type at $h_i^\tau \equiv (h^{a,\tau-1}, h_i^{s,\tau-1}, s_i^\tau)$. i gets different information at different h_i^τ, so he needs to update his belief. Players perceive that their beliefs may change as the game proceeds. One type should guess the other's type $(h_j^{s,\tau'-1}, s_j^{\tau'})$ and his concurrent action $a_j^{\tau'}$ varying moments of the opponent's past of future. A circulation among these types constructs the belief hierarchy.

The belief must be consistent with the prior and the signaling σ for every h_i^τ. Furthermore, each type at h_i^τ must follow a sequentially rational a_i such that it gives a higher expected payoff than any other available a_i', following reachable (h, s, a) based on the belief b_i.[13] We qualify (a, b) as the equilibrium that is sequentially rational and consistent with respect to information structure (p, S, σ) to produce PBE. Moreover, (S, σ) constitutes Bayes correlated equilibrium (Bergemann and Morris, 2013) in each h^τ.

In our model, we can reduce the type space. Player 1 observes his type before getting a signal.

[12] Given the signal s_i^τ, i is sure that j gets the signal s_j^τ. Then, we can introduce a single commonly observable signal in τ, that is, a public one.

[13] See Penta (2012) for the definition of the sequential rational choice of a belief type.

Also, at any h_1^2, 1 must choose his own dominant action regardless of h_1^2. So, his belief on Θ_1 and a signal s_1^2 are redundant. As player 2 knows this fact, $marg_{\Theta_1} b_2(h_2^\tau) = marg_{A_1^2} b_2(h_2^\tau)$ for every h_2^τ: her marginal belief on Θ_1 is sufficient to guess a_1^2. Extending Lemma 1, Dove type's sequentially rational choice is D in every h_1^τ. So, for any $a_1^1 = H$ in $h^{a,1}$, the beliefs collapse to $b_1(\cdot, a_2^2 = H \mid h^{a,1}, \cdot) = 1$ and $b_2(\theta_H, a_1^1 = H \mid h^{a,1}, \cdot) = 1$. Summarizing, it is sufficient to construct the set of signals $S_1^1 \times S_2^1 \times S_2^2$, the following signaling σ and belief structure b_i:

$$\begin{aligned}
\sigma^1(\theta) &\in \Delta(S_1^1 \times S_2^1 \mid \theta) \\
\sigma^2(\theta, s_1^1, s_2^1, (D, a_2^1)) &\in \Delta(S_2^2 \mid \theta, s_1^1, s_2^1, (D, a_2^1)) \\
\sigma^2(\theta, s_1^1, s_2^1, (H, \cdot)) &= \phi \text{ (Null signal)}
\end{aligned}$$

$$\begin{aligned}
b_1(\theta, s_1^1) &\in \Delta(S_2^1 \times \{D\} \times A_2^1 \times S_2^2 \times A_2^2 \mid \theta, s_1^1) \\
b_1(\theta, s_1^1, (D, a_2^1)) &\in \Delta(S_2^1 \times S_2^2 \times A_2^2 \mid \theta, s_1^1, (D, a_2^1)) \\
b_1(\theta, s_1^1, (H, \cdot)) &= \{(\cdot, s_2^2 = \phi, \cdot, a_2^2 = H)\}
\end{aligned}$$

$$\begin{aligned}
b_2(s_2^1) &\in \Delta(\Theta_1 \times S_1^1 \times A_1^1 \mid s_2^1) \\
b_2(s_2^1, (D, a_2^1), s_2^2) &\in \Delta(\Theta_1 \mid s_2^1, (D, a_2^1), s_2^2) \\
b_1(s_2^1, (H, \cdot)) &= \{(\theta_1 = \theta_H, \cdot, s_2^2 = \phi, a_1^2 = H)\}
\end{aligned}$$

The mediator's objective is to attain a maximum probability of $a^\tau = (D, D)$ in every period. Benchmarking the static persuasion model, two signals are required: one should pool of θ_D and θ_H sufficiently to induce player 2's D, and the other reveals θ_H with some probability. Abusing notation, like expressions of the mixed strategy, the scheme is expressed with "$\sigma^1(\theta_H) : x s_2^D + (1-x) s_2^H$", "$\sigma^1(\theta_D) : s_2^D$" where x denotes the probability of the signal s_2^D when θ_H. For given the signal s_2^D, 2 has a posterior belief $\frac{px}{px+1-p}$ on θ_H. Note that player 2's cutoff belief on $a_1 = H$ is

$$c_2 \equiv \frac{\theta_2 - \beta}{\gamma - \beta}$$

to choose D. To induce the belief, the probability of the signal s_2^D when θ_H must be

$$k_2 \equiv \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$$

It is the maximum probability, $\Pr(s_2^D \mid \theta_H)$, with which player 2 can tolerate to choose D. Let's call "$s_2^D \mid \theta_H$" as "false signal". It is not a coincidence that this probability is equal to m_{1H}^1, the mixed strategy probability of Hawk type's D, in PBE (1.1) in Proposition 1. The mediation necessarily reveals the bad case with some probability to make the other signals more trustful. When θ_H, it is impossible to attain the outcome $(a^1 = (D,D), a^2 = (H,D))$ with a probability 1 because the mediator should control the probability of false signal less than k_2. Otherwise, the signal s_2^D is too exaggerating of the peace outcome. So, the mediation problem is to determine the probabilities of outcomes $(a^1 = (D,D), a^2 = (H,D))$ or $(a^1 = (D,D), a^2 = (H,H))$ when θ_H.

Lemma 3 *In the signaling σ, the probability of the equilibrium outcome $(a^1 = (D,D), a^2 = (H,D))$ when θ_H is not greater than k_2.*

The consistency requirement restricts achievable probabilities of the desirable outcome. Even in dynamics, the probability of the false signal should not go beyond the cutoff eventually. However, we can appeal more to players' dynamic incentives. Fundamentally, player 2's goal is to coordinate her action perfectly with player 1's one. Even if she knows Hawk type, she is willing to play D when the opponent's action D is guaranteed. Hawk type wants to trick her to get a benefit β, but there is still a dilemma: after tricking her, he must get a worse payoff from (H,H) in all rest of the periods. He is willing to coordinate on his second best (D,D) presently if the best scenario (H,D) is possible with a chance in the future. In specific, Hawk type compares $0+\delta(r\beta-\theta_H)$ with $a_{1H}=D.H$ and $\beta-\theta_H+\delta(-\theta_H)$ with $a_{1H}=H.H$, against $a_2=D.rD+(1-r)H/H$, where r is the probability that 2 plays D in $\tau=2$ if he plays D in $\tau=1$. The probability must be at least

$$k_{1H}\equiv\frac{\beta-\theta_H}{\delta\beta}$$

It is the minimum probability of the false signal such that Hawk type requires to respond with $a^1_{1H}=D$. The probability is valid under the condition (3), and it is the same with m^2_2, the mixed strategy probability of 2's D in $\tau=2$ in PBE (1.1) in Proposition 1. Interestingly, Hawk type must be uncertain for player 2's strategy or signal. The condition $k_{1H}\leq k_2$ implies that there is an agreeable range of the false signal. The mediation works by implementing it.

For convenience, we will assume that players choose D in favor of the mediator's preference when they are indifferent between D and H. We mainly focus on maximization of the probability of $(a^1=(D,D),a^2=(H,D))$ when θ_H.

Proposition 4 *Assume $k_{1H}\leq k_2$. Consider the following public signaling:*

$$\begin{cases} S^1=\phi \\ S^2=\{s^{2,D},s^{2,H}\} \\ \sigma^2(\theta_H,(D,D)):k_2s^{2,D}+(1-k_2)s^{2,H} \\ \sigma^2(\theta_H,(H,D)):s^{2,H} \\ \sigma^2(\theta_H,(\cdot,H)):\phi \\ \sigma^2(\theta_D,(D,D)):s^{2,D} \\ \sigma^2(\theta_D,other\ a^1):\phi \end{cases} \tag{5}$$

It achieves the equilibrium outcomes: $(a^1 = (D, D), a^2 = (H, D))$ *and* $(a^1 = (D, D), a^2 = (H, H))$ *with probability* k_2 *and* $1 - k_2$*, respectively when* θ_H*, and* $(a^1 = (D, D), a^2 = (D, D))$ *with a probability 1 when* θ_D*.*

The public signaling (5) is optimal to achieve the maximum probability of $(a^1 = (D, D), a^2 = (H, D))$ in the sense of Lemma 3. Surprisingly, it attains the outcome (D, D) in $\tau = 1$ with a probability 1. Notably, there is no information at all in $\tau = 1$, but the signaling is delayed to $\tau = 2$ to appeal to dynamic incentives. Hawk type expects a false signal for some degree in the future so that he gets a chance to extract β. Otherwise, the mediator reveals his type truthfully, which plays as a punishment for his deviation. Player 2 expects more precise future information about the state, which plays a reward for her obedience.

We can construct another variation of it, private signaling.

Proposition 5 *Assume* $k_{1H} \leq k_2$*. Consider the following private signaling:*

$$\begin{cases} S_1^1 = S_2^2 = \phi \\ S_2^1 = \{s_2^{1,D}, s_2^{1,H}\} \\ \sigma^1(\theta_H) : k_2 s_2^{1,D} + (1 - k_2) s_2^{1,H} \\ \sigma^1(\theta_D) : s_2^{1,D} \end{cases} \tag{6}$$

It achieves the same equilibrium outcomes with (5): $(a^1 = (D, D), a^2 = (H, D))$ *and* $(a^1 = (D, D), a^2 = (H, H))$ *with probability* k_2 *and* $1 - k_2$*, respectively when* θ_H*, and* $(a^1 = (D, D), a^2 = (D, D))$ *with a probability 1 when* θ_D*.*

The private signaling (6) is also optimal in the sense that it yields the equivalent outcome to the public one (5). Both signalings share an essential property: Hawk type in $\tau = 1$ should be uncertain of player 2's belief. They manipulate Hawk type's second-order belief, that is, his belief on player 2's belief on (θ, a_1). In PBE and cheap talk, this kind of manipulation was impossible as player 1 was always omniscient: $h_1^\tau = h^\tau$ for every τ. Any messages exchanged in the negotiation table should become common knowledge to all players, too. The situation where all information or histories are shared in every stage is not advantageous to player 2 as she is also afraid of her information to be revealed. In the original game, Hawk type already knew and will know all parameters in every period. He would break up the peace when he learns that player 2 realizes his type. Thus, manipulating his higher-order belief through the information system is the only way to make a difference.

We can interpret the signaling geographically. Fix h^τ. Consider two variables: player 2's belief on θ_H, "$b_2(\theta_H)$", and Hawk type's second-order belief for it, "$b_1(b_2(\theta_H))$". In the space b_1-b_2, the signaling picks up points of signals (b_1, b_2) to induce individual belief. There is a constraint: the prior belief p, the point (p, p), should be expressed with a convex combination of the signals.[14] The public signaling should pick only points along the 45-degree line as each of them induces $b_1(b_2(\theta_H)) = b_2(\theta_H)$. (5) picks two points (c_2, c_2) and $(1, 1)$. The signaling expands the point (p, p) to others through a convex combination. The private signaling (6) picks only two points (p, c_2) and $(p, 1)$. That is, Hawk type has only distributions over two points whose mean is p. In both signalings, the weights for "(p, p)-$(1, 1)$" or "(p, p)-$(p, 1)$" is k_2.

Note again that the condition $k_{1H} \leq k_2$ makes it possible probabilities of the false signal and of truth-revelation acceptable to both players. In fact, the mediator can pick one probability in $[k_{1H}, k_2]$ according to her preference between $a^2 = (H, D)$ and $a^2 = (H, H)$.

Corollary 1 *Assume $k_{1H} \leq k_2$. There exist signalings that induces the equilibrium outcome* $(a^1 =$

[14] This constraint is called "Bayes plausibility."

$(D,D), a^2 = (H,D))$ with a probability $x \in [k_{1H}, k_2]$ when θ_H, and all other outcomes are same with (5) and (6).

One concern is that the outcome $a^2 = (D,H)$ inevitably occurs from the false signal. Interestingly, the damage $(-\gamma)$ from it to player 2 increases in γ while the probability of it decreases. The change of her expected payoff under the signaling is not straightforward. Recall that PBE (1.1) in Proposition 1 attains the highest probability of $a^1 = (D,D)$, and is the best for all players when δ and γ are moderate. We can compare the mediation with k_2 and PBE (1.1). All necessary conditions are possibly valid for the comparison.

Proposition 6 *In the interim stage <2>, all players' expected payoff in (5) or (6) is higher than PBE (1.1) in Proposition 1.*

Corollary 2 *Consider the private signaling (6) and the interim stage <3>. When player 2 realizes the signal $s_2^{1,D}$, her expected payoff is higher than (1.1) in Proposition 1.*

Once $s_2^{1,H}$ is given, player 2 may expect a lower payoff than PBE (1.1) because she now knows θ_H. For given $s_2^{1,D}$, she is concerned with the loss $-\gamma$, but the probability of $a^2 = (H,D)$ is getting endogenously lower enough to make her expect a higher payoff than PBE. Even Hawk type prefers hiding his type to revealing and getting a short-term benefit. All players trust the mediator's determination for a peaceful outcome, so the signals from her inspire players to expect better outcomes.

5. Variations

We will check signaling variations by changing parameter conditions and the mediator's preference. It must lead players to stay in peace patiently, imposing discrepant expectations in each one's favorable outcome. The key idea is how to spread the false signal distribution. Consider the following signaling that reveals θ_H truthfully or not:

$$\begin{cases} S_1^\tau = \phi \\ S_2^\tau = \{s_2^{\tau,D}, s_2^{\tau,H}\} \\ \sigma^1(\theta_H) : xs_2^{1,D} + (1-x)s_2^{1,H} \\ \sigma^2(\theta_H, (D,D), s_2^{1,D}) : ys_2^{2,D} + (1-y)s_2^{2,H} \\ \sigma^2(\theta_H, otherswise) : s_2^{2,H} \\ \sigma^\tau(\theta_D, h^{\tau-1}) : s_2^{\tau,D} \text{ for all } h^{\tau-1} \end{cases} \tag{7}$$

where x and y denote the probability of the false signals in each period. In $\tau = 2$, player 2's posterior beliefs are

$$\begin{cases} b_1(s_2^{1,D}, a_2 = D.D/H \mid \theta_H) = x \\ b_1(s_2^{1,H}, a_2 = D.H \mid \theta_H) = 1-x \\ b_1(s_2^{1,D}, s_2^{2,D}, a_2^2 = D \mid \theta_H, (D,D)) = xy \\ b_1(s_2^{1,D}, s_2^{2,H}, a_2^2 = H \mid \theta_H, (D,D)) = x(1-y) \\ b_1(s_2^{1,H}, s_2^{2,H}, a_2^2 = H \mid \theta_H, (D,H)) = 1 \end{cases} \tag{8}$$

$$
\begin{cases}
margb_2(\theta_H, a_1 = D.H \mid s_2^{1,D}) = \frac{px}{px+1-p} \\
margb_2(\theta_H, s_2^{2,D} \mid s_2^{1,D}) = \frac{pxy}{px+1-p} \\
margb_2(\theta_H, a_1 = D.H \mid s_2^{1,H}) = 1 \\
b_2(\theta_H, a_1^2 = H \mid s_2^{1,D}, (D,D), s_2^{2,D}) = \frac{pxy}{pxy+1-p} \\
b_1(s_2^{1,H}, s_2^{2,H}, a_2^2 = H \mid \theta_H, (D,H)) = 1 \\
b_2(\theta_H, a_1^2 = H \mid s_2^{1,D}, (D,D), s_2^{2,H}) = 1
\end{cases}
\tag{9}
$$

To attain the outcome $a = (D,D).(H.D)$ when θ_H with the maximum probability, it is necessary

$$k_{1H} \leq xy \leq k_2$$

following the previous result.

Corollary 3 *Assume $k_{1H} \leq k_2$. Consider the private signaling (7). If $xy \in [k_{1H}, k_2]$, then it achieves the same equilibrium outcomes with (6) replacing k_2 with xy.*

xy should be a fixed value. The dynamic follow of true-false signal distribution is intertemporally and endogenously controlled in the system. When the mediation sets a target probability $xy = k^* \in [k_{1H}, k_2]$, the period probability of the false signal in $\tau = 1$ should be at least k^*,

$$x \geq k^*$$

, that is, the mediator must exaggerate the present peace. The signaling appeals to players' incentives at different periods. The mediator entices Hawk type with more false signals in the present and appeases player 2 with more precise information in the future. Even if player 2's posterior belief is quite pessimistic, she takes a peaceful stance patiently, waiting for a smaller y, the more precise information.

The signaling is deemed static as the dynamic variations are spreading of the false signal distribution of the stage game. However, if we flip the condition $k_{1H} \leq k_2$, the signaling needs to be more subtle way. The problem arises with a higher p. Geographically, in the space b_1-b_2 again, any spreading of the point (p, p) with only two points can not obtain the desirable weight k^*. The point (p, p) is located too close to $(1, 1)$. We need more signals to attain k^*.

We can interpret one signal as one plan recommendation. So, we will find the probability distribution over the private action recommendations. Let a_i^τ be the recommendation (signal) for player i in τ, and it is trimmed to describe only on-the-equilibrium-paths. We can consider the following signaling sufficiently:

$$
\begin{cases}
\begin{array}{c|ccc}
\theta_H & D.D/H_2^1 & D.H_2^1 & H.H_2^1 \\ \hline
D.H_1^1 & x_1 & x_2 & x_3 \\
H.H_1^1 & x_4 & x_5 & x_6
\end{array} \\
\begin{array}{c|cc}
\theta_H, (D.H_1^1, D.D/H_2^1), (D, D) & D_2^2 & H_2^2 \\ \hline
 & y_1 & y_2
\end{array} \\
\text{For Hawk type's deviation, reveal } \theta_H \\
\text{For player 2's deviation, no signal}
\end{cases} \tag{10}
$$

where $\sum x_j = \sum y_j = 1$. "$D.D/H_2^1$" denotes a recommendation for player 2 in $\tau = 1$ such that she chooses $a_2^1 = D$ and $a_2^2 = D$ or H depending on the additional recommendation in $\tau = 2$ or action history in $\tau = 1$. "$D.H_2^1$" denotes $a_2^1 = D$ and $a_2^2 = H$ which is plausible when she knows Hawk type, but $a_1^1 = D$ is expected (like the signal $s_2^{1,H}$ in (6)). At the history $h^2 = (\theta_H, (D.H_1^1, D.D/H_2^1), (D, D))$, the additional recommendation will be given in $\tau = 2$. Otherwise, they will play only $a^2 = (H, H)$.

Since new outcomes are expected in the signaling, we must assume the mediator's preference for them. To skip the scale analysis between outcomes, let's consider a lexicographic preference. As a pacifist, she is eager to maximize $a(\theta_D) = ((D, D), (D, D))$ and $a(\theta_H) = ((D, D), (H, D))$ as the first best. $a(\theta_D) = ((D, D), (D, D))$ is possible with 100% by recommending $((D.H_1^1, D.D/H_2^1), D_2^2)$ with a probability 1. This probability is also helpful to be pooled with $a(\theta_H) = ((D, D), (H, D))$ to induce player 2's optimistic belief. Notice that $((D.H_1^1, D.D/H_2^1), H_2^2)$ and $(D.H_1^1, D.H_2^1)$ yield the same outcome $a(\theta_H) = ((D, D), (H, H))$ which is the second best for her. In specific, the probability of the first best $(a(\theta_H) = ((D, D), (H, D)))$ and the second best $(a(\theta_H) = ((D, D), (H, H)))$ is x_1y_1 and $x_1(1-y_1)+x_2$, respectively. Note that x_1y_1 and x_1+x_2 are bounded in the equilibrium. This means, even if x_1 is getting smaller (higher), we can compensate for it with a higher (smaller) y_1 and x_2, which does not hurt her utility as long as both x_1y_1 and $x_1 + x_2$ attain their maxima. We will postpone assuming the preference for other outcomes later. So, the mediator's lexicographic preference is expressed as

$$x_1y_1 \succ x_1 + x_2 \succ x_3, x_4, x_5, x_6$$

Lemma 4 *Assume $k_2 < k_{1H} < 1$. Then, $x_1 + x_2 \leq k_2/k_{1H}$ in the optimal signaling.*

The condition $k_2 < k_{1H}$ is possible when p is large. If the player's prior is so pessimistic, there is a conflict in the probability of the false signal. While player 2 requires a small of it to play $a_2 = D$, Hawk type is not persuaded by that probability. The range of probability of the fake signal is not agreeable. The signaling should give up some of $\Pr(D.H_1^1)$ so that the mediator can assign freely between x_1 and x_2, giving up the scheme of x_1 and $1-x_1$. The upper bound of x_1+x_2 is necessarily $k_2/k_{1H} < 1$, and we will see it is attainable. It is impossible to achieve $a^1 = (D, D)$ with 100%. The signaling must plan another outcome for the target outcome.

We can characterize desirable beliefs of the equilibrium. For given signal D/H_2^1, player 2 believes

$$\begin{aligned} margb_2(a_1 &= D.D \mid D.D/H_2^1) = \frac{px_1 + 1 - p}{px_1 + px_4 + 1 - p} \\ margb_2(\theta_H &\mid D.D/H_2^1) = \frac{px_1 + px_4}{px_1 + px_4 + 1 - p} \end{aligned}$$

She may have a biased belief toward θ_H beyond her critical belief to play D ($margb_2(\theta_H \mid D.D/H_2^1) \geq c_2$). However, she is supposed to get more information in $\tau = 2$. At $h_2^2 = (D.D/H_1^1, (D, D), D_2^2)$,

$$margb_2(\theta_H \mid D.D/H_1^1, (D, D), D_2^2) = \frac{px_1y_1}{px_1y_1 + 1 - p} = c_2$$

that attains her critical belief with $x_1y_1 = k_2$. For given $D.H_2^1$,

$$\begin{aligned} margb_2(a_1^1 &= D \mid D.H_2^1) = \frac{px_2}{px_2 + px_5} \\ margb_2(\theta_H &\mid D.H_2^1) = 1 \end{aligned}$$

The signal x_2 attracts her with the coordination incentive rather than the information one.

For given $D.H_1^1$,

$$margb_{1H}(a_2 = D.D \mid D.H_1^1) = \frac{x_1y_1}{x_1 + x_2} = \frac{k_2}{k_2/k_{1H}} = k_{1H}$$

which is the Hawk type's critical belief. Since the mediator can no longer control Hawk type's action in $\tau = 2$ anymore, she must target his belief in $\tau = 1$.

Proposition 7 *Assume $k_2 < k_{1H} < 1$, and the mediator prefers $x_4 \succ x_5, x_6$. Consider the signaling (10).*

If $k_{1H} \leq \frac{\gamma-\beta}{\gamma-\theta_2}\frac{k_2}{1-k_2}$*, then the following is optimal:*

$$\begin{cases} x_1 = \frac{k_2}{k_{1H}} \\ x_4 = 1 - \frac{k_2}{k_{1H}} \\ x_2 = x_3 = x_4 = x_5 = x_6 = 0 \\ y_1 = k_{1H} \end{cases} \tag{11}$$

Intuitively, it is necessary $x_3 = 0$ (Lemma 6) whose outcome is $a^1 = (D, H)$. The strategic action $a_1^1 = D$ may give player 2 a belief on Dove type, but this information is also possible from the mediator as well. Intuitively, $a_2^1 = H$ does not help induce Hawk to play $a_1^1 = D$.

When $k_2 < k_{1H}$, the key idea of signaling is how to assign the rest of probabilities to the signals in which incentive compatible condition (IC) slacks. The IC for $D.H_1^1$ implies that $x_1 + x_2$ must be binding to $\frac{k_2}{k_{1H}}$ in the optimum, but they are still free within the bound. Then, adjusting them, the mediator control ICs for $D.D/H_2^1$ and $D.H_2^1$ to be binding or slack. Assigning $\frac{k_2}{k_{1H}}$ on x_1, cast all the rest of probabilities to x_4, To make IC for $D.D/H_2^1$ still hold, the condition $k_{1H} \leq \frac{\gamma-\beta}{\gamma-\theta_2}\frac{k_2}{1-k_2}$ is technically needed. Otherwise, we need another signal to absorb the probabilities, which depends on her preference between x_5 and x_6. Let's skip this case. Instead, we will go through cases $x_5 > 0$ necessarily.

Note that both signals x_4 and x_5 implement the same outcome, $a = ((H, D), (H, H))$ inevitably with probability $1 - k_2/k_{1H}$. The difference is that player 2 learns Hawk type by herself from the action history under x_4 while she does it from the mediator's signal under x_5.

Proposition 8 *Assume* $k_2 < k_{1H} < 1$*, and the mediator prefers* $x_5 \succ x_6 \succ x_4$*. Consider the signaling (10).*

If $k_{1H} \leq \frac{k_2}{1-(1-k_2)c_2}$, *then the following is optimal:*

$$\begin{cases} x_2 \in [\frac{\gamma-\theta_2}{\theta_2-\beta} - \frac{1-p}{p}\frac{1}{k_{1H}}, k_2(\frac{1}{k_{1H}} - 1)] \\ x_1 = \frac{k_2}{k_{1H}} - x_2 \\ x_5 = 1 - \frac{k_2}{k_{1H}} \\ x_3 = x_4 = x_6 = 0 \\ y_1 = \frac{k_2}{x_1} \end{cases} \tag{12}$$

If $k_{1H} > \frac{k_2}{1-(1-k_2)c_2}$, *then the following is optimal:*

$$\begin{cases} x_2 = k_2(\frac{1}{k_{1H}} - 1) \\ x_1 = k_2 \\ x_5 = \frac{\theta_2-\beta}{\gamma-\theta_2} k_2(\frac{1}{k_{1H}} - 1) \\ x_6 = 1 - x_1 - x_2 - x_5 > 0 \\ x_3 = x_4 = 0 \\ y_1 = 1 \end{cases} \tag{13}$$

To assign a positive probability on x_5, we need a high x_2 to get valid IC for $D.H_2^1$. Subtracting some probability from x_1 is the solution, but it should not violate the intertemporal probability $x_1 y_1 = k_2$. If k_{1H} is not very high, then there is some margin for (x_1, x_2) to make x_5 absorb the rest of the probabilities, and then $x_6 = 0$. Otherwise, we need $x_6 > 0$.

When Hawk realizes that player 2 knows his type (under x_5), he will play $a_{1H} = H.H$. If the mediator decides to reveal player 2's second-order belief to him (with $H.H_1^1$), then player 2 needs more rewards, more precise information or coordinated actions in $\tau = 1$. x_2 appeals to both of them. The signaling should advance her incentives from the future to the present.

Proposition 9 *Consider a new game with a common prior* $\hat{p} \equiv \frac{p(1-x_6)}{p(1-x_6)+1-p} < p$ *on* θ_H*. Then, the signaling (12) replacing* p *with* $\hat{p}$ *is optimal as* $k_{1H} \leq \frac{\hat{k}_2}{1-(1-\hat{k}_2)c_2}$ *where* $\hat{k}_2 \equiv \frac{1-\hat{p}}{\hat{p}} \frac{\theta_2-\beta}{\gamma-\theta_2}$*.*

For the signal x_6, player 2 realizes the true state and action, $b_2(\theta_H, a_1 = H.H_1^1 \mid H.H_2^1) = 1$ while Hawk type believes $b_{1H}(a_2 = D.H_2^1 \mid H.H_1^1) = \frac{x_5}{x_5+x_6}$, $b_{1H}(a_2 = H.H_2^1 \mid H.H_1^1) = \frac{x_6}{x_5+x_6}$, and $a_1 = H.H_1^1$ is rational. The signal x_6 reveals θ_H to all players: player 2 knows θ_H, Hawk type knows that player 2 knows θ_H, and so on:

$$margb_2(\theta_H \mid H.H_2^1) = margb_{1H}(margb_2(\theta_H) = 1 \mid H.H_1^1) = 1$$

The role of the signal x_6 is to take up only the minimal probability and make the other signals more reliable. So, we can design an equivalent public signal, say s^H, that reveals θ_H truthfully with a probability p_6 in the previous stage before sending other private signals. At this stage, if they do "not" receive s^H, player 2's marginal belief on θ_H must be updated to

$$\hat{p} \equiv \frac{p(1-x_6)}{p(1-x_6)+1-p} < p$$

, and player 1 also should believe it. It is as if they are facing a new game with a lower prior $\hat{p}$ but the same information/belief structure. Note that k_2 depends on the prior, but k_{1H} and c_2 do not. The mediator can determine x_6 to switch the case $k_{1H} > \frac{k_2}{1-(1-k_2)c_2}$ to $k_{1H} \leq \frac{\hat{k}_2}{1-(1-\hat{k}_2)c_2}$ where $\hat{k}_2 \equiv \frac{1-\hat{p}}{\hat{p}} \frac{\theta_2-\beta}{\gamma-\theta_2}$. In the new game, we can pick the signaling (12) as the solution in which the signal $(H.H_1^1, H.H_2^1)$ is not needed anymore, provided s^H is not realized.

Proposition 10 *Assume* $k_2 < k_{1H} < 1$*, and the mediator prefers* $x_6 \succ x_4, x_5$*. Then, the following* *is optimal:*

$$\begin{cases} x_1 \in [k_2, k_2/k_{1H}] \\ x_2 = k_2/k_{1H} - x_1 \\ x_6 = 1 - k_2/k_{1H} \\ x_3 = x_4 = x_5 = 0 \\ y_1 = k_2/x_1 \end{cases}$$

The proof is straightforward: $x_1 + x_2$ attains the maximum, x_6 captures the rest of the probabilities, and all ICs hold. We can also regard this signaling with $x_6 > 0$ as the case switching getting a lower prior. Any signals except x_6 yield a new belief on θ_H such that

$$\hat{p} \equiv \frac{p(x_1 + x_2)}{p(x_1 + x_2) + 1 - p} = \frac{pk_2/k_{1H}}{pk_2/k_{1H} + 1 - p}$$

The belief is common knowledge since, at any h that excludes $(H.H_1^1, H.H_2^1)$,

$$\begin{aligned} margb_i(a_j &\neq H.H_j^1 \mid h_i) = 1 \\ margb_{1H}(\theta_H &\mid h_1) = margb_{1D}(\theta_D \mid h_1) = 1 \\ margb_2(\theta_H &\mid h_2) = \hat{p} \end{aligned}$$

The players are facing a new prior $\hat{p}$ with the same information/belief structure. Note that

$$\hat{k}_2 \equiv \frac{1-\hat{p}}{\hat{p}} \frac{\theta_2 - \beta}{\gamma - \theta_2} = \frac{1-p}{p} \frac{\theta_2 - \beta}{\gamma - \theta_2} \frac{k_{1H}}{k_2} = k_{1H}$$

, so the parameter condition of Proposition 5 is satisfied. It is sufficient to assign on $\hat{x}_1$ and $\hat{x}_2$. Following Corollary 3, the solution in the new game by replacing p with $\hat{p}$ is

$$\begin{aligned}
\hat{x}_1 &= \frac{k_{1H}}{k_2} x_1 \in [\hat{k}_2, 1] \\
\hat{x}_2 &= 1 - \hat{x}_1 = \frac{k_{1H}}{k_2} x_2 \\
\hat{x}_1 \hat{y}_1 &= \hat{k}_2 = \frac{k_{1H}}{k_2} x_1 y_1
\end{aligned}$$

The signaling separates the game into two common priors $\hat{p}$ or 1. The former has a probability k_2/k_{1H} and is sufficient to produce a desirable parameter condition that requires only two signals x_1 and x_2. When p is very high, war is inevitable. If the mediator always emphasizes only peace, her information will lose credibility. The possibility of declaring war helps the other cases bring them to an optimistic situation.

6. Discussion

6.1. Epistemic Structure

Based on the equilibrium belief system with the signaling, it is possible to construct an epistemic structure. At one history, or equivalently, one information set $h_i^\tau = (h^{a,\tau-1}, h_i^{s,\tau-1}, s_i^\tau)$, the player i has a belief about h_j^τ. At fixed h_i^τ but varying $h_j^{\tau'}$, i predicts j's beliefs for all moments (recall (4)) and determines a rational action plan $a_i^{\geq\tau} \in A_i^{\geq\tau} \equiv \prod_{\hat{\tau}\geq\tau} A_i^{\hat{\tau}}$. So, $(h_i^\tau, a_i^{\geq\tau})$ represents i's belief type that constructs a belief distribution over $H_j^{\tau'} \times A_j^{\geq\tau'}$. At each h^τ, players share the common knowledge $h^{a,\tau-1}$ concurrently, and players commonly predict each player i's rational plan for h_i^τ. Since one signal is assumed to produce only one plan, one belief type has one action plan. The mapping of plans represents the mapping of belief types. The collection of beliefs over the opponent's plans at each $h^{a,\tau-1}$ sufficiently describes the epistemic structure. That is what the table (10) describes.

For example, the signaling (13) constructs a circulation among belief types as following:

$h^1 = \theta_H$

$$
\begin{array}{ccccccccccc}
 & & D.D/H_2^1 & \rightarrow & D.H_1^1 & \rightarrow & ... & & & & \\
D.H_1^1 & \nearrow & & \searrow & & & & & & & \\
 & \searrow & & & D.D_1^1 & \rightarrow & D.D/H_2^1 & \rightarrow & ... & & \\
 & & D.H_2^1 & \rightarrow & D.H_1^1 & \rightarrow & ... & & & & \\
 & & & \searrow & H.H_1^1 & \rightarrow & D.H_2^1 & \rightarrow & ... & & \\
 & & & & & \searrow & H.H_2^1 & \rightarrow & H.H_1^1 & \rightarrow & ...
\end{array}
$$

$h^2 = (\theta_H, (D.H_1^1, D.D/H_2^1), (D, D))$

$$
\begin{array}{ccccccccc}
 & & D_2^2 & \rightarrow & H_1^2 & \rightarrow & ... & & \\
H_1^2 & \nearrow & & \searrow & & & & & \\
 & \searrow & & & D_1^2 & \rightarrow & D_2^2 & \rightarrow & ... \\
 & & H_2^2 & \rightarrow & H_1^2 & \rightarrow & ... & &
\end{array}
$$

Obviously, the same epistemic structures yield the same equilibrium outcome, but not vice versa. Recall Proposition 9: The signaling (13) can spread into two other signalings: One to reveal θ_H truthfully with a probability x_6 and the other such as (12) with $\hat{p} \equiv \frac{p(1-x_6)}{p(1-x_6)+1-p}$. Both yield the same outcomes, but they produce different epistemic structures. While x_6 (13) constructs non-trivial circulations of belief types, the revelation in the new signaling constructs common knowledge of belief types, that is, $H.H_1^1 \rightarrow H.H_2^1 \rightarrow H.H_1^1 \rightarrow ....$

6.2. Spy Espionage Model

We have assumed an omniscient mediator, and she could send private signals freely. The espionage and counter-espionage systems also can obtain true information about the opponent and play with both sides with different information. It operates covertly without sharing common information. We can transform the information mediation to an espionage model in which player 2 dispatches a spy who can discover true θ or D-day of his secret operation, whether $\tau = 1$ or 2.[15] In the beginning, the player 2 is very suspicious of Hawk type (a high p), and she prefers espionage rather than a peace negotiation because hiding her knowledge about the type is beneficial, taking advantage of the information.

No doubt, Dove type always stays in peace. However, there is a chance of spies being caught. Hawk type can convert and use them to disinform to player 2, such as the counter-espionage. It is also the best scenario for Hawk type, the maximization of the probability of $a = ((D, D), (H, D))$ with a false signal from the mediation model. Player 2 always doubts the report of Dove from the agent. Hawk type needs to design the report more trustfully by controlling xy to balance true and false information (British MI5's Double Cross system also used these strategies). The objective is to persuade her to change the decision to $a_2^1 = D$, hiding his secret project, and maximize the chance of tricking her in $\tau = 2$. In (10), achieving maximum x_1 should be the most successful counter-espionage operation.

Player 2 also utilizes disinformation with a patriotic agent. Her goal is to maximize $a = ((D, D), (H, H))$ when θ_H. The role of her spy is not limited to simply digging up the opponent's information, but it also plants false information for the enemy. The spy found θ_H and delivered it covertly to player 2. In addition, releasing disinformation to make Hawk believe she is

[15] $H.H_1^1$ means the d-day is $\tau = 1$ and $D.H_2^1$ does $\tau = 2$. Player 2 wants to respond to it coordinately.

being deceived to choose $a_2 = D.D$ (actually, she plans $a_2 = D.H$). If the chance is considerable ($xy \geq k_{1H}$), he is willing to switch his decision to $a^1_{1H} = D$.[16] The information system discovers player 1's true plan successfully, in addition, derives a favorable action toward her, such as Soviet Union NKVD's Operation Snow. This espionage is interpreted as maximizing x_2 of the model.

6.3. Testing Strategy

Now, let's endow the commitment to player 2. Then, she may want to derive player 1's true type by herself. Let's say, "testing strategy" that can reveal player 1's type truthfully in $\tau = 1$. Following Proposition 11, it must be $a_2 = H.D/H$. We can check an incentive of the strategy to player 2.

Proposition 11 *Suppose $k_{1H} \leq k_2$ and $-\frac{\theta_2}{\gamma-\theta_2} < (1-\delta)k_2$. Even with player 2 commitment power to her strategy, she prefers the mediator's signaling (6) or its variation (7) to her testing strategy.*

Intuitively, higher γ and δ give player 2 more incentive to play it. δ matters relatively more than γ. For the signaling under the condition $k_{1H} \leq k_2$, the probability of the false signal depends on γ. In other words, player 2's loss from being tricked is endogenously controlled. However, δ is not controlled by the signaling, so she is more afraid of future loss through δ rather than γ.

7. Conclusions

The optimal signaling must induce the peace-making signal falsely with some probability. For the less-informed player, it appeals to incentives of more future information. For the warlike type,

[16] In this operation, there should be a probability of x_1, too. Even if player 2 realizes θ_H, she should choose $a^1_2 = D$. The outcome $a = ((D.D),(D,H))$ can be interpreted as the failure of the operation, and it incurs the cost $-\gamma$. She admits that the operation may fail.

it perturbed his higher-order beliefs, so he is likely to hide his type. The peaceful outcome is attainable presently when the probability is agreeable for all players. Otherwise, the signaling must reduce the players' common belief on the bad state by revealing it to some degree. The model also can be transformed into an espionage operation model about strategic information acquisition and injection.

<References>

Acemoglu and Wolitzky (2014). Cycles of Conflict: An Economic Model. American Economic Review 2014, 104(4): 1350–1367.

Au (2015). Dynamic Information Disclosure. RAND Journal of Economics, Vol. 46, No. 4, Winter 2015, pp. 791–823.

Aumann (1987). Correlated Equilibrium as an Expression of Bayesian Rationality. Econometrica, Jan., 1987, Vol. 55, No. 1 (Jan., 1987), pp. 1-18.

Aumann and Maschler (1995). Repeated Games with Incomplete Information. Cambridge, MA: MIT Press.

Baliga and Sjöström (2004). Arms Races and Negotiations. Review of Economic Studies (2004) 71, 351–369.

Baliga and Sjöström (2010). The Hobbesian Trap, in Oxford Handbook of the Economics of Peace and Conflict, edited by M. GarÖnkel and S. Skaperdas. Oxford University Press.

Baliga and Sjöström (2012). The Strategy of Manipulating Conflict. American Economic Review 2012, 102(6), 2897–2922.

Baliga and Sjöström (2022). Causes of War. mimeo.

Bergemann and Morris (2013). Robust Predictions in Games with Incomplete Information. Econometrica, Vol. 81, No. 4 (July, 2013), 1251-1308.

Brandenburger and Dekel (1993). Hierarchies of Beliefs and Common Knowledge. Journal of Economic Theory, 59, pp.189-198.

Carlsson and van Damme (1993). Global Games and Equilibrium Selection. Econometrica, Vol.61, No.5 (Sep., 1993), pp.989-1018.

Che, Kim, and Mierendorff (2022). Keeping the Listener Engaged: A Dynamic Model of Bayesian Persuasion. Journal of Political Economy (forthcoming).

Crawford and Sobel (1982). Strategic Information Transmission. Econometrica, Vol. 50, No. 6 (Nov., 1982), pp. 1431-1451.

Dekel and Siniscalchi (2015). Chapter 12 - Epistemic Game Theory. Handbook of Game Theory with Economic Applications, Volume 4, 2015, pp. 619-702.

Fearon (2013). Fighting rather than Bargaining. mimeo.

Forges (1986). An Approach to Communication Equilibria, Econometrica, Nov., 1986, Vol.54, No.6, pp.1375-1385.

Forges (1988). Communication Equilibria in Repeated Games with Incomplete Information. Mathematics of Operations Research, Vol.13, No. 2 (May, 1988), pp.191-231.

Forges (1993). Five Legitimate Definitions of Correlated Equilibrium in Games with Incomplete Information. Theory and Decision, Nov 1, 1993; 35, 3.

Forges (2020). Games with Incomplete Information: From Repetition to Cheap Talk and Persuasion. Annals of Economics and Statistics , No.137 (March 2020), pp.3-30

Harsanyi (1967). Games with Incomplete Information Played by "Bayesian" Players, Part I. The Basic Model. Management Science, 14(3), 159-182.

Hörner, Morelli, and Squintani (2015). Mediation and Peace. Review of Economic Studies (2015) 82, 1483–1501.

Kamenica and Gentzkow (2011). Bayesian Persuasion. American Economic Review 101 (October 2011): 2590–2615.

Mathevet, Perego, and Taneva (2020). On Information Design in Games. Journal of Political Economy, 128(4): 1370–1404.

Mertens and Zamir (1985). Formulation of Bayesian Analysis for Games with Incomplete Information. International Journal of Game Theory, Vol.14, No.1, pp.1-29.

Myerson (1986). Multistage Games with Communication. Econometrica, Vol.54, No.2 (Mar., 1986), pp.323-358.

Penta (2012). Higher Order Uncertainty and Information Static and Dynamic Games. Econometrica, Vol.80, No.2, pp.631-660.

Perea (2012). Epistemic Game Theory: Reasoning and Choice. New York: Cambridge University Press.

Perea (2023). From Decision Theory to Game Theory: Reasoning about Others' Decision Problems. In progress.

Powell (1996). Uncertainty, Shifting Power, and Appeasement. The American Political Science Review, Dec., 1996, Vol.90, No.4, pp.749-764.

Powell (1999). In the Shadow of Power. Princeton, NJ: Princeton University Press.

Rayo and Segal (2010). Optimal Information Disclosure. Journal of Political Economy, Vol. 118 (2010), pp. 949–987.

Taneva (2019). Information Design. American Economic Journal: Microeconomics 2019, 11(4), pp.151–185.

<Appendix: Proofs>

Proof. Lemma 1. Suppose that there exists a sequentially rational strategy $(a_{1D}^{\tau})_{1\leq\tau\leq T}$ with a positive probability of $a_{1D}^{\tau'}(h_{1D}^{\tau'-1}) = H$ where $\tau' < T$ and player 1's type is not fully revealed at $h^{\tau'-1}$ (If his type is fully revealed, then H is never be a best response.) There is an incentive for Dove type to endure a short-term cost to reveal his type. Since H gives a lower period payoff than D in τ', the strategy must give a higher total payoff from τ' than the deviation to $(\hat{a}_{1D}^{\tau}(h_{1D}^{\tau-1}))_{\tau'\leq\tau\leq T}$ such that $\hat{a}_{1D}^{\tau}(h_{1D}^{\tau-1}) = D$ for all $\tau \geq \tau'$. That is, $(a_{1D}^{\tau}(h_{1D}^{\tau-1}))_{\tau'\leq\tau\leq T}$ must yield 2's strictly higher probabilities of choosing D than $(\hat{a}_{1D}^{\tau}(h_{1D}^{\tau-1}))_{\tau'\leq\tau\leq T}$ for some future periods in the view of τ'. Notice that there exists a history $h_{1H}^{\tau'-1}$ Hawk type can face such that it includes the same action history with $(a_{1D}^{\tau})_{1\leq\tau\leq\tau'-1}$. If he chooses D at $h_{1H}^{\tau'-1}$ when his type is not revealed yet, then it yields player 2's same belief as $\hat{a}_{1D}^{\tau'} = D$ does. So, he must choose only H in the equilibrium to strictly increase both his period payoff in τ' and the probabilities of 2's playing D for future periods. Then, the consistency requires that 2 must believe θ_D with a probability 1 for given $a_1^{\tau} = D$ at $h_2^{\tau'} = (h_2^{\tau'-1}, (D, a_2^{\tau'}))$. Then, she must choose D in all the rest of subgames after $h_2^{\tau'} = (h_2^{\tau'-1}, (D, a_2^{\tau'}))$. Dove type will deviate to $a_{1D}^{\tau'}(h_{1D}^{\tau'-1}) = D$. Contradiction. ■

Proof. Proposition 1. Note that $a_{1H}^2(\cdot) = H$ and $a_{1D}^2(\cdot) = D$ for any histories. By Lemma 1, only $a_{1D} = D.D$ is played in PBE.

If Hawk type chooses only $a_{1H}^1 = D$ in the equilibrium, then player 2 has a belief $b_2^2(\theta_H; (D, \cdot)) = p$. As she wants to coordinate with player 1's action, (2) implies that her best response in the last period with the belief $b_2^2(\theta_H; (D, \cdot)) = p$ is $a_2^2(D, \cdot) = H$. So, $a_2 = D.H/H$ is sequentially rational. Thus, the pure strategy $a_{1H}^1 = D$ is not the equilibrium strategy.

Now, check $a_{1H}^1 = H$. In $\tau = 2$, the player 2 realizes 1's true type: $b_2^2(\theta_H; (D, \cdot)) = 0$ and

$b_2^2(\theta_H; (H,\cdot)) = 1$, so $a_2^2(D,\cdot) = D$ and $a_2^2(H,\cdot) = H$. In $\tau = 1$, 2's belief on θ_H is p. So, only $a_2 = H.D/H$ is sequentially rational. For given this, Hawk type expects $-\theta_H + \delta[-\theta_H]$ with $a_{1H} = H.H$ and $-\gamma + \delta[\beta - \theta_H]$ with $a_{1H} = D.H$. If $\theta_H < \gamma - \delta\beta$, then $a_{1H} = H.H$ is sequentially rational. It constructs a PBE. If $\theta_H > \gamma - \delta\beta$, then such PBE does not exist with a circulation of best responding pure strategies.

Now, check players' mixed strategy. Consider Hawk type's mixed strategy $a_{1H}^1 = m_{1H}^1 D + (1 - m_{1H}^1)H$. In $\tau = 1$, the player 2 believes the state and 1's strategy as $(\theta_1, a_1) = (\theta_H, H.H)$, $(\theta_H, D.H)$, $(\theta_D, D.D)$ with a probability $p(1 - m_{1H}^1)$, pm_{1H}^1, $1 - p$, respectively. She expects in $\tau = 1$, that she will have a belief on θ_H in $\tau = 2$, either 1 for given $a_1^1 = H$ or $\frac{pm_{1H}^1}{pm_{1H}^1 + 1 - p}$ for given $a_1^1 = D$. For given $a_1^1 = H$, 2's best response in $\tau = 2$ is H. For given $a_1^1 = D$, she expects $\frac{pm_{1H}^1}{pm_{1H}^1+1-p}(-\gamma)$ with D and $(1 - \frac{pm_{1H}^1}{pm_{1H}^1+1-p})\beta - \theta_2$ with H in $\tau = 2$. Notice that $a_2^2(D,\cdot) = D$ is the best response in $\tau = 2$ if and only if $m_{1H}^1 \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$. There is no reason for Hawk type to choose $m_{1H}^1 > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ which induces only $a_2^2(D,\cdot) = H$ in the equilibrium. Furthermore, m_{1H}^1 also should yield 2's action in $\tau = 1$. Without any further information in $\tau = 1$, 2 expects, in $\tau = 1$,

$$
\begin{aligned}
a_2 &= D.D/H : p(1 - m_{1H}^1)(-\gamma + \delta[-\theta_2]) + pm_{1H}^1(0 + \delta[-\gamma]) + (1 - p)(0 + \delta[0]) \\
a_2 &= H.D/H : p(1 - m_{1H}^1)(-\theta_2 + \delta[-\theta_2]) + pm_{1H}^1(\beta - \theta_2 + \delta[-\gamma]) + (1 - p)(\beta - \theta_2 + \delta[0]) \\
a_2 &= D.H/H : p(1 - m_{1H}^1)(-\gamma + \delta[-\theta_2]) + pm_{1H}^1(0 + \delta[-\theta_2]) + (1 - p)(0 + \delta[\beta - \theta_2]) \\
a_2 &= H.H/H : p(1 - m_{1H}^1)(-\theta_2 + \delta[-\theta_2]) + pm_{1H}^1(\beta - \theta_2 + \delta[-\theta_2]) + (1 - p)(\beta - \theta_2 + \delta[\beta - \theta_2])
\end{aligned}
$$

$a_2 = D.D/H$ is better than $a_2 = D.H/H$ if and only if $m_{1H}^1 \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$.
$a_2 = H.D/H$ is better than $a_2 = H.H/H$ if and only if $m_{1H}^1 \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$.
$a_2 = D.D/H$ is better than $a_2 = H.D/H$ if and only if $m_{1H}^1 \geq 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$. Thus, conditional on $m_{1H}^1 < \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$, $a_2 = D.D/H$ is sequentially rational if and only if $m_{1H}^1 \geq 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$.

Consider following cases varying two conditions $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_H \leq \gamma - \delta\beta$.

Case 1. $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_H \leq \gamma - \delta\beta$

Case 1.1. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H$ where $m_{1H}^1 \in (1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}, \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2})$

Then only $a_2 = D.D/H$ is sequentially rational. For given that, check

$$\begin{aligned} a_{1H} &= D.H : 0 + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : \beta - \theta_H + \delta[-\theta_H] \end{aligned}$$

Since (3), only $a_{1H} = H.H$ is sequentially rational. Contradiction.

Case 1.2. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H$ where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2} > 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$

Player 2 is indifferent between $a_2^2 = D$ and $a_2^2 = H$ against $a_{1H}^1 = m_{1H}^1 D + (1 - m_{1H}^1)H$ and given $h_2^1 = (D, \cdot)$. Consider $a_2 = D.(m_2^2 D + (1 - m_2^2)H)/H$. To make D and H indifferent in $\tau = 1$ for Hawk type, check

$$\begin{aligned} a_{1H} &= D.H : 0 + \delta[m_2^2(\beta - \theta_H) + (1 - m_2^2)(-\theta_H)] \\ a_{1H} &= H.H : \beta - \theta_H + \delta[-\theta_H] \end{aligned}$$

The condition (3) implies $m_2^2 = \frac{\beta-\theta_H}{\delta\beta} < 1$. Thus, $(a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H, a_{1D} = D.D, a_2 = D.(m_2^2 D + (1 - m_2^2)H)/H$ where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $m_2^2 = \frac{\beta-\theta_H}{\delta\beta}$ is PBE.

Case 1.3. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H$ where $m_{1H}^1 = 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} < \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$

Player 2 is indifferent between $D.D/H$ and $H.D/H$ in $\tau = 1$. Consider $a_2 = m_2^1 D + (1 - m_2^1)H.D/H$. To make D and H indifferent for Hawk type in $\tau = 1$, check

$$\begin{aligned} a_{1H} &= D.H : m_2^1 0 + (1 - m_2^1)(-\gamma) + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : m_2^1(\beta - \theta_H) + (1 - m_2^1)(-\theta_H) + \delta[-\theta_H] \end{aligned}$$

The conditions (3) and $\theta_H \leq \gamma - \delta\beta$ imply $0 \leq m_2^1 = \frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta} < 1$. Thus, $(a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H, a_{1D} = D.D, a_2 = m_2^1 D + (1-m_2^1)H.D/H)$ where $m_{1H}^1 = 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$ and $m_2^1 = \frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$ is PBE.

Case 1.4. $a_{1H} = H.H$

Then, $a_2 = H.D/H$ is sequentially rational. Check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

Since $\theta_H \leq \gamma - \delta\beta$, $a_{1H} = H.H$ is sequentially rational against $a_2 = H.D/H$. Thus, $(a_{1H} = H.H, a_{1D} = D.D, a_2 = H.D/H)$ is PBE.

Case 2. $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_H > \gamma - \delta\beta$

Following the logic in Case 1.1, $a_{1H} = (m_{1H}^1 D + (1-m_{1H}^1)H).H$ where $m_{1H}^1 \in (1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}, \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2})$ can not be equilibrium strategy.

Case 2.2. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H$ where $m_{1H}^1 = 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} < \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$

Consider $a_2 = m_2^1 D + (1 - m_2^1)H.D/H$. Since $\theta_H > \gamma - \delta\beta$, only $a_{1H} = D.H$ is sequentially rational. Contradiction.

Case 2.3. $a_{1H} = (m_{1H}^1 D + (1 - m_{1H}^1)H).H$ where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2} > 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$

Consider $a_2 = D.(m_2^2 D + (1 - m_2^2)H)/H$. Then, we need $m_2^2 = \frac{\beta-\theta_H}{\delta\beta}$. Thus, $(a_{1H} = (m_{1H}^1 D + (1-m_{1H}^1)H).H, a_{1D} = D.D, a_2 = D.(m_2^2 D + (1-m_2^2)H)/H$ where $m_{1H}^1 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $m_2^2 = \frac{\beta-\theta_H}{\delta\beta}$ is PBE.

Case 2.4. $a_{1H} = H.H$

Then, $a_2 = H.D/H$ is sequentially rational. Check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

$\theta_H > \gamma - \delta\beta$ implies that $a_{1H} = D.H$ is sequentially rational against $a_2 = H.D/H$. Contradiction.

Case 3. $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_H \leq \gamma - \delta\beta$

Recall that the probability of $a^1_{1H} = D$ should not be greater than $\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ in the equilibrium.

Case 3.1. $a_{1H} = (m^1_{1H}D + (1 - m^1_{1H})H).H$ where $m^1_{1H} \in (0, \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2})$. Since $m^1_{1H} < 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$, only $a_2 = H.D/H$ is sequentially rational. Then, check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

To make them indifferent, we need a knife-edge condition: $\theta_H = \gamma - \delta\beta$. Let's rule out this case.

Case 3.2. $a_{1H} = (m^1_{1H}D + (1 - m^1_{1H})H).H$ where $m^1_{1H} = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$. Since $m^1_{1H} < 1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$, both $a_2 = H.D/H$ and $a_2 = H.H/H$ are sequentially rational. Consider $a_2 = H.(m^2_2 D + (1 - m^2_2)H)/H$. Then, check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[m^2_2(\beta - \theta_H) + (1 - m^2_2)(-\theta_H)] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

The condition $\theta_H \leq \gamma - \delta\beta$ implies that only $a_{1H} = H.H$ is sequentially rational. Contradiction.

Case 3.3. $a_{1H} = H.H$. Only $a_2 = H.D/H$ is sequentially rational. Then, check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

The condition $\theta_H \leq \gamma - \delta\beta$ implies $a_{1H} = H.H$ is sequentially rational. Thus, $(a_{1H} = H.H, a_{1D} = D.D, a_2 = H.D/H)$ is PBE.

Case 4. $1 - \frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_H > \gamma - \delta\beta$

Recall that the probability of $a^1_{1H} = D$ should not be greater than $\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ in the equilibrium.

Following the logic in Case 3.1., $a_{1H} = (m^1_{1H}D + (1 - m^1_{1H})H).H$ where $m^1_{1H} \in (0, \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2})$ can not be equilibrium strategy.

Case 4.2. $a_{1H} = (m^1_{1H}D + (1 - m^1_{1H})H).H$ where $m^1_{1H} = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$. Following the logic in Case 3.2., consider $a_2 = H.(m^2_2D + (1 - m^2_2)H/H$. Then, check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[m^2_2(\beta - \theta_H) + (1 - m^2_2)(-\theta_H)] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

The condition $\theta_H > \gamma - \delta\beta$ implies $m^2_2 = \frac{\gamma-\theta_H}{\delta\beta} < 1$. Thus, $(a_{1H} = (m^1_{1H}D + (1 - m^1_{1H})H).H, a_{1D} = D.D, a_2 = H.(m^2_2D + (1 - m^2_2)H)/H$ where $m^1_{1H} = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $m^2_2 = \frac{\gamma-\theta_H}{\delta\beta}$ is PBE.

Case 4.3. $a_{1H} = H.H$. Only $a_2 = H.D/H$ is sequentially rational. Then, check

$$\begin{aligned} a_{1H} &= D.H : -\gamma + \delta[\beta - \theta_H] \\ a_{1H} &= H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

The condition $\theta_H > \gamma - \delta\beta$ implies only $a_{1H} = D.H$ is sequentially rational. Contradiction. ■

Proof. Lemma 2. Each PBE yields a probability distribution over outcomes in each θ as following:

(1.1)

	θ_H		θ_D	
$a^1\backslash a^2$	(H,D)	(H,H)	(D,D)	(D,H)
(D,D)	$\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}\frac{\beta-\theta_H}{\delta\beta}$	$\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}(1-\frac{\beta-\theta_H}{\delta\beta})$	$\frac{\beta-\theta_H}{\delta\beta}$	$1-\frac{\beta-\theta_H}{\delta\beta}$
(D,H)	0	0	0	0
(H,D)	0	$1-\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$	0	0
(H,H)	0	0	0	0

(1.2)

	θ_H		θ_D	
$a^1\backslash a^2$	(H,D)	(H,H)	(D,D)	(D,H)
(D,D)	$(1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta})\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$	0	$\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$	0
(D,H)	$(1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta})(1-\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta})$	0	$1-\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$	0
(H,D)	0	$\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$	0	0
(H,H)	0	$\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}(1-\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta})$	0	0

(1.3)

	θ_H		θ_D	
$a^1\backslash a^2$	(H,D)	(H,H)	(D,D)	(D,H)
(D,D)	0	0	0	0
(D,H)	0	0	1	0
(H,D)	0	0	0	0
(H,H)	0	1	0	0

Dove type expects $-\gamma$, $(1-\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta})(-\gamma)$, $\delta(1-\frac{\beta-\theta_H}{\delta\beta})(-\gamma)$ under PBE (1.1), (1.2), and (1.3), respectively. Obviously, he prefers (1.2) to (1.3). He prefers (1.1) to (1.2) if and only if

$$\begin{aligned}
1-\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta} &> \delta\left(1-\frac{\beta-\theta_H}{\delta\beta}\right) \\
\frac{\theta_H-(1-\delta)\beta}{\gamma-\beta} &> \delta-\frac{\beta-\theta_H}{\beta} \\
\theta_H-(1-\delta)\beta &> \delta(\gamma-\beta)-\frac{(\beta-\theta_H)(\gamma-\beta)}{\beta} \\
\frac{(\beta-\theta_H)(\gamma-2\beta)}{\beta} &> \delta(\gamma-2\beta)
\end{aligned}$$

Since (3) implies $\frac{\beta-\theta_H}{\beta}<\delta$, the condition becomes

$$\gamma<2\beta$$

He prefers (1.1) to (1.3) if and only if $\delta(1-\frac{\beta-\theta_H}{\delta\beta})<1$, i.e.,

$$\delta<\frac{2\beta-\theta_H}{\beta}$$

Notice that another upper bound of δ satisfies $\frac{\gamma-\theta_H}{\beta}<\frac{2\beta-\theta_H}{\beta}$ if and only if $\gamma<2\beta$. Thus, Dove type's preference is as following:

If $\delta<\frac{\gamma-\theta_H}{\beta}<\frac{2\beta-\theta_H}{\beta}$, then $(1.1)\succ(1.2)\succ(1.3)$

If $\delta<\frac{2\beta-\theta_H}{\beta}<\frac{\gamma-\theta_H}{\beta}$, then $(1.2)\succ(1.1)\succ(1.3)$

If $\frac{2\beta-\theta_H}{\beta}<\delta<\frac{\gamma-\theta_H}{\beta}$, then $(1.2)\succ(1.3)\succ(1.1)$

Obviously, Hawk type prefers (1.1) to (1.3). Compare (1.1) and (1.2). Let $m_1\equiv 1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta}$, $m_2\equiv\frac{\gamma-\theta_H-\delta\beta}{\gamma-\beta}$, $m_1'\equiv\frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$, and $m_2'\equiv\frac{\beta-\theta_H}{\delta\beta}$. Under (1.2), Hawk type is indifferent between

$a_{1H} = D.H$ and $a_{1H} = H.H$ against $a_2 = m_2 D + (1 - m_2)H.D/H$. He expects, with $a_{1H} = H.H$,

$$\begin{aligned}
& m_2(\beta - \theta_H + \delta[-\theta_H]) + (1 - m_2)(-\theta_H + \delta[-\theta_H]) \\
= \; & m_2\beta - \theta_H + \delta[-\theta_H] \\
< \; & \beta - \theta_H + \delta[-\theta_H] \\
= \; & \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}\frac{\beta - \theta_H}{\delta\beta}\delta\beta + (1 - \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2})(\beta - \theta_H) + \delta(-\theta_H)
\end{aligned}$$

, that is, Hawk type prefers (1.1) to (1.2). He prefers (1.2) to (1.3) since $m_2\beta - \theta_H + \delta[-\theta_H] > -\theta_H + \delta[-\theta_H]$. Thus, Hawk type's preference is as following:

$$(1.1) \succ (1.2) \succ (1.3)$$

Player 2 expects $-\theta_2 + (1-p)\beta + p\delta[-\theta_2]$ in (1.3). In (1.2), she is indifferent between $a_2 = D.D/H$ and $a_2 = H.D/H$ against $a_{1H} = m_1 D + (1 - m_1)H.H$ and $a_{1D} = D.D$. She expects, with $a_2 = D.D/H$,

$$pm_1(0 + \delta[-\gamma]) + p(1 - m_1)(-\gamma + \delta[-\theta_2]) + (1 - p)(0 + \delta 0)$$

In (1.1), she is indifferent between $a_2 = D.D/H$ and $a_2 = D.H/H$ against $a_{1H} = m_1' D + (1 - m_1')H.H$ and $a_{1D} = D.D$. She expects, with $a_2 = D.D/H$,

$$pm_1'(0 + \delta[-\gamma]) + p(1 - m_1')(-\gamma + \delta[-\theta_2]) + (1 - p)(0 + \delta 0) \tag{14}$$

She prefers (1.1) to (1.2) if and only if $(m_1' - m_1)(-\gamma + \delta[-\theta_2]) < (m_1' - m_1)\delta[-\gamma]$. Since $m_1 < m_1'$, the condition becomes

$$\delta < \frac{\gamma}{\gamma - \theta_2}$$

Compare (1.2) and (1.3). Her payoff with $a_2 = H.D/H$ in (1.2) is

$$\begin{aligned} & pm_1(\beta - \theta_2 + \delta[-\gamma]) + p(1-m_1)(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2 + \delta 0) \\ = \quad & (p - \frac{\theta_2 - \beta}{\gamma - \beta})(\beta - \theta_2 + \delta[-\gamma]) + \frac{\theta_2 - \beta}{\gamma - \beta}(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2) \end{aligned}$$

She prefers (1.2) to (1.3) if and only if

$$\begin{aligned} & (p - \frac{\theta_2 - \beta}{\gamma - \beta})(\beta - \theta_2 + \delta[-\gamma]) + \frac{\theta_2 - \beta}{\gamma - \beta}(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2) \\ > \quad & p(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2) \end{aligned}$$

, i.e.,

$$\delta < \frac{\beta}{\gamma - \theta_2}$$

She prefers (1.1) to (1.3) if and only if

$$\begin{aligned} & pm_1'(0 + \delta[-\theta_2]) + p(1-m_1')(-\gamma + \delta[-\theta_2]) + (1-p)(0 + \delta[\beta - \theta_2]) \\ > \quad & p(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2) \end{aligned}$$

in which her payoff in (1.1) is calculated with $a_2 = D.H/H$. It becomes

$$(1-p)(\theta_2 - \beta)\delta < p(1-m_1')(-\gamma) + \theta_2 - (1-p)\beta = -p(\gamma - \beta) + pm_1'\gamma + \theta_2 - \beta$$

, which is equivalent with

$$\begin{aligned} \delta \quad < \quad & -\frac{p(\gamma - \beta)}{(1-p)(\theta_2 - \beta)} + pm_1'\gamma + \theta_2 - \beta \\ = \quad & pm_1'\gamma + \theta_2 - \beta - \frac{1}{m_1'} \\ = \quad & \frac{\gamma}{\gamma - \theta_2} + 1 - \frac{1}{m_1'} \\ = \quad & \frac{\gamma}{\gamma - \theta_2} + 1 - \frac{\gamma - \theta_2}{\theta_2 - \beta}\frac{p}{1-p} \end{aligned}$$

Notice that the condition $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ implies $p \leq \frac{(\theta_2-\beta)(2\gamma-\beta-\theta_2)}{(\gamma-\beta)^2}$. Then,

$$\begin{aligned}
\frac{\gamma}{\gamma-\theta_2}+1-\frac{\gamma-\theta_2}{\theta_2-\beta}\frac{p}{1-p} &\geq \frac{2\gamma-\theta_2}{\gamma-\theta_2}-\frac{\gamma-\theta_2}{\theta_2-\beta}\frac{\frac{(\theta_2-\beta)(2\gamma-\beta-\theta_2)}{(\gamma-\beta)^2}}{1-\frac{(\theta_2-\beta)(2\gamma-\beta-\theta_2)}{(\gamma-\beta)^2}} \\
&= \frac{2\gamma-\theta_2}{\gamma-\theta_2}-\frac{\gamma-\theta_2}{\theta_2-\beta}\frac{(\theta_2-\beta)(2\gamma-\beta-\theta_2)}{(\gamma-\theta_2)^2} \\
&= \frac{\beta}{\gamma-\theta_2}
\end{aligned}$$

Thus, player 2's preference is as following:

If $\delta < \frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2}$, then $(1.1) \succ (1.2) \succ (1.3)$

If $\frac{\beta}{\gamma-\theta_2} < \delta < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2}$, then $(1.1) \succ (1.3) \succ (1.2)$

If $\frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \delta < \frac{\gamma}{\gamma-\theta_2}$, then $(1.3) \succ (1.1) \succ (1.2)$

If $\frac{\beta}{\gamma-\theta_2} < \frac{\gamma}{\gamma-\theta_2} - \frac{p\gamma+(1-p)\beta-\theta_2}{(1-p)(\theta_2-\beta)} < \frac{\gamma}{\gamma-\theta_2} < \delta$, then $(1.3) \succ (1.2) \succ (1.1)$

■

Proof. Proposition 2.Recalling Proposition 1. PBE is unique when $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} > \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$. Suppose that there is a message from Dove type to separate his type more, that is, to reduce player 2's belief on θ_H so that it increases the probability of $a_2^\tau = D$. Then, Hawk type must send the same message in the equilibrium as well. Any possible messages from Dove type must be chosen by Hawk type, too. The only way to make the cheap talk equilibrium yield a different outcome in PBE is to separate Hawk type by himself with some probability. There is possibly such an incentive as the other signal allows 2's higher belief on θ_D so that players are more likely to choose D.

Notice that it is sufficient to consider a mixture of messages. They are represented by either one message that induces mixed strategies or multiple messages such that each of them induces a pure strategy. Conforming to the belief consistency, if one message from Hawk type reduces 2's belief on θ_H, then there must be another message that increases her belief on θ_H. Let c_i^τ denote i's cheap talk message in τ. Any messages in $\tau = 2$ $c_i^{2,a}$ must be non-informative to all players in the equilibrium.

Let's consider player 1's message strategy in $\tau = 1$: $q_{1H}^1 c^{1,D} + (1 - q_{1H}^1)c^{1,H}$ for Hawk type and $c^{1,D}$ for Dove type. Since $c^{1,H}$ reveals θ_H for sure, all players will choose H in every period. However, $c^{1,H}$ is helpful to reduce the common belief on θ_H when $c^{1,D}$ is sent as if all players face a new game with a prior $\hat{p} \equiv \frac{pq_{1H}^1}{pq_{1H}^1 + 1 - p} < p$.

Suppose $1 - \frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta} > \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$, and it is still true that $1 - \frac{1}{\hat{p}}\frac{\theta_2 - \beta}{\gamma - \beta} > \frac{1-\hat{p}}{\hat{p}}\frac{\theta_2 - \beta}{\gamma - \theta_2}$. Obviously, in case 3, the cheap talk does not change the equilibrium outcome at all. Consider the case 4 in which, for given $c^{1,D}$, all play $a_{1H} = (\hat{m}_{1H}^1 D + (1 - \hat{m}_{1H}^1)H).H$, $a_{1D} = D.D$, $a_2 = H.(m_2^2 D + (1 - m_2^2)H)/H$ where $\hat{m}_{1H}^1 \equiv \frac{1-\hat{p}}{\hat{p}}\frac{\theta_2 - \beta}{\gamma - \theta_2}$. Notice that $m_2^2 = \frac{\gamma - \theta_H}{\delta\beta}$ remains as it does not depend on p. Then, Hawk type expects, before sending a message,

$$
\begin{aligned}
& (1 - q_{1H}^1)u_{1H}((H,H),(H,H)) + q_{1H}^1[(\hat{m}_{1H}^1 m_2^2 u_{1H}((D,H),(H,D)) \qquad (15) \\
& + \hat{m}_{1H}^1(1 - m_2^2)u_{1H}((D,H),(H,H)) + (1 - \hat{m}_{1H}^1)u_{1H}((H,H),(H,H))] \\
= \; & [(1 - q_{1H}^1\hat{m}_{1H}^1))]u_{1H}((H,H),(H,H)) + q_{1H}^1\hat{m}_{1H}^1 m_2^2 u_{1H}((D,H),(H,D)) \\
& + q_{1H}^1\hat{m}_{1H}^1(1 - m_2^2)u_{1H}((D,H),(H,H)) \\
= \; & (1 - m_{1H}^1)u_{1H}((H,H),(H,H)) + m_{1H}^1 m_2^2 u_{1H}((D,H),(H,D)) + m_{1H}^1(1 - m_2^2)u_{1H}((D,H),(H,H))
\end{aligned}
$$

as the last equality holds because $q_{1H}^1\hat{m}_{1H}^1 = m_{1H}^1$. Similarly, in the view of player 2 in the same stage, a distribution of the equilibrium outcomes does not change.

Now, consider the case that $1 - \frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta} > \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$ switches to $1 - \frac{1}{\hat{p}}\frac{\theta_2 - \beta}{\gamma - \beta} \leq \frac{1-\hat{p}}{\hat{p}}\frac{\theta_2 - \beta}{\gamma - \theta_2}$. Recall Lemma 2: Any type of player 1 prefers (1.2) to (1.3). Moreover, player 2's preference may become less favored in (1.3) with a new $\hat{p}$. Thus, player 1 may mix some messages to achieve outcomes of (1.2) or (1.1) under $c^{1,D}$.

Case 1. consider the cheap talk strategy profile: $a_{1H} = (q^1_{1H}c^{1,D} + (1 - q^1_{1H})c^{1,H}).(r^1_{1H}D + (1 - r^1_{1H})H))/H.H$, $a_{1D} = c^{1,D}.D/D.D$, $a_2 = D/H.(r^1_2 D + (1 - r^1_2)H).D/H$.

After $c^{1,D}$ is realized, Hawk type must be indifferent $a_{1H} = D.H$ and $a_{1H} = H.H$ in $\tau = 1$. Check

$$
\begin{aligned}
a_{1H} &= D.H : r^1_2 0 + (1 - r^1_2)(-\gamma) + \delta[\beta - \theta_H] \\
a_{1H} &= H.H : r^1_2(\beta - \theta_H) + (1 - r^1_2)(-\theta_H) + \delta[-\theta_H]
\end{aligned}
$$

We need $r^1_2 = \frac{\gamma - \theta_H - \delta\beta}{\gamma - \beta}$. In the stage before sending the message, Hawk type must be indifferent between $c^{1,D}$ and $c^{1,H}$. Check

$$
\begin{aligned}
a_{1H} &= c^{1,D}.(r^1_{1H}D + (1 - r^1_{1H})H)).H : (1 - r^1_{1H})r^1_2(\beta - \theta_H) + r^1_{1H}(1 - r^1_2)(-\gamma) \\
&\quad +(1 - r^1_{1H})(1 - r^1_2)(-\theta_H) + \delta[r^1_{1H}\beta - \theta_H] \\
a_{1H} &= c^{1,H}.H.H : -\theta_H + \delta[-\theta_H]
\end{aligned}
$$

His first payoff becomes $r^1_{1H}(r^1_2(\gamma - \beta) + \theta_H - \gamma + \delta\beta) + r^1_2\beta - \theta_H + \delta[-\theta_H]$ which is strictly greater than the second one. Contradiction.

Case 2. consider the cheap talk strategy profile: $a_{1H} = (q^1_{1H}c^{1,D} + (1 - q^1_{1H})c^{1,H}).(r^1_{1H}D + (1 - r^1_{1H})H))/H.H$, $a_{1D} = c^{1,D}.D/D.D$, $a_2 = D/H.D.(r^2_2 D + (1 - r^2_2)H)/H$.

After $c^{1,D}$ realized, Hawk type must be indifferent $a_{1H} = D.H$ and $a_{1H} = H.H$ in $\tau = 1$. Check

$$
\begin{aligned}
a_{1H} &= D.H : 0 + \delta[r^2_2(\beta - \theta_H) + (1 - r^2_2)(-\theta_H)] \\
a_{1H} &= H.H : \beta - \theta_H + \delta[-\theta_H]
\end{aligned}
$$

We need $r^2_2 = \frac{\beta - \theta_H}{\delta\beta}$. In the stage before sending the message, Hawk type must be indifferent between $c^{1,D}$ and $c^{1,H}$. Check

$$\begin{aligned} a_{1H} &= c^{1,D}.(r^1_{1H}D + (1-r^1_{1H})H)).H : (1-r^1_{1H})(\beta-\theta_H) + \delta[r^1_{1H}r^2_2\beta - \theta_H] \\ a_{1H} &= c^{1,H}.H.H : -\theta_H + \delta[-\theta_H] \end{aligned}$$

His first payoff becomes $r^1_{1H}(-\beta+\theta_H+r^2_2\delta\beta)+\beta-\theta_H+\delta[-\theta_H]$ which is strictly greater than the second one. Contradiction.

When $1-\frac{1}{p}\frac{\theta_2-\beta}{\gamma-\beta} \le \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ (Case 1 or Case 2), fix the best PBE. $c^{1,D}$ changes only Hawk type's mixed strategy from m^1_{1H} to $\hat{m}^1_{1H}$. Following the same logic with (15), it does not change the ex-ante probabilities of the equilibrium outcome. ■

Proof. Lemma 3. There are paths of equilibrium signals that induces the outcome $((D,D),(H,D))$ when θ_H. Collect all such paths into $\hat{S}(\theta_H)$. Since σ is common knowledge and each path must construct an on-the-equilibrium-path, there is a common belief $\hat{\sigma}_H \equiv \Pr(\hat{S}(\theta_H))$, that is, the common ex-ante probability of the outcome $((D,D),(H,D))$ when θ_H. Pick $(s^1_2, s^2_2) \in S^1_2 \times S^1_2$ such that $(\cdot, s^1_2, \cdot, s^2_2) \in \hat{S}(\theta_H)$. For given it, players are expected to play $a^1 = (D,D)$. Let $\sigma_D \equiv \Pr(s^1_2,(D,D),s^2_2 \mid \theta_D)$, the probability of $(s^1_2,(D,D),s^2_2)$ from θ_D. It is also common knowledge. Following the equilibrium path, at $h^2_2 = (s^1_2,(D,D),s^2_2)$, we need $a^2_2 = D$. It is necessary

$$\frac{\theta_2-\beta}{\gamma-\beta} \ge b_2(\theta_H \mid s^1_2,(D,D),s^2_2) = \frac{p\hat{\sigma}_H}{p\hat{\sigma}_H + (1-p)\sigma_D} \ge \frac{p\hat{\sigma}_H}{p\sigma_1 + 1 - p}$$

Then,

$$\hat{\sigma}_H \le \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2} = k_2$$

■

Proof. Proposition 4. At $h^2_2 = ((D,D),s^{2,D})$, $b_2(\theta_H \mid (D,D),s^{2,D}) = \frac{pk_2}{pk_2+1-p} = \frac{\theta_2-\beta}{\gamma-\beta}$ and

$b_2(\theta_D \mid (D,D), s^{2,D}) = 1 - \frac{\theta_2-\beta}{\gamma-\beta}$, so her rational choice is $a_2^2 = D$. At $\hat{h}_2^2$ with $s^{2,H}$ or $a_1^1 = H$, then $b_2(\theta_H \mid \hat{h}_2^2) = 1$, so $a_2^2 = H$. At the other $\widetilde{h_2^2}$ with $a_2^1 = H$, there is no more information: $b_2(\theta_H \mid \widetilde{h_2^2}) = p$, so $a_2^2 = H$. In $\tau = 1$, if she chooses $a_2^1 = D$, then she expects the signal $s^{2,D}$ with probability k_2: $b_2(\theta_H, (D,D), s^{2,D}) = pk_2$, $b_2(\theta_H, (D,D), s^{2,H}) = p(1-k_2)$, and $b_2(\theta_D, (D,D), s^{2,D}) = 1-p$. If she chooses $a_2^1 = H$, then she will not get any further information: $b_2(\theta_H, (D,H)) = p$ and $b_2(\theta_D, (D,H)) = 1-p$. Check

$$\begin{aligned} a_2 &= D.D/H : pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2] + (1-p)\cdot 0 \\ a_2 &= H.H : \beta - \theta_2 + \delta[(1-p)\beta - \theta_2] \end{aligned}$$

It is necessary $pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2] \geq \beta - \theta_2 + \delta[(1-p)\beta - \theta_2]$, i.e.,

$$\theta_2 - \beta \geq \delta[pk_2(\gamma - \theta_2) + (1-p)(\beta - \theta_2)]$$

which is true since $k_2 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $\theta_2 - \beta > 0$.

Dove type always choose $a_{1D} = D.D$, following Lemma 1. At any histories in $\tau = 2$, Hawk type chooses $a_{1H}^2 = H$. At h_{1H}^1, if he chooses $a_{1H}^1 = D$, then he expects the signal $s^{2,D}$ with probability k_2: $b_{1H}((D,D), s^{2,D}) = k_2$ and $b_{1H}((D,D), s^{2,H}) = 1-k_2$. If $a_{1H}^1 = H$, then $s^{2,H}$ is expected for sure: $b_{1H}((H,D), s^{2,H}, a_2^2 = H) = 1$. Check

$$\begin{aligned} a_{1H} &= D.H : 0 + \delta[k_2\beta - \theta_H] \\ a_{1H} &= H.H : \beta - \theta_H + \delta[-\theta_H] \end{aligned}$$

Since $k_2 \geq k_{1H} = \frac{\beta-\theta_H}{\delta\beta}$, $a_{1H} = D.H$ is sequentially rational. ■

Proof. Proposition 5. In $\tau = 2$, for any histories with $s_2^{1,H}$ or $a_1^1 = H$, player 2 must believe θ_H for sure, so $a_2^2 = H$. At $h_2^2 = (s_2^{1,D}, (D,D))$, $b_2(\theta_H \mid s_2^{1,D}, (D,D)) = \frac{pk_2}{pk_2+1-p} = \frac{\theta_2-\beta}{\gamma-\beta}$ and

$b_2(\theta_D \mid s_2^{1,D},(D,D)) = 1 - \frac{\theta_2-\beta}{\gamma-\beta}$. Her rational choice is $a_2^2 = D$. In $\tau = 1$, for given $s_2^{1,H}$, $b_2(\theta_H, a_1 = D.H \mid s_2^{1,H}) = 1$ so that her rational choice is $a_2 = D.H$. For given $s_1^{1,D}$, $b_2(\theta_H, a_1 = D.H \mid s_2^{1,D}) = \frac{\theta_2-\beta}{\gamma-\beta}$ and $b_2(\theta_D, a_1 = D.D \mid s_2^{1,D}) = 1 - \frac{\theta_2-\beta}{\gamma-\beta}$. Her rational choice is $a_2 = D.D$. Dove type always choose $a_{1D} = D.D$, following Lemma 1. At any histories in $\tau = 2$, Hawk type chooses $a_{1H}^2 = H$. At h_{1H}^1, Hawk believes $b_{1H}(s_2^{1,D}, a_2 = D.D) = k_2$ and $b_{1H}(s_2^{1,H}, a_2 = D.H) = 1 - k_2$. Check

$$\begin{aligned} a_{1H} &= D.H : 0 + \delta[k_2\beta - \theta_H] \\ a_{1H} &= H.H : \beta - \theta_H + \delta[-\theta_H] \end{aligned}$$

Since $k_2 \geq k_{1H} = \frac{\beta-\theta_H}{\delta\beta}$, $a_{1H} = D.H$ is sequentially rational. ■

Proof. Corollary 1. In proofs of Proposition 4 and 5, replace k_2 with x. If $x = k_{1H}$, then Hawk type's incentive compatible conditions become binding while player 2's ones does slack. ■

Proof. Proposition 6. Recall that PBE (1.1) in Proposition 1 produces a probability distribution over the outcome in the interim stage <2> as following:

	θ_H		θ_D	
$a^1 \backslash a^2$	(H,D)	(H,H)	(D,D)	(D,H)
(D,D)	k_2k_{1H}	$k_2(1-k_{1H})$	k_{1H}	$1-k_{1H}$
(D,H)	0	0	0	0
(H,D)	0	$1-k_2$	0	0
(H,H)	0	0	0	0

(5) (or (6)) produces it as following:

	θ^H		θ^D	
$a^1 \backslash a^2$	(H,D)	(H,H)	(D,D)	(D,H)
(D,D)	k_2	$1-k_2$	1	0
(D,H)	0	0	0	0
(H,D)	0	0	0	0
(H,H)	0	0	0	0

Obviously, Dove type betters off in the latter. Hawk type expects, in each case,

$$(1.1): \qquad k_2 k_{1H}\delta[\beta-\theta_H] + k_2(1-k_{1H})\delta[-\theta_H] + (1-k_2)(\beta-\theta_H+\delta[-\theta_H])$$

$$(5): \qquad k_2\delta[\beta-\theta_H] + (1-k_2)\delta[-\theta_H]$$

We need to show that $k_2 k_{1H}\delta\beta + (1-k_2)(\beta-\theta_H) \leq k_2\delta\beta$, i.e.,

$$\frac{k_{1H}}{1-k_{1H}} \leq \frac{k_2}{1-k_2}$$

It is true since $k_{1H} \leq k_2$.

Now, consider player 2's expected payoffs:

$$(1.1): \qquad pk_2k_{1H}\delta[-\gamma] + pk_2(1-k_{1H})\delta[-\theta_2] + p(1-k_2)(-\gamma+\delta[-\theta_2]) + (1-p)(1-k_{1H})\delta[\beta-\theta_2]$$

$$(5): \qquad pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2]$$

We need to show that

$$(1-k_{1H})\delta(pk_2(\gamma-\theta_2) + (1-p)(\beta-\theta_2)) \leq p(1-k_2)\gamma$$

which is true since $k_2 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$. ■

Proof. Corollary 2. In the interim stage <3>, player 2 expects

$$\begin{aligned}
(1.1): &\qquad pk_2\delta[-\gamma] + p(1-k_2)(-\gamma + \delta[-\theta_2]) \\
(5),\ s_2^{1,D}: &\qquad \frac{pk_2}{pk_2 + 1 - p}\delta[-\gamma]
\end{aligned}$$

Note that the payoff (1.1) is obtained with $a_2 = D.D/H$ against $a_{1H} = k_2 D + (1-k_2)H.H$ and $a_{1D} = D.D$ following (14). We need to show that

$$pk_2\delta[-\gamma] + p(1-k_2)(-\gamma + \delta[-\theta_2]) \leq \frac{\theta_2 - \beta}{\gamma - \beta}\delta[-\gamma]$$

It is sufficient to show that $pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2] \leq \frac{\theta_2-\beta}{\gamma-\beta}\delta[-\gamma]$, i.e.,

$$\begin{aligned}
\frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta}\gamma &\leq k_2(\gamma - \theta_2) + \theta_2 \\
\frac{1}{p}\frac{\theta_2 - \beta}{\gamma - \beta}\gamma &\leq \frac{1-p}{p}(\theta_2 - \beta) + \theta_2 \\
\frac{\gamma}{\gamma - \beta} &\leq 1 + \frac{p\beta}{\theta_2 - \beta} \\
\frac{1}{\gamma - \beta} &\leq \frac{p}{\theta_2 - \beta} \\
\theta_2 &\leq p\gamma + (1-p)\beta
\end{aligned}$$

which is true because (2). ■

Proof. Corollary 3. Replace k_2 with xy in proofs of Proposition 4 and 5. Beliefs (8) and (9) are consistent, and the strategies are sequentially rational in every history. ■

Lemma 5 *If*

$$x_4 \leq \frac{\gamma - \beta}{\gamma - \theta_2}k_2 \tag{16}$$

, then the signal "$D.D/H_2^1$" is incentive compatible.

Proof. For given D_2^2 on the equilibrium path, that is, at $h_2^2 = (D.D/H_2^1, (D,D), D_2^2)$, $b_2(\theta_H \mid h_2^2) = \frac{px_1y_1}{px_1y_1+1-p}$. To make $a_2^2 = D$ best response, we need $x_1y_1 \leq k_2 = \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}$ and $y_1 \leq 1$. In optimum,

$$x_1y_1 = k_2 \text{ and } x_1 \geq k_2 \tag{17}$$

If player 2 deviates to $a_2^1(D.D/H_2^1) = H$, then there is no more information in $\tau = 2$. So, her belief given $a_1^1 = D$, is

$$a_2^2(\theta_H \mid D.D/H_2^1, (D,H)) = \frac{px_1}{px_1+1-p} > \frac{pk_2}{pk_2+1-p} = c_2$$

$a_2^2 = H$ is best response since $x_1 \geq k_2$.

To check $a_2 = D.D/H_2^1$'s sequential rationality, it is sufficient to compare $D.D/H_2^1$ and $H.H/H_2^1$. We need

$$\begin{aligned}
& \frac{px_1}{px_1+px_4+1-p}\delta[y_1(-\gamma)+(1-y_1)(-\theta_2)] + \frac{px_4}{px_1+px_4+1-p}(-\gamma+\delta[-\theta_2]) \\
\geq\ & \frac{px_1}{px_1+px_4+1-p}(\beta-\theta_2+\delta[-\theta_2]) + \frac{px_4}{px_1+px_4+1-p}(-\theta_2+\delta[-\theta_2]) \\
& + \frac{1-p}{px_1+px_4+1-p}(\beta-\theta_2+\delta[\beta-\theta_2]) \\
& px_1(\delta y_1(\gamma-\theta_2)-(\theta_2-\beta)) + px_4(\gamma-\theta_2) \leq (1-p)(\theta_2-\beta+\delta[\theta_2-\beta])
\end{aligned}$$

, i.e.,

$$x_1(\delta y_1 - \frac{\theta_2-\beta}{\gamma-\theta_2}) + x_4 \leq \frac{1-p}{p}\frac{\theta_2-\beta}{\gamma-\theta_2}(1+\delta)$$

Since $x_1y_1 = k_2$ in the equilibrium,

$$x_4 \leq \frac{\theta_2-\beta}{\gamma-\theta_2}x_1 + k_2 \tag{18}$$

The minimum of the right hand side is $\frac{\theta_2-\beta}{\gamma-\theta_2}k_2 + k_2 = \frac{\gamma-\beta}{\gamma-\theta_2}k_2$. If $x_4 \leq \frac{\gamma-\beta}{\gamma-\theta_2}k_2$, then $a_2 = D.D/H_2^1$

is sequentially rational. ■

Summarizing, incentive compatibility conditions (IC) are as following:

Recall the incentive compatible condition for D_2^2 at $h_2^2 = (D.D/H_2^1, (D, D), D_2^2)$, (17):

$$x_1 y_1 = k_2 \text{ and } x_1 \geq k_2$$

For given H_2^2 on the equilibrium path, that is, at $\hat{h}_2^2 = (D.D/H_2^1, (D, D), H_2^2)$, $b_2(\theta_H \mid \hat{h}_2^2) = 1$. $a_2^2 = H$ is best response. For other histories $\widetilde{h_2^2}$, $b_2(\theta_H \mid \widetilde{h_2^2}) = 1$. $a_2^2 = H$ is best response, too.

For given $D.H_1^1$, $\frac{x_1 y_1}{x_1+x_2+x_3}\delta\beta + \frac{x_3}{x_1+x_2+x_3}(-\gamma) + \delta[-\theta_H] \geq -\theta_H + \frac{x_1+x_2}{x_1+x_2+x_3}\beta + \delta[-\theta_H]$, i.e.,

$$\frac{x_1 y_1}{x_1 + x_2 + x_3} \geq \frac{\beta - \theta_H}{\delta\beta} + \frac{x_3}{x_1 + x_2 + x_3}\frac{\gamma - \beta}{\delta\beta} \tag{19}$$

For given $D.D/H_2^1$, recall (18)

$$x_4 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2} x_1 + k_2$$

and the sufficient condition (16) in Lemma 5:

$$x_4 \leq \frac{\gamma - \beta}{\gamma - \theta_2} k_2$$

For given $D.H_2^1$, $\frac{x_5}{x_2+x_5}(-\gamma) + \delta[-\theta_2] \geq -\theta_2 + \frac{x_2}{x_2+x_5}\beta + \delta[-\theta_2]$, i.e.,

$$\frac{x_5}{x_2 + x_5} \leq \frac{\theta_2 - \beta}{\gamma - \beta} \tag{20}$$

For given $H.H_1^1$, $-\theta_H + \frac{x_4+x_5}{x_4+x_5+x_6}\beta + \delta[-\theta_H] \geq \frac{x_6}{x_4+x_5+x_6}(-\gamma) + \delta[-\theta_H]$, i.e.,

$$\frac{x_6}{x_4 + x_5 + x_6} \geq \frac{\theta_H - \beta}{\gamma - \beta} \tag{21}$$

For given $H.H_2^1$, $-\theta_2 + \frac{x_3}{x_3+x_6}\beta + \delta[-\theta_2] \geq \frac{x_6}{x_3+x_6}(-\gamma) + \delta[-\theta_2]$, i.e.,

$$\frac{x_6}{x_3+x_6} \geq \frac{\theta_2-\beta}{\gamma-\beta} \tag{22}$$

Lemma 6 *In the optimal signaling, $x_3 = 0$.*

Proof.

θ_H	$D.D/H_2^1$	$D.H_2^1$	$H.H_2^1$
$D.H_1^1$	x_1	x_2	x_3
$H.H_1^1$	x_4	x_5	x_6

$\theta_H, (D.H_1^1, D.D/H_2^1), (D, D)$	D_2^2	H_2^2
	y_1	y_2

Suppose $x_3 > 0$. Consider a new signaling with $\hat{x}_3 = 0$, $\hat{x}_2 = x_2 + x_3$, and all others are same. The mediator prefers the new one. Then, (19) implies

$$\frac{\hat{x}_1\hat{y}_1}{x_1+\hat{x}_2+\hat{x}_3} = \frac{x_1y_1}{x_1+x_2+x_3} \geq \frac{\beta-\theta_H}{\delta\beta} + \frac{x_3}{x_1+x_2+x_3}\frac{\gamma-\beta}{\delta\beta} > \frac{\beta-\theta_H}{\delta\beta}$$

(20) implies

$$\frac{\hat{x}_5}{\hat{x}_2+\hat{x}_5} < \frac{x_5}{x_2+x_5} \leq \frac{\theta_2-\beta}{\gamma-\beta}$$

(22) implies

$$\frac{\hat{x}_6}{\hat{x}_3+x_6} = 1 > \frac{x_6}{x_3+x_6} \geq \frac{\theta_2-\beta}{\gamma-\beta}$$

All other ICs are satisfied. Thus, $x_3 = 0$ in the optimum. ■

Proof. Lemma 4. By Lemma 6, $x_3 = 0$ in the optimum. Recall (19):

$$x_1 y_1 \geq \frac{\beta - \theta_H}{\delta\beta}(x_1 + x_2)$$

At the history $h_2^2 = (D.D/H_2^1, (D,D), D_2^2)$, recall (17):

$$x_1 y_1 \leq \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}$$

Then,

$$x_1 + x_2 \leq x_1 y_1 \frac{\delta\beta}{\beta - \theta_H} \leq \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \theta_2}\frac{\delta\beta}{\beta - \theta_H} = k_2/k_{1H}$$

■

Proof. Proposition 7. By Lemma 6, $x_3 = 0$.

In $\tau = 2$, IC (17) holds since

$$x_1 y_1 = k_2 \text{ and } x_1 = \frac{k_2}{k_{1H}} \geq k_2$$

For given $D.H_1^1$, Hawk type is sure "$D.D/H_2^1$". Then, IC (19) holds since

$$\frac{x_1 y_1}{x_1}\delta\beta = k_{1H}\delta\beta = \beta - \theta_H$$

For given $D.D/H_2^1$, IC (18) implies that we need

$$1 - \frac{k_2}{k_{1H}} = x_4 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2}\frac{k_2}{k_{1H}} + k_2$$

,i.e.,

$$k_{1H} \leq \frac{\gamma - \beta}{\gamma - \theta_2}\frac{k_2}{1 - k_2}$$

For given $H.H_1^1$, the signaling will reveals Hawk type in $\tau = 2$ if he deviates. IC (21) holds trivially.

■

Lemma 7 *If $x_4 + x_5$ is not large enough, in specific,*

$$x_4 + x_5 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2}(x_1 + x_2 - x_1 y_1) \tag{23}$$

, then there exists an equivalent optimal signaling with $x_4 = 0$.

Proof. Suppose $x_4 > 0$ in the optimum and $x_4 + x_5 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2}(x_1 + x_2 - x_1 y_1)$. Consider a new signaling with $\hat{x}_4 = 0$, $\hat{x}_1 = x_1 - f(x_4)$, $\hat{x}_2 = x_2 + f(x_4)$, $\hat{x}_5 = x_4 + x_5$, $\hat{x}_6 = x_6$, and $\hat{y}_1 = \frac{x_1 y_1}{\hat{x}_1} > y_1$, where $f(x_4) \equiv \frac{\gamma - \theta_2}{\theta_2 - \beta}(x_4 + x_5) - x_2$ so that $f(x_4) < 1$ and

$$\frac{\hat{x}_5}{\hat{x}_2 + \hat{x}_5} = \frac{x_4 + x_5}{x_2 + f(x_4) + x_4 + x_5} = \frac{\theta_2 - \beta}{\gamma - \beta} \tag{24}$$

Notice that it is still optimal. The new signaling becomes

θ_H	$D.D/H_2^1$	$D.H_2^1$	$H.H_2^1$
$D.H_1^1$	$x_1 - f(x_4)$	$x_2 + f(x_4)$	0
$H.H_1^1$	0	$x_4 + x_5$	x_6

Check that $\hat{x}_1 \geq k_2$:

$$\begin{aligned}
\hat{x}_1 &= x_1 - \frac{\gamma - \theta_2}{\theta_2 - \beta}(x_4 + x_5) + x_2 \\
&\geq x_1 - \frac{\gamma - \theta_2}{\theta_2 - \beta}\frac{\theta_2 - \beta}{\gamma - \theta_2}(x_1 + x_2 - x_1 y_1) + x_2 \\
&= x_1 y_1 \\
&= k_2
\end{aligned}$$

IC (19) holds since

$$\frac{\hat{x}_1\hat{y}_1}{\hat{x}_1+\hat{x}_2} = \frac{x_1y_1}{x_1+x_2} \geq \frac{\beta-\theta_H}{\delta\beta}$$

Obviously, the sufficient condition (16) for $D.D/H_2^1$ holds.

IC (20) is true from (24). IC (21) holds since

$$\frac{\hat{x}_6}{\hat{x}_4+\hat{x}_5+\hat{x}_6} = \frac{x_6}{x_4+x_5+x_6} \geq \frac{\theta_H-\beta}{\gamma-\beta}$$

IC (22) holds since

$$\frac{\hat{x}_6}{\hat{x}_3+\hat{x}_6} = \frac{x_6}{0+x_6} > \frac{\theta_2-\beta}{\gamma-\beta}$$

Now, check validity of probabilities generated by $f(x_4)$. The condition $x_4+x_5 \leq \frac{\theta_2-\beta}{\gamma-\theta_2}(x_1+x_2-x_1y_1)$ and $f(x_4)<1$ imply

$$\begin{aligned}
0 &\leq \hat{x}_1 = x_1 - \frac{\gamma-\theta_2}{\theta_2-\beta}(x_4+x_5)+x_2 \leq 1 \\
0 &\leq \hat{x}_2 = \frac{\gamma-\theta_2}{\theta_2-\beta}(x_4+x_5) \leq 1 \\
0 &\leq \hat{y}_1 = \frac{x_1y_1}{x_1-\frac{\gamma-\theta_2}{\theta_2-\beta}(x_4+x_5)+x_2} \leq 1
\end{aligned}$$

■

Proof. Proposition 8. By Lemma 6, $x_3=0$. Assume $x_4=0$.

In $\tau=2$, IC (17) holds since $x_1y_1=k_2$ and

$$x_1 = \frac{k_2}{k_{1H}} - x_2 \geq \frac{k_2}{k_{1H}} - k_2\left(\frac{1}{k_{1H}}-1\right) = k_2$$

IC (19) holds since

$$\frac{x_1y_1}{x_1+x_2} = \frac{k_2}{k_2/k_{1H}} = \frac{\beta-\theta_H}{\delta\beta}$$

IC (18) and (21) hold trivially.

In the optimum, $x_2 = \frac{k_2}{k_{1H}} - x_1 \leq k_2(\frac{1}{k_{1H}} - 1) < 1$ since $x_1 \geq k_2$ and $\frac{k_2}{k_{1H}} < 1 < 1 + k_2$. As $x_1 + x_2 = \frac{k_2}{k_{1H}}$ in the optimum, $x_5 \leq 1 - \frac{k_2}{k_{1H}}$. We also need to check IC (20), i.e., $x_5 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2} x_2$. Summarizing,

$$\begin{aligned} x_2 &\leq k_2(\frac{1}{k_{1H}} - 1) \\ x_5 &\leq \min\{\frac{\theta_2 - \beta}{\gamma - \theta_2} x_2, 1 - \frac{k_2}{k_{1H}}\} \end{aligned}$$

We need to pick possible maximum x_2 and x_5.

If we pick $x_2 \geq \frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}}$, i.e., $\frac{\theta_2 - \beta}{\gamma - \theta_2} x_2 \geq 1 - \frac{k_2}{k_{1H}}$, then we must pick $x_5 \leq 1 - \frac{k_2}{k_{1H}}$.

If we pick $x_2 < \frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}}$, i.e., $\frac{\theta_2 - \beta}{\gamma - \theta_2} x_2 < 1 - \frac{k_2}{k_{1H}}$, then we must pick $x_5 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2} x_2$.

Suppose $k_{1H} \leq \frac{k_2}{1-(1-k_2)c_2}$, i.e., $\frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}} \leq k_2(\frac{1}{k_{1H}} - 1)$[17]. If we pick $x_2 \in [\frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}}, k_2(\frac{1}{k_{1H}} - 1)]$, then we must pick $x_5 \leq 1 - \frac{k_2}{k_{1H}} < \frac{\theta_2 - \beta}{\gamma - \theta_2} x_2$. So, $x_5 = 1 - \frac{k_2}{k_{1H}}$ to make $x_4 = x_6 = 0$.

If we pick $x_2 < \frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}}$, then we must pick $x_5 \leq \frac{\theta_2 - \beta}{\gamma - \theta_2} x_2 < 1 - \frac{k_2}{k_{1H}}$. So, $x_6 > 0$. The

17

$$\begin{aligned} k_{1H} &\leq \frac{k_2}{1-(1-k_2)c_2} \\ k_{1H} - k_{1H}c_2 + k_{1H}k_2c_2 &\leq k_2 \\ 1 - c_2 &\leq (\frac{1}{k_{1H}} - 1 + 1 - c_2)k_2 \\ (1-c_2)(1-k_2) &\leq (\frac{1}{k_{1H}} - 1)k_2 \\ (1-c_2)(1-k_2)\frac{\gamma - \beta}{\theta_2 - \beta} &\leq (\frac{1}{k_{1H}} - 1)k_2(1 + \frac{\gamma - \theta_2}{\theta_2 - \beta}) \\ \frac{\gamma - \theta_2}{\gamma - \beta}\frac{\gamma - \beta}{\theta_2 - \beta} - \frac{1-p}{p}\frac{\theta_2 - \beta}{\gamma - \beta}\frac{\gamma - \beta}{\theta_2 - \beta} &\leq (\frac{1}{k_{1H}} - 1)k_2(1 + \frac{\gamma - \theta_2}{\theta_2 - \beta}) \\ \frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p} - (\frac{1}{k_{1H}} - 1)\frac{1-p}{p} &\leq (\frac{1}{k_{1H}} - 1)k_2 \\ \frac{\gamma - \theta_2}{\theta_2 - \beta} - \frac{1-p}{p}\frac{1}{k_{1H}} &\leq (\frac{1}{k_{1H}} - 1)k_2 \end{aligned}$$

optimum is to pick $x_2 \in [\frac{\gamma-\theta_2}{\theta_2-\beta} - \frac{1-p}{p}\frac{1}{k_{1H}}, k_2(\frac{1}{k_{1H}} - 1)]$ to make $x_5 = 1 - k_2\frac{1}{k_{1H}}$ $(x_6 = 0)$.

$$\begin{aligned}
x_2 &\in [\frac{\gamma-\theta_2}{\theta_2-\beta} - \frac{1-p}{p}\frac{1}{k_{1H}}, k_2(\frac{1}{k_{1H}} - 1)] \\
x_1 &= \frac{k_2}{k_{1H}} - x_2 \\
x_5 &= 1 - \frac{k_2}{k_{1H}} \\
x_6 &= 0 \\
y_1 &= \frac{k_2}{x_1}
\end{aligned}$$

Suppose $k_{1H} > \frac{k_2}{1-(1-k_2)c_2}$, i.e.,

$$k_2(\frac{1}{k_{1H}} - 1) < \frac{\gamma-\theta_2}{\theta_2-\beta} - \frac{1-p}{p}\frac{1}{k_{1H}} \tag{25}$$

As it is necessary $x_2 \leq k_2(\frac{1}{k_{1H}} - 1)$, we need to pick $x_5 = \frac{\theta_2-\beta}{\gamma-\theta_2}x_2 < 1 - \frac{k_2}{k_{1H}}$ and assign the rest of probabilities to x_4 or x_6. Picking the possible maximum $x_2 = k_2(\frac{1}{k_{1H}} - 1)$, we get a maximum $x_5 = \frac{\theta_2-\beta}{\gamma-\theta_2}k_2(\frac{1}{k_{1H}} - 1)$. Also, $x_1 = \frac{k_2}{k_{1H}} - x_2 = k_2$ and then $y_1 = \frac{k_2}{x_1} = 1$. Then, check the condition (23) with $x_4 = 0$:

$$x_4 + x_5 = \frac{\theta_2-\beta}{\gamma-\theta_2}k_2(\frac{1}{k_{1H}} - 1) = \frac{\theta_2-\beta}{\gamma-\theta_2}(x_1 + x_2 - x_1y_1)$$

which holds with an equality. Lemma 7 implies that we can have $x_4 = 0$. Assign all the rest of probabilities to x_6.

$$\begin{aligned}
x_2 &= k_2(\frac{1}{k_{1H}} - 1) \\
x_1 &= k_2 \\
x_5 &= \frac{\theta_2 - \beta}{\gamma - \theta_2} k_2(\frac{1}{k_{1H}} - 1) < 1 - \frac{k_2}{k_{1H}} \\
x_6 &= 1 - x_1 - x_2 - x_5 \\
y_1 &= 1
\end{aligned}$$

Check

$$\begin{aligned}
x_6 &= 1 - x_1 - x_2 - x_5 \\
&= 1 - \frac{k_2}{k_{1H}} - \frac{\theta_2 - \beta}{\gamma - \theta_2} k_2(\frac{1}{k_{1H}} - 1) \\
&= 1 - \frac{1}{k_{1H}} \frac{1-p}{p} \frac{\theta_2 - \beta}{\gamma - \theta_2} - \frac{\theta_2 - \beta}{\gamma - \theta_2} k_2(\frac{1}{k_{1H}} - 1) \\
&= 1 - \frac{\theta_2 - \beta}{\gamma - \theta_2} (\frac{1-p}{p} \frac{1}{k_{1H}} + k_2(\frac{1}{k_{1H}} - 1)) \\
&> 0
\end{aligned}$$

The last inequality is true because (25). ■

Proof. Proposition 9. For any signals except $H.H_i^1$, players share a new marginal belief $\hat{p} =$

$\frac{p(x_1+x_2+x_5)}{p(x_1+x_2+x_5)+1-p}$ on θ^H. From (13), check

$$\begin{aligned}
\hat{p} &= \frac{p(\frac{k_2}{k_{1H}}+\frac{\theta_2-\beta}{\gamma-\theta_2}k_2(\frac{1}{k_{1H}}-1))}{p(\frac{k_2}{k_{1H}}+\frac{\theta_2-\beta}{\gamma-\theta_2}k_2(\frac{1}{k_{1H}}-1))+1-p} \\
&= \frac{pk_2(\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2})}{pk_2(\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2})+1-p} \\
&= \frac{\frac{\theta_2-\beta}{\gamma-\theta_2}(\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2})}{\frac{\theta_2-\beta}{\gamma-\theta_2}(\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2})+1} \\
&= \frac{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}}{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}+\frac{\gamma-\theta_2}{\theta_2-\beta}}
\end{aligned}$$

Now, check the parameter condition. Note that $k_{1H} \leq \frac{\hat{k}_2}{1-(1-\hat{k}_2)c_2}$ if and only if $\hat{k}_2 \geq \frac{(1-c_2)k_{1H}}{1-c_2k_{1H}}$.

$$\begin{aligned}
\hat{k}_2 &= \frac{1-\hat{p}}{\hat{p}}\frac{\theta_2-\beta}{\gamma-\theta_2} \\
&= \frac{1-\frac{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}}{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}+\frac{\gamma-\theta_2}{\theta_2-\beta}}}{\frac{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}}{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}+\frac{\gamma-\theta_2}{\theta_2-\beta}}}\frac{\theta_2-\beta}{\gamma-\theta_2} \\
&= \frac{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}+\frac{\gamma-\theta_2}{\theta_2-\beta}-\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}+\frac{\theta_2-\beta}{\gamma-\theta_2}}{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}}\frac{\theta_2-\beta}{\gamma-\theta_2} \\
&= \frac{\frac{\gamma-\theta_2}{\theta_2-\beta}}{\frac{1}{k_{1H}}\frac{\gamma-\beta}{\gamma-\theta_2}-\frac{\theta_2-\beta}{\gamma-\theta_2}}\frac{\theta_2-\beta}{\gamma-\theta_2} \\
&= \frac{\frac{\gamma-\theta_2}{\gamma-\beta}k_{1H}}{1-\frac{\theta_2-\beta}{\gamma-\beta}k_{1H}} \\
&= \frac{(1-c_2)k_{1H}}{1-c_2k_{1H}}
\end{aligned}$$

The condition holds with an equality. ■

Proof. Proposition 11. Suppose that player 2's testing strategy works. She must realize player 1's true type before choosing action in $\tau = 2$ based on the signals or action history. Obviously, she commits to $a_2^2 = H$ whenever she receives $s_2^{\tau,H}$ or detects $a_1^1 = H$. If Hawk type is supposed to play $a_{1H}^1 = D$ with a positive probability, then 2 may not detect his true type with a chance of $s_2^{1,D}$

and $s_2^{2,D}$. It is necessary $a_{1H}^1 = H$. Then, $a_2^2 = D/H$.

Suppose $a_2^1(s_2^{1,D}) = a_2^1(s_2^{1,H}) = D$. Then, Hawk type will choose $a_{1H}^1 = D$ because $\delta[-\theta_H + xy\beta] = \delta[-\theta_H + k_2\beta] \geq \beta - \theta_H + \delta[-\theta_H]$ from $k_2 \geq k_{1H}$. Contradiction.

Compare $(a_2^1(s_2^{1,D}) = D, a_2^1(s_2^{1,H}) = H)$ and $(a_2^1(s_2^{1,D}) = a_2^1(s_2^{1,H}) = H)$. Player 2's ex-ante expected payoffs are $px(-\gamma) + p(1-x)(-\theta_2) + \delta[p(-\theta_2)]$ and $px(-\theta_2) + p(1-x)(-\theta_2) + (1-p)(\beta - \theta_2) + \delta[p(-\theta_2)]$, respectively. Notice that she will get the same payoff in $\tau = 2$ since she will realize the true type.

$$px(-\gamma) + p(1-x)(-\theta_2) \leq px(-\theta_2) + p(1-x)(-\theta_2) + (1-p)(\beta - \theta_2)$$

because $x \geq k_2$ in the optimal mediation. The testing strategy must play $a_2 = H.D/H$, that is, $a_2^1 = H$ regardless of s_2^1. She expects $p(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2)$. If she chooses the strategy corresponding to the signals, then she expects $pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2]$. Check

$$\begin{aligned}
p(-\theta_2 + \delta[-\theta_2]) + (1-p)(\beta - \theta_2) &< pk_2\delta[-\gamma] + p(1-k_2)\delta[-\theta_2] \\
pk_2\delta[\gamma - \theta_2] &< p\theta_2 + (1-p)(\theta_2 - \beta) \\
-\frac{\theta_2}{\gamma - \theta_2} &< (1-\delta)k_2
\end{aligned}$$

Then, she prefers the outcome from the mediation signaling. ■