Formal Contracts Without Courts. Scoring Suppliers To Build Trust*

Juan José Ganuza†and Fernando Gomez‡

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Abstract

Relevant empirical findings point at the actual use in business contracting of explicit (but imperfectly enforceable) formal contracts alongside substantial informal dimensions in the relationship. In this paper we formally show the supporting role that formal contracts play for relational interactions. Contractual documents, even in the extreme case when the parties know they are not meant to be enforced in reality, may have an important and positive influence on reputational or reciprocity-based sanctions firms may impose to sustain cooperation. We demonstrate that setting compliance with certain tasks in a formal document reduces the cost of reputational punishments that firms may need to inflict in order to ensure the right incentives. We also show that formal contracts impact the way in which reputational punishments will be structured: Formal contracts optimally induce a more eschewed pattern of sanctioning, compared to a benchmark case in which no formal document setting observable tasks exists. Thus, when dealing with its counterparties a firm will be, when the relational contract comes together with a formal one, less forgiving with those counterparties who have not performed the tasks under the formal contract, and more forgiving with those other ones who have not infringed the provisions of the formal document. We extend the basic setting to imperfect-but positive-enforcement and explore optimal investment in setting explicit tasks in the contractual document.

JEL classification numbers: K13, K23, L51, H24.

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†Department of Economics, Universitat Pompeu Fabra and Barcelona GSE. E-mail: juanjo.ganuza@upf.edu.
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‡Department of Law, Universitat Pompeu Fabra. E-mail: fernando.gomez@upf.edu.
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1 Introduction

Since the landmark findings of Macaulay (1963) concerning views and practices on commercial contracting among US business people, nonformal dealings have reached the forefront of the economic understanding of inter-firm cooperation. A recent survey of businesses in various industries confirms Macaulay's fundamental observation that firms do not essentially rely on legal enforcement in order to ensure cooperation with contracting partners (Bozovic and Hadfield, 2016). This evidence shows, at least in respect of those business sectors that possess an important innovative dimension requiring external contracting with other firms, that formal contracting is widely used, despite explicit recognition by market participants also of the fact that legal enforcement of the contract is often not a realistic outcome.

Consider the following illustrative case: A consulting firm advises a client in antitrust matters. Client cares only about the outcome of antitrust investigations. The effort of the advisor is not observable and incentives are provided mainly by the threat of termination of the relationship. Nevertheless, both firms incur the time and expense of signing a formal document setting certain terms and obligations for the parties. This "formal contract" will rarely be enforced in Court and it may specify tasks that are not directly relevant for the client, but has an influence on the decision by the latter to continue or discontinue the relationship.[1] Similar illustrations could be found in other contexts of manufacturer-supplier relationships, or outsourcing of multi-stage R&D projects. In this paper, we try to explain this pattern.

We provide a formal model showing that the existence of a formal contractual document, even if known to the parties that it will be enforced only imperfectly, or perhaps not at all,[2] in case

[1] Bozovic and Hadfield (2016) report the following answer in their survey::"*Every time someone comes and says to me, I think we need to terminate this relationship, or revisit this relationship, or asses this relationship, the first thing I do is look and see, what is the relationship?....You pull out the contract....*"

[2] The total absence of legal enforcement in a given case may be explained by different reasons that will be presented below in the text. Moreover, in a setting of a large buyer dealing with a number of suppliers, the fact that it is common knowledge that the formal document with its terms and conditions will not be enforced in the future may also have the effect of alleviating the concerns of other suppliers in a buyer's network of being held-up by the buyer.

something goes wrong, provides valuable information or generates asymmetric costs that help sustain the relational contract: Both mechanisms improve the incentives built upon sanctions that are purely relational, not legal. In our setting, it is not the ”enforcement” side of the legal document which is of particular value –because, in our framework, legal enforcement may even be fully absent. It is the improvement in the functioning of the informal contracting generated by the ”legal” contract that makes it attractive for the parties to draft and sign.

In particular, we explore a setting with a relational contract between two firms, where there is asymmetric information concerning the effort invested by the seller or producing party in a given project, and where the observable outcome of the project provides only an imperfect signal of the underlying effort. We then consider that the parties are able to draft a formal contract, although one that is only imperfectly enforceable by an adjudicator. In our basic setting, we use the extreme case in which the agreed upon contractual document is wholly unenforceable, and this is common knowledge to both firms. In an extension of our basic framework, we consider a more general imperfect enforcement scenario, where our main results also hold.

The extreme case of written contracts that are non-enforceable is used as the initial setup not just for heuristic motives, but also because it is an interesting case. There may be a variety of reasons why a formal written contractual document will not be enforced, and this is known beforehand: (i) The performance or breach actions under the formal contract may be unverifiable before a Court or other third party enforcer, or they may be insufficiently definite so as to merit legal enforceability, given that Contract Law refuses to enforce obligations that lack sufficient ”definiteness”[3]; (ii) the costs of pursuing enforcement of the formal contract before a Court or an arbitrator may be too costly, given the expected benefits in terms of damages or other transfers or remedies that may be imposed by the adjudicator, being this excess of litigation costs over expected value of litigation common knowledge; (iii) the parties themselves draft the formal document but deprive it of legal enforceability by declaring it non-binding (a mere “gentlemen’s

[3]See Schwartz (1992).

agreement").

Moreover, we start with the full non-enforcement assumption in order to emphasize the virtues of formal contracting unrelated to legal enforcement of the contractual obligations, which are clearly more striking in such a setting, where legal enforcement is altogether absent, infeasible or prohibitively costly. Later in the paper, we show that our results hold when the formal contract is partially or weakly enforceable through legal institutions, and provide an analysis of how the degree of enforceability impacts the relational contract.

Although there may be alternative or complementary explanations to ours, we wish to emphasize a rationale for the observation that parties resort to drafting a formal contract spelling out a number of tasks for the agent, even when they do not foresee the use of courts to enforce it or, in a weaker form, they foresee the courts will enforce the explicit agreement only imperfectly. Our explanation relies on the performance or breach of the obligations set out in the formal contract being better observable than the underlying effort (the relevant choice variable that the parties would optimally wish to influence) and correlated, however imperfectly, with the level of effort which constitutes the true variable of interest.[4] We denote this effect as the trust-building information effect. For instance, the parties, aware of the fact that the effort taken by the agent will only be very imperfectly observable through its effect on the likelihood of success of the project, although the latter may also be influenced by a wide range of other factors, may prefer to sign a formal contract that imposes upon the agent a set of tasks whose costs, or whose likelihood of performance, are correlated with the level of effort chosen. One could think of a variety of such correlated tasks: building and submitting a model or sample, writing and submitting progress reports (monthly, or quarterly, or with a different timing), making presentations from time to time so to update the other party on the development of the project, submitting detailed information on costs incurred, or conveying information to the counterparty on the milestones of the project. These ancillary tasks do not provide an actual benefit on the other side, or only a minor one

[4]In this respect, our idea has a flavor of those models where the contract can be a signal: Spier (1992); Hermalin (2002).

given the value of the project as a whole. But the parties, nonetheless, may prefer to invest time and effort in putting them in writing in a legally drafted document. Not because the customer or principal intends to bring the other party to Court if one of those ancillary tasks is not performed (probably both are aware that the costs of the lawsuit widely exceed the expected damages to be obtained from the breach of an ancillary obligation), but to improve the quality of the signal about the level of effort taken by the producing firm.

In such a setting we formally show two results of the use of formal contracts on the reputational or reciprocity-based sanctions that firms may impose upon their suppliers of goods and services. One is that formal contracts reduce the cost of reputational punishments that firms may need to inflict upon their counterparties in order to keep them under the right incentives to provide effort. Obviously, given that those reputational sanctions that may be applied are socially costly, formal contracts may provide net (of the transaction costs necessary to draft them) welfare benefits. The other is that formal contracts make reputational sanctions more eschewed than they would otherwise be. In other words, the customer will be, when the relational contract comes together with a formal contract, less forgiving with the counterparties who have not performed the formal contract, and more forgiving with those who have not infringed the provisions of the formal document. In fact, the optimal reputational sanctioning policy may be completely dichotomous. On the one hand, one should be fully forgiving with those contractors who perform the formal contract, even if the project did not succeed, when punishing the non-performing contractors provides enough incentives. On the other hand, one should be fully unforgiving -that is, strike them out forever from the list of potential suppliers- with those counterparties who fail to deliver the project, and at the same time do not perform the formal contract.

Our basic model is based on the probability of performance of the contractually foreseen tasks being affected by the underlying effort. When it is the cost of such tasks that is influenced by the level of effort, we show that if the difference between the cost of undertaking the formal tasks under high and under low effort is large enough, a formal (even if non-enforceable) contract may improve the relational contract. We denote this as the cost channel. It basically depends on which of two effects will dominate. On the one hand, as the cost of the task is higher under low effort, the incentive compatibility constraint is softened (+ incentive effect). On the other hand, introducing additional costs –performing non-intrinsically valuable contract tasks- reduces the value of the relationship, which has a negative impact over incentives (-loss of value effect). We characterize the conditions under which the positive incentive effect dominates the negative loss of value effect, and how a formal contract operating through the cost channel is effective in improving the efficiency of the relational contract. We also characterize, in terms of the main parameters (value of the relationship, asymmetry of information, discount factor), the optimal process of including tasks to be performed in the formal agreement. With all the above we do not only develop the idea that supplementing the relational exchange with the use of signals that may make the relational sanctions better tailored to the unobserved behavior of the agent improves contract surplus, but we also provide conditions for this to work, characterize the resulting outcomes and analyze the optimal decisions depending on the values of the relevant parameters.

One may still wonder if we have provided a theory for the use by parties, in settings that are largely relational, of formal contracts in the legal sense of the term, or rather we have simply shown how setting intermediate tasks for the agent -however these are expressed or formalized by the parties- allows a more refined calibration of the relational sanctions to the underlying behavior.

We think we have achieved both. No doubt there are various reasons why parties prefer, even when they know that the chances of actually using it in court are slim, to express and record the parties' future actions in a document that is drafted by lawyers, displays legal notions and jargon, and in sum can be properly assessed as a legal contract. For instance, parties may be subject to inertia or simply stick to contracting practices prevailing in other areas of business where relational sanctions may not be effective and thus legal enforcement is critical, may be responsive to the allure of "neutrality" and "authority" permeating legal concepts -such as obligation, legal duty and legal remedy- or intend to benefit from the coordinating function of the stock of legal precedents and experience that underlie contracts drafted with legal expertise. Additionally, the formal contract brings advantages in terms of enforceability, which may not be a completely irrelevant factor even when the chances of relying on the legal enforcement apparatus it are not large at the time of contracting. Moreover, as we show in Section 6, an imperfectly enforceable formal contract is a better complement to the scheme of relational sanctions the higher is the "quality" or accuracy of the enforceable content of the contract, which is correlated, in our view, with the skill and professionalism with which the contract has been drafted, and the display of legal notions and the contribution from legal professionals would seem to enhance those factors that are linked with a higher level of quality. Thus, a combination of all those factors, some that we explore in our paper, others that we do not explicitly consider, but are not inconsistent with the ones we do, clearly point at the existence of relevant advantages of formalizing the set of actions whose signals improve the functioning of the relational interaction in a legal document.

The persistence and relevance of the phenomenon we are trying to explain –explicit formal contracts setting certain tasks within an interaction that is essentially relational- seems to be well documented. In a recent paper, Bernstein (2015) presents a detailed analysis of the interaction between large industrial buyers and their suppliers in the US Midwest. That paper also shows the use of scorecards for suppliers, rating them on various objective performance metrics, future business depending on consistent results on such scores.[5] The explicit agreements include contractual provisions trying to improve the buyer's assessment of the suppliers' performance when it is not perfectly observable, in order to avoid making a given failure the trigger for termination or for other negative reactions affecting existing relational contract. This coexistence of formal and relational contracting has been also shown by Gilson, Sabel and Scott (2009, 2010, 2013) and Bozovic and Hadfield (2016).

Our paper relates to an already large strand of the economic literature that looks, theoretically

[5] The literature on the use of scorecards is vast. See, Kaplan and Norton (2001), and Gibbons and Kaplan (2015).

and empirically, into relational contracts.[6] The role of informal contract relationships, social norms in business networks, and reputation within the networks has also been identified and explored in various historical and economic circumstances.[7]

The literature has also considered the use of formal explicit contracts (with the accompanying legal consequences) on the functioning of implicit contracts. Kornhauser and MacLeod (2012) explore relational contracting within the full range of legal enforcement remedies (damages, specific performance, liquidated damages) that modern Contract Law displays.[8] Other papers in the economic theory of relational contracts analyze the effects of having a formal explicit contract written over some subset of verifiable actions on the relational contract: Baker, Gibbons and Murphy (1994, 2011); Schmidt and Schnitzer (1995). In particular, Baker, Gibbons and Murphy (1994) show that when a-fully enforceable- explicit contract is sufficiently "good" in order to approximate the relevant action, the relational and the formal contract are substitutes. When this is not the case, only an appropriate combination of both can generate desirable outcomes, and they act as complements. Our paper extends their insights to a setting of imperfect enforcement of the explicit contract.

Close to our paper is also Iossa and Spagnolo (2011), who analyze as well a setting in which firms use formal contracts in order to improve relational contracting. The driving forces of their results are, however, very different from ours. Their main idea is that the formal contract may specify some irrelevant or inefficient tasks, and used as "threat" to discipline informal agreements over efficient and non contractible tasks. Thus, in their model, formal tasks are fully enforceable but are not undertaken in equilibrium, while in our setting, formal tasks are imperfectly enforceable but they are carried out in equilibrium.

Our paper is also related to Gil and Zanarone (2014), although their focus of interest is to present a model of the optimal use of informal and formal contracts that would provide a relatively

[6]See, MacLeod (2007) and Malcomson (2012) for useful surveys of the literature.

[7]See, Greif (1989, 2012); Bernstein (1992, 2001); Baker, Gibbons and Murphy (2002), Baker and Hubbard (2004).

[8]They also analyze other functions (fact-finding, interpretation, adjudication) that legal institutions play in contracting.

simple set of empirically testable implications as to the factors affecting the use of just one or the other option (formal or informal) or a combined use (formal and informal) in terms of contracting strategies.

In the Law and Economics literature, in an important series of papers, Gilson, Sabel and Scott (2009, 2010, 2013) provide complementary analyses of the phenomenon they label as "braiding", the use of legal contracts to support informal contracting specially in technology-intensive industries. They place their lens on provisions of formal agreements that commit to exchange information between the parties, and those that establish conflict-solving schemes, bodies and procedures. With them, the parties not only may eventually improve observability of effort and preferences, but also increase the joint understanding of the parties concerning the development of their relationship (bringing the parties' beliefs closer), learn about the capabilities and the co-operative or non-cooperative features of the partner, and build increased trust among the parties. They also correctly underline how courts, with different interpretive and enforcement strategies (essentially what they label "low-powered enforcement", such as imposing obligations to negotiate in good faith, but not delivery and payment over the main subject matter of the contract) may help the parties to fruitfully use the formal side of contracting to support the informal dimensions of their relationship, and how Court's mistaken choices may interfere with the desirable helping hand function of legal contracts and Contract Law.

Bozovic and Hadfield (2016) provide important empirical findings on whether the standard Macaulay's narrative remains valid 50 years later. They add the interesting twist that within innovation-oriented industries and relationships there is a pervasive phenomenon: parties are still reluctant to go to court to adjudicate disputes and determine outcomes, but they make heavy use of formal documents, involving extensive legal advice, in the negotiation and agreement of contracts. They also contribute a theory trying to explain the observed dichotomy of contracting practices in industries with or without relevant innovation-related external contracts, that they label "scaffolding". In their theory, when uncertainty about the future desirability of certain actions is very high, the parties cannot rely on formal contracts to determine them, but cannot count either on informal but shared understandings by the parties. Thus, they may profitably opt to use the strong ”classification” properties and abilities of Contract Law, by signing a formal contract, and relying on the rules, doctrines, and interpretive strategies of Contract Law to determine further down the road if there is performance or breach in concretely realized contingencies. Moreover, by relying on a trusted set of classification properties, parties are able to better align what is the future understanding of parties concerning that future action in that future set of circumstances. Thus, formal contracting and Contract Law, even absent litigation and court intervention, will be able to bridge the gap in beliefs and underBoth the “braiding” and the “scaffolding” theories of the interplay between relational and formal contracting have in common that they advance a “complementarity” view of how both dimensions relate to each other. Recently it has been argued that formal contract provisions enforced through Contract Law serve to disrupt “too much” relational implication between the parties: Jennejohn (2016, 2018). In this “disruptive” theory of the interplay formal-relational, it is argued that relational contracts may in certain circumstances be too much of a good thing, and produce informational spillovers that parties do not desire, or give rise to redundant ties between the parties that hinder meaningful cooperation with other parties. Both constitute a source of inefficiencies. Jennejohn puts forward the hypothesis that enforceable contract provisions that parties insert in their largely relational interaction may address these sources of inefficiency in relational exchange by “disrupting” those elements of the relationship where these inefficiencies are more likely to emerge. Then, the hypothesis is tested with the use of empirical evidence, both qualitative and quantitative, from the biopharmaceutical industry: Jennejohn (2018).standings of the parties. This is what they name as ”scaffolding”.

Both the “braiding” and the “scaffolding” theories of the interplay between relational and formal contracting have in common that they advance a “complementarity” view of how both dimensions relate to each other. Recently it has been argued that formal contract provisions enforced through Contract Law serve to disrupt "too much" relational implication between the parties: Jennejohn (2016, 2018). In this "disruptive" theory of the interplay formal-relational, it is argued that relational contracts may in certain circumstances be too much of a good thing, and produce informational spillovers that parties do not desire, or give rise to redundant ties between the parties that hinder meaningful cooperation with other parties. Both constitute a source of inefficiencies. Jennejohn puts forward the hypothesis that enforceable contract provisions that parties insert in their largely relational interaction may address these sources of inefficiency in relational exchange by "disrupting" those elements of the relationship where these inefficiencies are more likely to emerge. Then, the hypothesis is tested with the use of empirical evidence, both qualitative and quantitative, from the biopharmaceutical industry: Jennejohn (2018).

Baker and Choi (2014), in turn, analyze a setting of a relational contract in which court enforcement is possible, that is, parties may resort to reputational and to legal sanctions, both of them costly. Legal sanctions provide two advantages compared to the setting of a pure relational contract. First, contract damages -that are different in nature and size from the future benefit of the transaction to the breaching party, which provides the size of the reputational sanction as well as the cost of it- allow parties to decouple the benefit of the legal sanction in terms of deterring undesirable breach, from the cost of implementing the sanction, which is given by litigation costs. Second, a formal contract breach may allow the parties to uncover more evidence of the true behavior of the other parties, and thus to better tailor the reputational sanctions.

The structure of the paper is as follows: Section 2 will present the basic setting. Section 3 will model the relational contract without formal contracting. Section 4 presents how the relational contract will be influenced by the parties drafting a formal contractual document, but under our assumption that it will not be enforced before an external adjudicator. Section 5 extends the binary performance/non-performance outcome to a richer setting of scores on the behavior of the contractor. Section 6 considers a more general setting of imperfect enforcement of the formal agreement. Section 7 extends our basic model when the costs of performing the contractually agreed tasks are considered. Section 8 tentatively analyzes the optimal investment in contracting. Section 9 briefly concludes. All proofs are relegated to the Appendix.

2 The Model

A producing firm (PF) undertakes a project for a customer firm (CF). The outcome of the project is uncertain but the probability of the project (e.g developing an innovative process or product) being successful depends on the effort exerted by PF. In particular, we assume that PF decides between two possible levels of effort, $e \in \{\underline{e}, \overline{e}\}$. The choice of effort is private information (not observable by CF) and not directly contractible. Exerting effort is costly, $c_{\underline{e}} < c_{\overline{e}}$, and determines the probability of success or completion of the project, $p_{\underline{e}} < p_{\overline{e}}$. For simplicity and without loss of generality, we take $c_{\underline{e}} = 0, c_{\overline{e}} = c$, $p_{\underline{e}} = 0$ and $p_{\overline{e}} = \pi$.

If the project is successful, it delivers profits $V > 0$. High effort is socially efficient, $\pi V - c > 0$. PF is financially constrained and it cannot bear the risk of financing the project. CF pays an exogenous price P[9] to PF for undertaking the project, as $\pi V > P > c$.[10] Given these assumptions, CF would be willing to contract with PF if effort is high ($\pi V > P$), but not otherwise. In a static framework, CF correctly anticipates that given that the effort is not observable and contractible, PF has strong incentives to shirk and therefore there will be no trade. Parties can overcome this market failure when the interaction is repeated by using a relational contract.

[9] We take P as exogenous in order to simplify the presentation. We could add a bargaining mechanism to the game in order to endogenize P and obtain the same results. For example, we could endogenize the price by giving full bargaining power to PF. Then, the latter would set the price at the level in which CF is indifferent between buying or not, $P = \pi V$. Then, one could replace P by πV in all the subsequences expressions and verify that our results hold.

[10] We only consider payments from CF to PF. We are implicitly assuming that PF is protected by limited liability, and we cannot use the results of Levin (2003).

3 Building Trust

Now we consider an infinite horizon framework with an infinitely lived PF and an infinitely lived CF, in which the basic game above is repeated over and over again. As in the static game, we still assume that contracts cannot be verified by a third party who could enforce the explicit provisions of a formal contract.

This repeated game has multiple equilibria, including the repetition of the solution to the static game. We will focus on equilibria supporting cooperation between PF and CF. In particular, we consider the following grim strategy subgame perfect equilibrium inspired by Green and Porter (1984):[11] When project fails, the no-trade equilibrium, in which the PF chooses no effort and CF does not buy, takes place for T periods. After expiration of these T periods, the cooperation phase is reinstated.

- CF starts trusting PF in period 1, and financing the project by paying price P, starting a cooperation phase.
- Cooperation phase. There is trade, PF chooses high effort and CF trusts PF by financing the project by paying the price P until a project failure occurs, starting a punishment phase.
- Punishment phase. When CF observes project failure, she reacts by discontinuing to finance projects with PF for T periods. After expiration of the T periods, CF is willing to trade with PF again. The cooperation phase may be reinstated.

We are in a setting of ex-post imperfect information: the fact that the project has failed is an imperfect signal of PF's level of effort. If the signal were perfect, then T could be infinite and the cost of punishment would be 0, since punishment would never be imposed in equilibrium. In our setting, the imperfect information leads agents to incur punishment cost. Both parties would be better off if they did not stop trading during the punishment phase (T periods). However,

[11]In fact, our way of modeling relational contracting is a simplified version of the collusion model by Green and Porter (1984) as presented in Tirole (1988) and Cabral (2005).

punishment is necessary to preserve incentives. We will focus on the "optimal" relational contract, the one that maximizes the number of periods in which trade occurs, or, equivalently, minimizes the number of periods in which the costly reputational sanction is imposed.

This relational contract is optimal within the set of "Green and Porter" grim strategies described above, but it is not globally optimal. In the Green and Porter model (and also in our setting) there exist alternative and more complex strategies generating equilibria in which parties get higher surplus.[12] However, the "Green and Porter " optimal grim strategy is appealing, since it is simple and easy to implement and, more importantly for the present paper, it summarizes in a single parameter T the inefficiencies of the relational contract due to the imperfect monitoring of effort.

We assume that both agents face the same discount factor, $\delta \in (0,1)$. When CF and PF play the strategy described above, let V^+ and V^- be the present discounted value of PF's profits in the cooperation and punishment phase, respectively. We have:

$$\begin{aligned} V^+ &= P - c + \pi\delta V^+ + (1-\pi)\,\delta V^-, \\ V^- &= \delta^T V^+. \end{aligned}$$

Solving the equation system we obtain both present values in terms of the parameters of the model

$$V^+ = \frac{P-c}{1-\pi\delta-(1-\pi)\,\delta^{T+1}}, \tag{1}$$

$$V^- = \delta^T V^+ = \frac{\delta^T(P-c)}{1-\pi\delta-(1-\pi)\,\delta^{T+1}}. \tag{2}$$

Finally, to achieve this equilibrium we must add an incentive compatibility constraint. The following inequality captures the lack of incentives of PF to choose low effort:

$$V^+ \geq P + \delta V^-$$

[12] See for example Abreu, Pearce and Stacchetti (1996) for a characterization of the optimal discrete strategies in the Green and Porter model.

Using the definition of $V^{+} = P - c + \pi\delta V^{+} + (1-\pi)\,\delta V^{-}$, the incentive compatibility constraint can also be written as:

$$\pi\delta\left(V^{+} - V^{-}\right) \geq c. \tag{3}$$

We are interested in another equivalent expression for the inequality above, which can be found using the solution to the equation system V^{+} and V^{-} (we plug equations (1) and (2) into (3)):

$$\pi\delta\frac{(1-\delta^{T})(P-c)}{1-\pi\delta-(1-\pi)\,\delta^{T+1}} \geq c.$$

Let $\Phi(T)$ be the left side of the incentive compatibility constraint above. For our purposes, this function has a useful property:

Lemma 1 *$\Phi(T)$ is increasing in T.*

Hence, to solve optimally the infinitely repeated game, we want to choose T in order to maximize V^{+} :

$$\max_{T}\ V^{+} = \max_{T}\ \frac{P-c}{1-\pi\delta-(1-\pi)\,\delta^{T+1}}$$

subject to the following constraint:

$$\Phi(T) \geq c.$$

Given that our function satisfies $\frac{\partial V^{+}}{\partial T} < 0$, then the optimal T^{*} for our problem will be the minimum T that satisfies the identity $\Phi(T^{*}) = c$. But this equation has a unique solution, by Lemma 1.[13]

The optimal punishment T^{*} has been characterized for a given value of the discount factor δ, probability of success of the project under high effort π, and marginal profit $P-c$. Next Lemma establishes how the optimal punishment T^{*} depends on this set of parameters.

Lemma 2 *The optimal punishment T^{*} is decreasing in π, $P-c$ and δ.*

[13] For expositional convenience we treat T as a continuous variable. If T were a discrete variable (the number of no-trade periods), the optimal punishment T^{*} should be defined by the following conditions: $\Phi(T^{*}-1) < c$ and $\Phi(T^{*}) \geq c$.

The intuition of Lemma 2 is as follows. The optimal punishment decreases with π since it is a measure of the level of imperfect information (higher π implies lower asymmetric information, that is project failure is a more informative signal of low effort by PF). It also decreases with $P - c$ and δ, since they increase the cost for PF of the missing trade following project failure.

4 Trust with Formal but Non Enforceable Contracts

In this section we explore the role that a formal contractual agreement that will not be enforced. For instance, because it lacks sufficient certainty in the agreement, and thus does not reach the status of a legally enforceable contract, or, being theoretically enforceable, the anticipated enforcement costs are so high that it will not be actually enforced, this outcome being common knowledge. In section 6 we will explore the case when the contract detailed below is weakly enforceable.

In our previous setup we introduce a formal contractual document or agreement between CF and RF which we label, C. This formal agreement, for any of the reasons pointed out above, will not receive legal enforcement in case of "breach". C specifies for PF some intermediate tasks $a \in A$ that may be imperfectly correlated with the true effort invested by PF and thus, provide incentives to the agent. We assume that CF does not obtain any direct benefits from the tasks specified in C other than giving incentives to PF. The way in which the formal contract $C(a)$ may provide incentives can be approached in various ways. One possibility (that we denote as the cost channel) is that the cost of undertaking the contractually stipulated tasks is smaller if PF has exerted effort: $c(a|\overline{e}) < c(a|\underline{e})$. Another path (the probability channel) is that the probability of success in discharging the contractual tasks is larger when PF has taken effort, $p(a|\overline{e}) > p(a|\underline{e})$. Obviously, both channels may be at work at the same time. In this section we will focus on the probability channel, and in a later extension we will look into the cost channel.

The formal contract, through the probability channel, would work as follows. If the project succeeds, CF learns that high effort has been exerted and the formal contract plays no role. If the project fails, the contract provides imperfect information on effort exerted by PF. Formally, at the end of the project (or the relevant project phase) the formal contract allows the production of a signal $s \in \{s_P, s_{NP}\}$, where the sub-index means performance and non-performance of the formal contract, respectively. The realization of the signal is observable by PF and CF. We initially assume that the contract signal is informative, and we take its informativeness as given. As the signal is informative, the probability of a good signal realization is higher when PF has exerted effort: $P(s_P|\overline{e}) = \alpha > \beta = P(s_P|\underline{e}) = \beta$.

The next step is to analyze the interaction between the formal (albeit, remember, not to be enforced)contract and the relational/informal interaction. The main idea is that parties can use the information provided by the formal agreement for improving the functioning of the informal contract by tailoring the reputational or relational punishment more tightly to the expost probability that no effort has been exerted.

Formally, we define a new infinite horizon game in which CF makes the relational sanction dependent on the performance of the formal contract. After observing project failure, CF also observes the signal from the formal contract, and then CF sets the relational sanction accordingly. We proceed as in the previous case by computing the Present Discounted Value of PF's profits given the punishment by CF, now based on the observed performance of the formal contract (we have included an upper index C to refer to the formal contract). Notice that the formal contract reduces but does not eliminate the asymmetry of information (otherwise this would be equivalent to make the production effort contractible). Thus, there may be Type I errors (situations in which PF exerts effort and the formal contract is not satisfied), as well as Type II errors (situations in which PF does not exert effort but the formal contract is satisfied). These errors will be key in determining the optimal punishment. In sum, in case of project failure, a relational sanction is triggered, but the length of the punishment depends on PF's performance of the formal contract

$$\begin{aligned}
V^{C+} &= P - c + \pi\delta V^{C+} + (1-\pi)\,\delta\alpha V_P^{C-} + (1-\pi)\,\delta(1-\alpha)V_{NP}^{C-},\\
V_P^{C-} &= \delta^{T_P} V^{C+}\\
V_{NP}^{C-} &= \delta^{T_{NP}} V^{C+}
\end{aligned}$$

Solving the equation system, we obtain:

$$V^{C+} = \frac{P-c}{1-\pi\delta-(1-\pi)(\alpha\delta^{T_P+1}+(1-\alpha)\delta^{T_{NP}+1})},$$

The formal contract also affects the incentive compatibility constraint, so in order to express that the firm has no incentive to exert low effort, now we have:

$$V^{C+} \geq P + \delta[\beta V_P^{C-} + (1-\beta)V_{NP}^{C-}]$$

Following similar computations than in the previous section, we obtain the incentive compatibility constraint under formal contracting as the inequality given by:

$$\Psi^C(T_P, T_{NP}, \alpha, \beta) \geqslant c$$

where this new function is:

$$\Psi^C(T_P, T_{NP}, \alpha, \beta) = \frac{\delta\left[\pi + (1-\pi)(\alpha\delta^{T_P} + (1-\alpha)\delta^{T_{NP}}) - (\beta\delta^{T_P} + (1-\beta)\delta^{T_{NP}})\right](P-c)}{1-\pi\delta-(1-\pi)(\alpha\delta^{T_P+1}+(1-\alpha)\delta^{T_{NP}+1})}$$

Notice that if we impose that penalties are independent of the performance of the formal contract, $T_P = T_{NP} = T$, and by construction $\Psi^C(T, T, \alpha, \beta) = \Phi(T)$.

We are interested in characterizing the optimal relational sanctions with formal but non enforced contracting, which will be the solution to the following problem

$$\max_{T_P, T_{NP}} V^{C+} = \frac{P-c}{1-\pi\delta-(1-\pi)(\alpha\delta^{T_P+1}+(1-\alpha)\delta^{T_{NP}+1})}$$

subject to the incentive constraint:

$$\Psi^C(T_P, T_{NP}, \alpha, \beta) \geq c.$$

Then, we need to determine the optimal relational punishment when the firm satisfied the formal contract, T_P, and when the formal contract is not performed, T_{NP}. In order to compare the solution to this problem (with two punishment variables (T_P, T_{NP})) with the optimal relational punishment in the previous framework with only one instrument T, we focus on the impact of the punishment on the objective function. We say that (T_P, T_{NP}) generates lower expected relational punishment costs than T if $\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1} > \delta^T$. In fact, the solution to the problem is the pair (T_P^*, T_{NP}^*) that satisfies the incentive compatibility constraint and maximizes $\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1}$ (minimizes the expected relational punishment costs).

First, we characterize what is the optimal punishment policy when the information provided by the contract is used.

Proposition 1 *The optimal punishment with formal contracting feedback maximizes the relational punishment in case of non performance of the formal contract (minimizes the punishment in case of performance of the formal contract). This implies that the optimal relational contract (T_P^*, T_{NP}^*) may have two formats: i) Never again, $(T_P^* = T, T_{NP}^* = \infty)$ and ii) Full forgiveness $(T_P^* = 0, T_{NP}^* = T)$.*

A sketch of the proof is as follows. There is a set of pairs (T_P, T_{NP}) that generate the same expected punishment when PF exerts effort, $\alpha\delta^{T_P} + (1-\alpha)\delta^{T_{NP}} = U$. The optimal solution is characterized by finding the maximum U^* that satisfies the incentive compatibility condition $\Psi^C(T_P, T_{NP}, \alpha, \beta) \geq c$. Using a change of variable we can rewrite Ψ^C as a function of U and $\delta^{T_{NP}^*}$. We show in the proof that Ψ^C is decreasing in U and increasing in δ^{T_P}. The higher the relational punishment (the lower U) the higher the incentives to exert effort, since by doing so PF reduces the probability of that punishment. This result implies that the incentive compatibility constraint must be binding $\Psi^C(U^*, \delta^{T_P^*}, \alpha, \beta) = c$. Among all the pairs that generate the same expected punishment U^*, choosing the one with higher δ^{T_P} maximizes the incentives of PF to exert effort. Conditional on being punished, the difference between exerting effort and not exerting it is $(\alpha - \beta)(\delta^{T_P} - \delta^{T_{NP}})$, which is maximized with the highest δ^{T_P}. Then, the optimal punishment requires to maximize δ^{T_P}, implying that if conditions do not require a tough punishment, PF is forgiven if there is project failure but the formal contract has been performed. Otherwise, PF is punished in case of performance, but the relationship with CF is completely severed for ever in case of non-performance ("never again").

The underlying intuition is based on the costly nature of the relational punishment and on how, once incentives have been ensured, minimizing the punishment enhances the contract surplus. Thus, we would optimally want to focus the costly punishment on those cases where signals reveal the "worst news" concerning the underlying effort that the contract is interested in inducing.

PROPOSITION 2 *The optimal relational punishment with formal contracting feedback,* (T_P^*, T_{NP}^*) *, generates lower expected relational punishment costs than without it,* T^**, i.e.* $\alpha\delta^{T_L^*}+(1-\alpha)\delta^{T_{NL}^*} > \delta^{T^*}$.

Technically, Proposition 2 is implied by the previous result. Disregarding the information provided by the performance or non-performance of the formal contract, that is, using the same punishment in case of performance and non performance, $T_P = T_{NP} = T^*$, is feasible, but Proposition 1 shows that it is not the optimal solution. By using formal contracting feedback, (T_P^*, T_{NP}^*), lower expected relational punishment cost can be achieved by imposing higher penalties in the case of non performance of the formal contract, which is relatively more likely to arise if PF has not exerted effort.[14]

PROPOSITION 3 *The optimal relational punishment with formal contracting feedback* (T_P^*, T_{NP}^*) *is decreasing in the informativeness of the formal contract, decreasing in* α*, and increasing in* β.

All previous results depend on the assumption that the formal contract is informative as to the effort decision of PF, $\alpha > \beta$. Proposition 3 provides the intuitive result that the more informative

[14] In a way, this result rewrites the Holmström (1979) Informativeness Principle for relational contracting. The Informativeness Principle reads: "any measure of performance (ASSOCIATED TO THE FORMAL CONTRACT in our relational set up) that reveals information about the effort level chosen by the agent should be included (TAKEN INTO ACCOUNT in our relational setup) in the compensation (RELATIONAL) contract".

the formal contract is, the better the tailoring of the punishment and consequently, the more efficient the relational contract. Higher informativeness (higher α or lower β) makes the optimal relational contract more effective, since minimizing the punishment in case of performance has larger impact over incentives, the higher is the informativeness of the formal contract, as clearly captured by the term $(\alpha - \beta)(\delta^{T_P} - \delta^{T_{NP}})$.

5 Formal Contracts with Scoring.

Up to now we have considered that the outcome of actions under the formal contract is binary. This seems to be a natural assumption for enforceable contracts that can be brought before courts: Contracts are performed or breached, standards are satisfied or not, and so on. It is not by chance that legal terms and legal reasoning is to a large extent binary. This would carry over to formal contracts drafted by lawyers trained in this dyadic mode of thinking even when parties know that court enforcement will rarely be the case. However, the role of formal contract in our setting is to provide information regarding the underlying effort decision by PF, not the traditional legal role of securing enforcement. Then, it makes sense to consider that the outcome of the formalized contract relationship is a score, a "grade", rather than a simple binary performance/ non-performance outcome. This seems important, since the reader may suspect that behind some of our previous results the binary structure of the formal contract is at work. In related mood, Gibbons and Kaplan (2015) emphasize the importance of appropriately balanced scoring for formal and informal decisions in agency settings and discuss both formal and informal weights on such scores. As observed in the introduction, Bernstein (2015) thoroughly documents the importance of the actual use of scorecards.

Therefore, we now generalize the model by allowing the formal contract between CF and PF (C), to deliver a non-binary outcome. In particular, we assume that at the end of the project or relevant phase thereof, the formal contract generates a score signal s that is observable by PF and CF. To ensure that taking high effort translates into more evidence (a higher score) that the agent took high effort, we assume that the signal is monotone, that is, $f(s|e)$ satisfies the Monotone Likelihood Ratio Property (MLRP):

$$\frac{f(s|\overline{e})}{f(s|\underline{e})} \text{ is increasing in } s.$$

This condition ensures that more evidence is "good news" about effort (Milgrom (1981)), that is, $\Pr(\overline{e}|s)$ is increasing in s. To prove the results, it is convenient to take the score s as a discrete variable, $s_1 < s_2 < .... < s_N$. The Monotone Likelihood Ratio Property (MLRP) implies in that case:

$$\frac{\Pr(s_j|\overline{e})}{\Pr(s_j|\underline{e})} > \frac{\Pr(s_i|\overline{e})}{\Pr(s_i|\underline{e})} \text{ if } j > i,$$

equivalently,

$$\frac{\Pr(s_j|\overline{e})}{\Pr(s_i|\overline{e})} > \frac{\Pr(s_j|\underline{e})}{\Pr(s_i|\underline{e})} \text{ if } j > i$$

Following similar computations than in the previous sections, we can rewrite the problem as follows

$$\max_{T(s)} V^{C+} = \frac{P-c}{1-\pi\delta-(1-\pi)(\sum \Pr(s_i|\overline{e})\delta^{T(s_i)})}$$

subject to the incentive constraint:

$$\Psi^C(T(s), \Pr(s|\underline{e}), \Pr(s_i|\overline{e})) \geq c.$$

where $\Psi^C(T(s), \Pr(s|\underline{e}), \Pr(s_i|\overline{e}))$ is equal to

$$\frac{\delta\left[\pi+(1-\pi)(\sum_{i=1}^{N}\Pr(s_i|\overline{e})\delta^{T(s_i)})-(\sum_{i=1}^{N}\Pr(s|\underline{e})\delta^{T(s_i)})\right](P-c)}{1-\pi\delta-(1-\pi)(\sum_{i=1}^{N}\Pr(s_i|\overline{e})\delta^{T(s_i)})}$$

Now, $T(s_i)$ is a punishment function that depends on the score obtained by PF in the formal contract C, and $\Pr(s_i|\underline{e})$ and $\Pr(s_i|\overline{e})$ are the distributions of the score that depend on whether or not the PF firm has exerted care. Notice that our previous binary setting is just a particular case of the present formulation.

Proposition 4 *Let* $U^* = \sum_{i=1}^{N} \Pr(s_i|\overline{e})\delta^{T^*(s_i)}$ *be the optimal punishment, then there exists a score* $s^* \in \{s_1, s_2, ..., s_N\}$ *such that if* $s_i < s^*$ *then* $T(s) = \infty$ *(never again), and if* $s_i > s^*$ *then* $T(s) = 0$ *(total forgiveness).*

In other words, there is an optimal standard or minimum score, s^*, such that if the outcome of the formal contract is higher than s^*, PF is forgiven if the project fails. Otherwise, when the score is lower than s^* and the project fails, the relationship is terminated by CF forever. The ground of this result lies in the point that between two scores s_i and a larger one, s_{i+1}, one wants to maximize the punishment in s_i (if at all needed), because by doing so the MLRP ($\frac{\Pr(s_{i+1}|\overline{e})}{\Pr(s_i|\overline{e})} > \frac{\Pr(s_{i+1}|\underline{e})}{\Pr(s_i|\underline{e})}$) implies that the punishment in s_i increases more, in relative terms, the punishment of the firm when it has exerted low effort. As a consequence the punishment increases the incentives to exert high effort. Finally, it is important to point out that there are no restrictions over the number of elements and structure of $s^* \in \{s_1, s_2, ..., s_N\}$.Thus, in the limit, the scoring set could be continuous. Although the proof is now technically more challenging, the intuition at work here is similar to that behind Proposition 1: since punishment is costly, we desire to calibrate it in such a way that focuses the relational sanctions -and thus saves costs that otherwise would be incurred- on the cases revealing the worst signals concerning the variable of interest as to incentives, that is, agent's effort. It is remarkable that the structure of the optimal reputational punishment looks like a standard negligence type of rule. One should notice, though, that the standard neglicence rule is a purposeful threshold rule that intends to improve incentives by creating a discontinuity in the pay-off function of the agent (the potential injurer, for instance). Here, it is the optimal incentive structure that determines the existence of a threshold in the scoring function.

Proposition 4 generalizes Proposition 1 given that the binary signal was a particular case of the set of signals that we consider in this section. As in the previous section, Proposition 4 implies that disregarding the information provided by the score resulting from the formal contract is not optimal. Then, in a way, it generalizes Proposition 2.

Proposition 3 established that the optimal relational punishment with feedback from the formal contracting is decreasing in the informativeness of the formal agreement. To generalize it, we need a criterion of informativeness that we can apply to scores resulting from formal contracts.

DEFINITION 1 *The formal contract, C_1, is more informative than C_2, if $F_1(s|\overline{e}) \leq F_2(s|\overline{e})$ ∇s($\sum_{i=1}^{x} \Pr(s_i|\overline{e})_1 \leq \sum_{i=1}^{x} \Pr(s_i|\overline{e})_2$ ∇x) and $F_1(s|\underline{e}) \geq F_2(s|\underline{e})$ ∇s($\sum_{i=1}^{x} \Pr(s_i|\underline{e})_1 \geq \sum_{i=1}^{x} \Pr(s_i|\underline{e})_2$ ∇x)*

Next Proposition states that the informativeness order of the scores based on formal contracts implies all common informativeness criteria based in the value of information for a decision maker (Blackwell sufficiency and Lehmann efficiency.).[15] Those informativeness criteria are built in terms of the value of information in decision making problems: a signal X is more informative than some other signal Y if every decision-maker with preferences in a particular class prefers X to Y. Thus, a signal is more informative if it allows decision-makers to make better decisions and to reduce type I and II decision errors.

PROPOSITION 5 *If the scoring based on contract C_1 is more informative than the scoring from contract C_2, according to definition 1, then C_1 is more informative than C_2 according to Blackwell sufficiency and Lehmann efficiency, and it generates less decision errors.*

Finally, using our concept of formal contract's informativeness, we can state that more informative formal contracts translate into a more productive relationship and lower reputational sanctions.

PROPOSITION 6 *If contract C_1 is more informative than contract C_2, according to definition 1, then optimal relational punishment under C_1, $\sum_{i=1}^{N} \Pr(s_i|\overline{e})_1 \delta^{T_1^*(s_i)}$ is lower than under C_2, $\sum_{i=1}^{N} \Pr(s_i|\overline{e})_2 \delta^{T_2^*(s_i)}$.*

This result generalizes Proposition 3 and states that a more informative formal contract, by reducing decision errors, allows to decrease the equilibrium punishment while keeping incentives.

[15] Ganuza and Penalva (2010) provide alternative criteria of informativeness based on the dispersion of posterior conditional expectations. These criteria have the advantage that the dispersion of conditional expectations is easily verified. Ganuza and Penalva (2010) show that the weakest of these criteria, integral precision (based on the convex order) is equivalent to Lehmann efficiency in dichotomous settings and then it is also implied by the defined contract informativeness order.

6 Weakly Enforceable Contracts.

In the previous sections we have assumed that the formal contract between CF and RF (C), will lack any enforcement by an adjudicator. In this subsection, we reconsider our binary framework when the contract is weakly (or imperfectly) enforceable, meaning with this that if the project fails and the contract generates a non performance signal, s_{NP}, CF is entitled to receive some monetary compensation D (maybe related to the price paid by CF at the start of the project, P) by PF with probability γ. This γD is the expected compensation to CF in case of project failure and non-performance, and the expected monetary sanction on PF for breach of the obligations under the formal document. We introduce two conditions over the "quality" of the (now enforceable) contract: i) $(1-\beta)D - (1-\pi)(1-\alpha)D > c$; and ii) $P - c > (1-\pi)(1-\alpha)D$. These two conditions are easier to interpret under full enforceability $\gamma = 1$ of the contract. In such a case, i) guarantees that the enforceable contract provides enough incentives to PF to exert effort; ii) ensures that PF gets some surplus and is willing to trade. Notice that these conditions are related to the "quality" of the contract and the contract legal enforcement apparatus, since both conditions are easier to meet if a better or more informative contract for the adjudicator (higher α or lower β) is in place.

How does this complication change the problem of the optimal relational contract? The design of the optimal relational contract with a weakly enforceable formal contractual agreement requires to recompute the Present Discounted Value of PF's profits in order to include the expected legal monetary sanction, $(1-\pi)(1-\alpha)\gamma D$.

$$\begin{aligned}
V^{C+} &= P - c - (1-\pi)(1-\alpha)\gamma D + \pi\delta V^{C+} + (1-\pi)\delta\alpha V_P^{C-} + (1-\pi)\delta(1-\alpha)V_{NP}^{C-}, \\
V_P^{C-} &= \delta^{T_P} V^{C+} \\
V_{NP}^{C-} &= \delta^{T_{NP}} V^{C+}
\end{aligned}$$

Solving the equation system, we obtain:

$$V^{C+} = \frac{P - c - (1-\pi)(1-\alpha)\gamma D}{1 - \pi\delta - (1-\pi)(\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1})},$$

Most importantly, weak enforceability of the contract also affects the incentive compatibility constraint:

$$V^{C+} \geq P - (1-\beta)\gamma D + \delta[\beta V_P^{C-} + (1-\beta)V_{NP}^{C-}],$$

We can rewrite the IC as:

$$\Psi^{WE}(T_P, T_{NP}, \alpha, \beta, \gamma) \geqslant c$$

where $\Psi^{WE}(T_P, T_{NP}, \alpha, \beta, \gamma)$ is:

$$\frac{\delta\left[\pi + (1-\pi)(\alpha\delta^{T_P} + (1-\alpha)\delta^{T_{NP}}) - (\beta\delta^{T_P} + (1-\beta)\delta^{T_{NP}})\right](P - c - (1-\pi)(1-\alpha)\gamma D)}{1 - \pi\delta - (1-\pi)(\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1})} +$$
$$+ (\beta - \alpha - (1-\pi)\alpha)\gamma D$$

By construction, if $\gamma = 0$ then $\Psi^C = \Psi^E$. Thus, the optimal relational sanction with weakly enforceable formal contracting will be the solution to the following problem:

$$\max_{T_P, T_{NP}} V^{C+} = \frac{P - c - (1-\pi)(1-\alpha)\gamma D}{1 - \pi\delta - (1-\pi)(\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1})}$$

subject to the incentive constraint:

$$\Psi^{WE}(T_P, T_{NP}, \alpha, \beta, \gamma) \geq c.$$

As before, the solution to the problem is the pair $(T_P^{WE*}, T_{NP}^{WE*})$ that satisfies the incentive compatibility constraint and maximizes $\alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1}$ (minimizes the expected punishment costs). Thus, all our previous results hold with weakly enforceable contracts: The optimal relational contract $(T_P^{WE*}, T_{NP}^{WE*})$ has the familiar double format: (i) never again, $(T_P^{WE*} = T, T_{NP}^{WE*} = \infty)$, or (ii) full forgiveness $(T_P^{WE*} = 0, T_{NP}^{WE*} = T)$. The optimal relational punishment $(T_P^{WE*}, T_{NP}^{WE*})$ is decreasing in the informativeness of the formal contract, decreasing in α and increasing in β.

In addition, we can state a new result regarding the impact of the degree of enforcement of the formal contract on the efficiency of the optimal relational contract.

PROPOSITION 7 *The optimal relational punishment* $(T_P^{WE*}, T_{NP}^{WE*})$ *is decreasing in the degree of enforcement of the formal contract* γ.

This result goes in line with previous work of ours (Ganuza et al. 2016) showing how, in settings of product markets with asymmetric information about product quality, the legal system may simultaneously reduce the cost of market sanctions, and sustain cooperation between firm and consumers for a larger set of relevant parameter values. Here, we have showed that if the enforcement of the formal contract increases, the optimal reputational sanction decreases, and thus there is a substitution effect between the two dimensions. Moreover, as in Ganuza et al. 2016, enforceability makes it possible for cooperation to emerge for a larger set of parameter values. Along such dimension, formal and relational contracting are complements.

It is important to notice that the proof of Proposition 7 relies on the stated minimal conditions over the quality of the contract: (i) $(1-\beta)D-(1-\pi)(1-\alpha)D > c$; and (ii) $P-c > (1-\pi)(1-\alpha)D$. For example, consider that the quality of the contract is low (high β and low α), and previous conditions are not satisfied. Then, it is possible that, contrary to Proposition 7, optimal relational punishment increases when the degree of enforceability of the formal contract goes up. This is due to the fact that in such a case, the legal consequences of the formal contract are severely misaligned with the underlying relevant effort, and increasing enforcement reduces the value of the relationship without substantially improving incentives, having an overall negative impact on the efficiency of the relational contract. Thus, if the "quality" of the formal contract and of the legal apparatus entrusted with its enforcement are sufficiently poor, doing away with the formal contract and solely relying on the relational sanctions is a more advantageous action plan.

7 Exploring the Cost-Channel.

The cost channel exists when the cost of undertaking the intermediate tasks $a \in A$ specified in the formal contract vary in the level of underlying effort, $c(a|\overline{e}) < c(a|\underline{e})$. We will show that in this case, even if we shut down the probability channel by assuming that the intermediate tasks are always undertaken, $P(s_P|\overline{e}) = P(s_P|\underline{e}) = 1$, a formal document $C(a)$ that will not be enforced may improve the relational contract.[16] The idea is simple: as PF has to incur a higher cost to perform the tasks when it chooses low effort, introducing the formal contract softens the incentive compatibility constraint $\pi\delta\left(V^{+} - V^{-}\right) \geq c + c(a|\overline{e}) - c(a|\underline{e})$ (+ incentive effect). However, introducing the formal contract also has a downside, since it burdens PF with an extra cost that does not provide benefits (remember, the tasks are not valuable per se to CF) and thus reduces the value of the relationship, which has a negative impact on incentives (-loss of value effect). Then the problem becomes

$$\max_{T} V^{+} = \max_{T} \frac{P - c - c(a|\overline{e})}{1 - \pi\delta - (1-\pi)\,\delta^{T+1}}$$

subject to the following constraint:

$$\pi\delta \frac{(1-\delta^{T})}{1 - \pi\delta - (1-\pi)\,\delta^{T+1}} \geq \frac{c - (c(a|\underline{e}) - c(a|\overline{e}))}{P - c - c(a|\overline{e})}.$$

As in our baseline model, the optimal relational contract T^{*} will be the minimum T that satisfies the incentive compatibility constraint. Lemma 1 and its proof in the appendix states that the left hand side of the incentive compatibility constraint is increasing in T. Then, as we discussed above, the optimal T^{*} will be decreasing in the cost difference between undertaking the effort and not undertaking it, $c(a|\underline{e}) - c(a|\overline{e})$ (+ incentive effect), and increasing in the contract performance costs under high effort $c(a|\overline{e})$ (- loss of value effect). These comparative statics can be summarized in the following proposition.

[16]Notice that when we are assuming $P(s_P|\overline{e}) = P(s_P|\underline{e}) = 1$, we mean that there is no risk in undertaking the task, but we are also implicitely assuming that failure to undertake the task will be understood as cheating (exerting $\underline{e}$) and then it is out of the equilibrium path.

Proposition 8 *(i) The optimal punishment T^*_a is decreasing in $c(a|\bar{e})$ and increasing in $c(a|\underline{e})$. (ii) Let two contracts C_1 and C_2 with two different sets of tasks a and a' and two optimal punishment T^*_a and $T^*_{a'}$, then $T^*_a \leq T^*_{a'}$ iff $\frac{c-(c(a|\underline{e})-c(a|\bar{e}))}{P-c-c(a|\bar{e})} \leq \frac{c-(c(a'|\underline{e})-c(a'|\bar{e}))}{P-c-c(a'|\bar{e})}$.*

It is interesting to illustrate the result with the following example. Consider the tasks as a continuous variable, where $c(a|\underline{e}) = \kappa a$ and $c(a|\bar{e}) = a$. Then, simple computations show that the optimal punishment T^* is decreasing in the number of tasks a if and only if $\kappa \geq \frac{P}{P-c} \geq 1$. In words, including costly non-productive tasks in a contract that will not be enforced may increase the efficiency of the relational contract as long as the cost difference of undertaking these tasks between exerting and not exerting effort is large enough. Notice that the previous condition is only a necessary one: deciding optimally the number of tasks included in the contract requires to jointly consider the reduction in punishment costs and how $c(a|\bar{e}) = a$ also reduces V^+. This problem is briefly analyzed in the next section.

8 Investing in Contracting.

In previous sections we had taken the contract between CF and PF as exogenous, and we have explored the probability and the cost channels independently. Now, we want to consider that the contract (the set of tasks) is chosen optimally in order to maximize the value of the relationship. In addition, the tasks, $a \in A$, are characterized by different costs, $\{c(a|\bar{e}), c(a|\underline{e})\}$, and probabilities, $\{p(s_P|\bar{e}), p(s_P|\underline{e})\}$, and are likely to have an impact on the relational contract through both the probability and the cost channels simultaneously. The precise characterization of the optimal contract should depend on the particular structure of the set of tasks. We take a more parsimonious approach and we define an investment parameter in formal contracting λ, in such a way that the inverse measure of the equilibrium punishment $IP(\lambda) = \alpha\delta^{T_P+1} + (1-\alpha)\delta^{T_{NP}+1}$ increases with λ. Tasks are included in the contract optimally, in a way such that the overall effect of the investment in contracting (the increase in the contract performance costs due to the new task is compensated by the increase in the effectiveness of the relational contract through the reduction of the equilibrium punishment) increase the value of the relationship. Under such characterization, we can define the optimal level of contract investment λ^* as the solution to the following problem

$$\lambda^* \in \arg\max \frac{P - c - \lambda}{1 - \pi\delta - (1-\pi)IP(\lambda)}$$

Consider the previous example in which we focus on the cost channel, $c(a|\underline{e}) = \kappa a$ and $c(a|\overline{e}) = a$. We showed that if $\kappa \geq \frac{P}{P-c} \geq 1$, the larger the number of tasks the lower the equilibrium punishment, $IP(a)$. Therefore, if only those sorts of tasks are available, the optimal investment in contracting is given by the optimal number of tasks, $\lambda = a$, and the optimal contracting is characterized by $a^* \in \arg\max \frac{P-c-a}{1-\pi\delta-(1-\pi)IP(a)}$.

A simple comparative statics analysis over λ^* provides interesting results.

Proposition 9 *The optimal investment in contracting λ^* is increasing in $P-c$ and may increase or decrease with π.*

The intuition of Proposition 9 is as follows. The optimal formal contracting investment λ^* increases with $P-c$ since the larger the trade surplus, the costlier the relational punishment is, and consequently, the higher the investment in decreasing it should be. The effect of π over contractual investment λ^* is ambiguous, because an increase in π reduces the asymmetric information and, with it, the need to resort to relational punishment in order to preserve incentives for effort, which leads to lower optimal contracting investment. But a higher π also increases the value of the relationship and the cost of punishment, which enhances the productivity of investing in formal contracting. In the proof of Proposition 9 it is shown that if δ is low enough, and consequently, the positive impact of π on the value of the relationship becomes less relevant, the optimal investment λ^* in formal contracting decreases in π.

9 Conclusions

Several observers have noticed the complexity and the multi-faceted nature of the coexistence of informality and formality in business contracting. Very few would dispute that relational elements are pervasive in inter-firm contractual exchanges, and that future dealings -with the same contract partner or with others- play a large role in securing adequate behavior in such interactions. Formal contracting and legal enforcement of the verifiable actions within the relationship, obviously at the core of the legal understanding of contracting phenomena, have been often considered by some strands of the economic thinking on contracting as clearly subordinate, when not irrelevant, or even a source of obnoxious interference or crowding out of the less costly and more effective reputational or reciprocity-based mechanisms.

Recently, the supporting role of formal contracting and Contract Law seems to have seen a revival. Our paper belongs to this school of thought concerning the link between the relational and the legal sides of contracting. We have formally shown that formal contractual documents, despite their being far from perfect levels of enforcement (and even when they are not at all expected to be enforced) exert a positive effect on the reputational or reciprocity-based sanctions that firms may impose upon their counterparties. On the one side, setting compliance with certain tasks in a formal explicit contract alongside the informal contracting on the "core" performance, reduces the cost of reputational punishments that firms may need to inflict upon their contract partners in order to keep them under the right incentives to provide "core" effort. Given that reputational sanctions are costly, formal contracts may provide net (of the drafting and performance or other costs of having a formal contract in place) welfare benefits for the contracting parties. On the other side, formal contracts impact the way in which reputational punishments will be structured by the sanctioning contract party. This party will use a more eschewed pattern of sanctioning than when no formal contract has been agreed: when the relational contract comes together with a formal contract, a firm will be less forgiving with those counterparties who have not performed the formal contract, and more forgiving with those other ones who have not infringed the provisions of the formal agreement.

Formal contracts, thus, are not just gates to future litigation when things go sour. Formal contracts play an important role in improving the informal dealings of business parties by providing a desirable guidance to the relational actions that the parties may take. We do not intend to downplay the importance of legal systems and formal enforcement institutions for business contracting. But even when parties cannot (or prefer not to) rely on legal enforcement, still investing in some degree of contract formality, documentation, and specified legal obligations may improve relationships that remain largely sustained by reciprocity and reputation.

There is substantial evidence, summarized in the introductory section, on the use of formal contracting even when legal enforcement is a remote possibility. Our findings shed light into the reasons why firms rely both on relational sanctions and on intensive formal drafting of legal contracts, though surely as we have acknowledged above, there are other reasons for having formal contracts in repeated relationship even when they are not intended to be brought to court, reasons such as double sided moral hazard or even behavioral factors. For instance, since a formal contract provides a more precise trigger for the relational contract, it reduces the cost of imposing relational sanctions when there are behavioral elements such as the feelings of entitlement that the written contract inspires in the parties, as in Hart and Moore (2008) and Halonene-Akatwijuka and Hart (2013).

We think that the results from our model may provide testable hypothesis for the empirical exploration of the interplay between relational and formal contracting, and eventually confirm the role of what we identify as the driver of the observed pattern of using formal documents specifying ancillary tasks in environments largely relying on relational sanctions to provide incentives. For instance, we predict that the size of the reputational sanctions should be negatively correlated with the use of formal contracts and that the investment in such formal contracting would be increasing in the gains from trade. Another implication from our view about the role of formal contracts in settings of repeated interactions is that the terms included in the formal contract would not fundamentally focus on issues linked to enforcement (liquidated damages, choice of forum, other provisions facilitating external verification and enforcement) but rather would concentrate on the specification of tasks, and the adoption and improvement of contractual devices that may enhance the observability of actions by the counterparty.

Finally, the theoretical analysis of optimal investment in formal contracting is inherently limited as to the description of the particular tasks and terms that the parties would wish to include in the contract, since the "flesh in the bone" depends on the feasible set of available tasks in the relevant contracting environment or sector. We believe this opens interesting avenues for empirical research on formal contracts used in repeated relationships in order to identify the most efficient ancillary tasks that are actually deployed by the parties to improve the functioning of the relational contract.

A Appendix

Proof of Lemma 1: From the main text, $\Phi(T) = \pi\delta\frac{(1-\delta^T)(P-c)}{1-\pi\delta-(1-\pi)\delta^{T+1}}$. Let $\varphi(x) = \frac{1-x}{1-\pi\delta-(1-\pi)x\delta}$. Then, we have $\Phi(T) = \pi\delta\varphi(x(T))$, for $x(T) = \delta^T$. As $x(T)$ is decreasing, in order to show that Φ is increasing in T, we have to show that $\varphi(x)$ is decreasing in x.

$$\begin{aligned}
\varphi'(x) &= \frac{-(1-\pi\delta-(1-\pi)x\delta)+(1-x)(1-\pi)\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} \\
&= \frac{-(1-\pi\delta)+(1-\pi)\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} \\
&= \frac{-1+\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} < 0
\end{aligned}$$

this concludes the proof.

Proof of Lemma 2: We write the binding incentive compatibility condition that characterizes the optimal punishments as follows, $\Phi(T^*(a), a) - c = 0$, where $a \in \{\pi, \delta, P - C\}$. By the implicit function theorem we obtain $T^{*\prime}(a) = -\frac{\frac{\partial\Phi(T^*,a)}{\partial a}}{\frac{\partial\Phi(T^*,a)}{\partial T^*}}$. Given that for Lemma 1 $\frac{\partial\Phi(T^*,a)}{\partial T^*} > 0$, the $sign\{T^{*\prime}(a)\} = -sign\{\frac{\partial\Phi(T^*,a)}{\partial a}\}$. Given that, i) $\frac{\partial\Phi(T^*,P-c)}{\partial P-c} = \pi\delta\frac{(1-\delta^T)}{1-\pi\delta-(1-\pi)\delta^{T+1}} > 0$ and $\frac{\partial T^*}{\partial P-c} < 0$.

ii)

$$\begin{aligned}
\frac{\partial \Phi(T^*,\pi)}{\partial \pi} &= (P-c)(1-\delta^T)\delta \left[\frac{1-\pi\delta-(1-\pi)\delta^{T+1}+\pi(\delta-\delta^{T+1})}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] \\
&= (P-c)(1-\delta^T)\delta \left[\frac{1-\delta^{T+1}}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] > 0
\end{aligned}$$

and $\frac{\partial T^*}{\partial \pi} < 0$. Finally,

$$\begin{aligned}
\frac{\partial \Phi(T^*,\delta)}{\partial \delta} &= (P-c)\pi \left[\frac{(1-(T+1)\delta^T)\left(1-\pi\delta-(1-\pi)\delta^{T+1}\right)+(\delta-\delta^{T+1})(\pi+(1-\pi)(T+1)\delta^T)}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] \\
&= (P-c)\pi \left[\frac{(1-(T+1)\delta^T)\left(1-\delta^{T+1}\right)+(\delta-\delta^{T+1})\ (T+1)\delta^T}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] \\
&= (P-c)\pi \left[\frac{(1-\delta^{T+1}-(T+1)\delta^T+(T+1)\delta^{T+1})}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] \\
&= (P-c)\pi \left[\frac{(1-(T+1)\delta^T+T\delta^{T+1})}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2}\right] > 0
\end{aligned}$$

Where the positive sign comes from the fact that $1-(T+1)\delta^T+T\delta^{T+1}$ is strictly decreasing, and 0 when $\delta=1$, therefore for all $\delta<1$, the expression is positive. Then $\frac{\partial \Phi(T^*,\delta)}{\partial \delta} > 0$ and $\frac{\partial T^*}{\partial \delta} < 0$. ■.

Proof of Proposition 1:

(i) We rewrite the incentive compatibility constraint.

$$\begin{aligned}
\frac{\delta\left[\pi+(1-\pi)(\alpha\delta^{T_P}+(1-\alpha)\delta^{T_{NP}})-(\beta\delta^{T_P}+(1-\beta)\delta^{T_{NP}})\right](P-c)}{1-\pi\delta-(1-\pi)(\alpha\delta^{T_P+1}+(1-\alpha)\delta^{T_{NP}+1})} &\geq c \\
\frac{\delta\left[\pi(1-(\alpha\delta^{T_P}+(1-\alpha)\delta^{T_{NP}}))+(\alpha-\beta)(\delta^{T_P}-\delta^{T_{NP}})\right](P-c)}{1-\pi\delta-(1-\pi)(\alpha\delta^{T_P+1}+(1-\alpha)\delta^{T_{NP}+1})} &\geq c
\end{aligned}$$

Consider the following change of variable $U=\alpha\delta^{T_P}+(1-\alpha)\delta^{T_{NP}}$, which implies $\delta^{T_{NP}} = \frac{U}{(1-\alpha)}-\frac{\alpha}{(1-\alpha)}\delta^{T_P}$, and then $\delta^{T_P}-\delta^{T_{NP}}=\frac{\delta^{T_P}}{(1-\alpha)}-\frac{U}{(1-\alpha)}$.

$$\frac{\delta\left[\pi(1-U)+(\alpha-\beta)(\frac{\delta^{T_P}}{1-\alpha}-\frac{U}{1-\alpha})\right](P-c)}{1-\pi\delta-(1-\pi)\delta U} \geq c$$

Let $\chi(x)=\frac{\left[\pi(1-x)+(\alpha-\beta)(\frac{\delta^{T_P}}{1-\alpha}-\frac{x}{1-\alpha})\right]}{1-\pi\delta-(1-\pi)\delta x}$. Now, we want to show that $\chi(x)$ is decreasing in x.

$$\begin{aligned}
\chi'(x) &= \frac{(-(\frac{\alpha-\beta}{1-\alpha}+\pi)(1-\pi\delta-(1-\pi)\delta x)+(1-\pi)\delta\left[\pi(1-x)+(\alpha-\beta)(\frac{\delta^{T_P}}{1-\alpha}-\frac{x}{1-\alpha})\right]}{(1-\pi\delta-(1-\pi)\delta x)^2} \\
&= \frac{-\pi(1-\delta)-\frac{(\alpha-\beta)}{1-\alpha}(1-\pi\delta-(1-\pi)\delta^{T_{NP}+1})}{(1-\pi\delta-(1-\pi)\delta x)^2} \leq 0
\end{aligned}$$

As the optimal relational punishment policy is characterized by the maximum $U = \alpha\delta^{T_L} + (1-\alpha)\delta^{T_{NL}}$ that satisfied the incentive compatibility constraint, and $\chi(x)$ is decreasing, this implies that the incentive compatibility constraint must be binding.

Then

$$\frac{\delta\left[\pi(1-U^*) + (\alpha-\beta)(\frac{\delta^{T_P}}{1-\alpha} - \frac{U^*}{1-\alpha})\right](P-c)}{1-\pi\delta-(1-\pi)\delta U^*} = c$$

As the left hand side of the equality is decreasing in U^*, and increasing in δ^{T_P}, this implies that $\frac{\partial U^*}{\partial \delta^{T_P}} > 0$. Then, the optimal policy requires to maximize δ^{T_P} (minimize T_P). This implies that in the optimal solution, $T^*_{NP} \neq \infty \rightarrow T^*_P = 0$, or alternatively $T^*_P \neq 0 \rightarrow T^*_{TP} = \infty$. This concludes the proof. ■

Proof of Proposition 2

As we mention in the main text, the proof that the optimal punishment with formal contracting feedback, (T^*_P, T^*_{NP}), generates lower expected punishment cost than without it, T^*, i.e. $U^* = \alpha\delta^{T^*_P} + (1-\alpha)\delta^{T^*_{NP}} > \delta^{T^*}$, just requires to notice that $T_P = T_{NP} = T^*$ was feasible and it is not optimal. We can also verify this by comparing the two binding incentive compatibility constraints.

$$\frac{\delta\pi(1-U^*)(P-c)}{1-\pi\delta-(1-\pi)\delta U^*} = c - \frac{\delta\left[(\alpha-\beta)(\frac{\delta^{T_P}}{1-\alpha} - \frac{U^*}{1-\alpha})\right](P-c)}{1-\pi\delta-(1-\pi)\delta U^*} \tag{4}$$

$$\frac{\delta\pi(1-\delta^{T^*})(P-c)}{1-\pi\delta-(1-\pi)\,\delta\delta^{T^*}} = c \tag{5}$$

Notice that the left-hand side of both equalities is the same decreasing function of $U^*$and δ^{T^*}, respectively. The right hand side of the first equality (4) is lower (the second term is negative) than the right-hand side of (5) and this implies that $U^* = \alpha\delta^{T^*_P} + (1-\alpha)\delta^{T^*_{NP}} > \delta^{T^*}$. ■

Proof of Proposition 3: By the implicit function theorem and $\Psi^C(U^*, \delta^{T^*_P}, \alpha, \beta) = c$, we obtain $\frac{\partial U^*}{\partial \alpha} = -\frac{\frac{\partial \Psi^C}{\partial \alpha}}{\frac{\partial \Psi^C}{\partial U^*}} = -\frac{\geq 0}{<0} > 0$. Similarly, $\frac{\partial U^*}{\partial \beta} = -\frac{\frac{\partial \Psi^C}{\partial \beta}}{\frac{\partial \Psi^C}{\partial U^*}} = -\frac{\leq 0}{<0} < 0$. Finally, notice that higher $U^* = \alpha\delta^{T^*_P} + (1-\alpha)\delta^{T^*_{NP}}$ means a lower expected relational punishment. ■

Proof of Proposition 4: As in the previous section, the value of the relationship between PF and CF is captured by V^{C+}

$$\max_{T(s)} V^{C+} = \frac{P-c}{1-\pi\delta-(1-\pi)(\sum \Pr(s_i|\overline{e})\delta^{T(s_i)})}$$

that is increasing in $\sum \Pr(s_i|\overline{e})\delta^{T(s_i)}$. Then, similarly to previous results, the optimal relational punishment $T^*(s_i)$ maximizes $U^* = \sum_{i=1}^{N} \Pr(s_i|\overline{e})\delta^{T^*(s_i)}$ subject to satisfy the incentive compatibility constraint:

$$
\begin{aligned}
\Psi^C(T(s), \Pr(s|\underline{e}), \Pr(s_i|\overline{e})) &\geqslant c \\
\frac{\delta\left[\pi + (1-\pi)(\sum\limits_{i=1}^{N}\Pr(s_i|\overline{e})\delta^{T(s_i)}) - (\sum\limits_{i=1}^{N}\Pr(s_i|\underline{e})\delta^{T(s_i)})\right](P-c)}{1-\pi\delta-(1-\pi)(\sum\limits_{i=1}^{N}\Pr(s_i|\overline{e})\delta^{T(s_i)})} &\geqslant c
\end{aligned}
$$

In order to prove the result, take as given the punishment of all scores but the two first ones: $\sum\limits_{i=1}^{N}\Pr(s_i|\overline{e})\delta^{T(s_i)} = \alpha_1\delta^{T_1}+\alpha_2\delta^{T_2}+A$ and $\sum\limits_{i=1}^{N}\Pr(s|\underline{e})\delta^{T(s_i)} = \beta_1\delta^{T_1}+\beta_2\delta^{T_2}+B$. Then, the function $\Psi^C(T(s), \Pr(s|\underline{e}), \Pr(s_i|\overline{e}))$ becomes:

$$
\frac{\delta\left[\pi+(1-\pi)(\alpha_1\delta^{T_1}+\alpha_2\delta^{T_2}+A)-(\beta_1\delta^{T_1}+\beta_2\delta^{T_2}+B)\right](P-c)}{1-\pi\delta-(1-\pi)(\alpha_1\delta^{T_1}+\alpha_2\delta^{T_2}+A)}
$$

We make the following change of variable $\overline{U} = \alpha_1\delta^{T_1}+\alpha_2\delta^{T_2}$ and $\delta^{T_2} = \frac{\overline{U}}{\alpha_2} - \frac{\alpha_1\delta^{T_1}}{\alpha_2}$, and we rewrite $\Psi^C(T(s), \Pr(s|\underline{e}), \Pr(s_i|\overline{e}))$ as

$$
\begin{aligned}
&\frac{\delta\left[\pi+(1-\pi)(\overline{U}+A)-(\beta_1\delta^{T_1}+\beta_2(\frac{\overline{U}}{\alpha_2}-\frac{\alpha_1\delta^{T_1}}{\alpha_2})+B)\right](P-c)}{1-\pi\delta-(1-\pi)(\overline{U}+A)} \\
=\ &\frac{\delta\left[\pi+(1-\pi)(\overline{U}+A)-(\beta_2\frac{\overline{U}}{\alpha_2}+B)-\beta_2\delta^{T_1}(\frac{\beta_1}{\beta_2}-\frac{\alpha_1}{\alpha_2}))\right](P-c)}{1-\pi\delta-(1-\pi)(\overline{U}+A)}
\end{aligned}
$$

For the Monotone Likelihood Ratio Property (MLRP), $\frac{\beta_1}{\beta_2}-\frac{\alpha_1}{\alpha_2}>0$, which implies that for a given $\overline{U}$, Ψ^C is decreasing in δ^{T_1}. In other words, we want to maximize T_1 punishment with respect to T_2 . This implies $\delta^{T_1} = \min\{0, \frac{\overline{U}-\alpha_2}{\alpha_1}\}$ We can repeat this proof for all pairs, T_i and T_{i+1} and obtaining the same result. Then, the global solution has to be $\delta^{T_i}=0$ ($T_i=\infty$) for all initial scores until we can guaranty that $\Psi^C>0$. ■

Proof of Proposition 5:

For simplifying the notation we will prove the results using a continuous distribution of signals. Then consider two signals $F_1(s|e)$ and $F_2(s|e)$ that we want to rank according to their informativeness. Jewitt (2007) shows the equivalence of Lehmann efficiency and Blackwell sufficiency in a dichotomous setting as ours, $e\in\{\underline{e},\overline{e}\}$ in which signals satisfy MLRP. Lehmann criterion establish that a signal F_1 is more informative than another F_2 if, the following condition over quantiles holds:

$$
\forall p\in[0,1],\quad F_1\left(F_1^{-1}(p|\underline{e})\,|\overline{e}\right)\leq F_2\left(F_2^{-1}(p|\underline{e})\,|\overline{e}\right). \tag{6}
$$

By definition, the c.d.f.s $F_1(x|e)$ and $F_2(x|e)$ are nondecreasing functions, so that

$$
\forall x,\ F_1(x|e)\geq F_2(x|e) \iff \forall p,\ F_1^{-1}(p|e)\leq F_2^{-1}(p|e).
$$

By definition 1, $F_1\left(x|\underline{e}\right) \geq F_2\left(x|\underline{e}\right)$ and hence, for any p,

$$\begin{aligned} F_1^{-1}(p|\underline{e}) &\leq F_2^{-1}(p|\underline{e}) \Rightarrow \\ F_2(F_1^{-1}(p|\underline{e})|\overline{e}) &\leq F_2(F_2^{-1}(p|\underline{e})|\overline{e}) \end{aligned}$$

By definition 1, $F_1\left(x|\overline{e}\right) \leq F_2\left(x|\overline{e}\right)$, then replacing $F_2(F_1^{-1}(p|\underline{e})|\overline{e})$ by $F_1(F_1^{-1}(p|\underline{e})|\overline{e})$ then, we obtain

$$F_1(F_1^{-1}(p|\underline{e})|\overline{e}) \leq F_2(F_2^{-1}(p|\underline{e})|\overline{e}).$$

Then, our criterion of informativeness captured by definition 1 implies Lehmann efficiency and using Jewitt's result, also Blackwell sufficiency. ■

Proof of Proposition 6: Let $\sum\limits_{i=1}^{N} \Pr_1(s_i|\overline{e})\delta^{T_1^*(s_i)}$ and $\sum\limits_{i=1}^{N} \Pr_2(s_i|\overline{e})\delta^{T_2^*(s_i)}$ be the optimal punishment under C_1 and C_2.First, we show that $T_2^*(s_i)$ is feasible under C_1.

$$\Psi^{C_1}(T_2^*(s_i), \Pr(s|\underline{e})_1, \Pr(s_i|\overline{e})_1) \geq \Psi^{C_2}(T_2^*(s_i), \Pr(s|\underline{e})_2, \Pr(s_i|\overline{e})_2)$$

This is due to the following reasons: (i) Ψ^C increases with $\sum \Pr(s_i|\overline{e})\delta^{T(s_i)}$ and decreases with $\sum \Pr(s_i|\underline{e})\delta^{T(s_i)}$; (ii) $\delta^{T(s_i)}$ is an increasing function of s_i; (iii) Scoring distributions are ordered according to the first order stochastic dominance, $(\sum\limits_{i=1}^{x} \Pr(s_i|\overline{e})_1 \leq \sum\limits_{i=1}^{x} \Pr(s_i|\overline{e})_2 \ \nabla x)$ and $(\sum\limits_{i=1}^{x} \Pr(s_i|\underline{e})_1 \geq \sum\limits_{i=1}^{x} \Pr(s_i|\underline{e})_2 \ \nabla x)$. Then, by (ii) and (iii) $\sum\limits_{i=1}^{N} \Pr(s_i|\overline{e})_1\delta^{T_2^*(s_i)} > \sum\limits_{i=1}^{N} \Pr(s_i|\overline{e})_2\delta^{T_2^*(s_i)}$ and $\sum\limits_{i=1}^{N} \Pr(s_i|\underline{e})_1\delta^{T_2^*(s_i)} < \sum\limits_{i=1}^{N} \Pr(s_i|\underline{e})_2\delta^{T_2^*(s_i)}$, which jointly with (i) implies the inequality above. Finally, as $T_2^*(s_i)$ is feasible under C_1, we can state that

$$V^{C_1+}(\sum_{i=1}^{N} \Pr(s_i|\overline{e})_1\delta^{T_1^*(s_i)}) > V^{C_1+}(\sum_{i=1}^{N} \Pr(s_i|\overline{e})_1\delta^{T_2^*(s_i)}) > V^{C_2+}(\sum_{i=1}^{N} \Pr(s_i|\overline{e})_2\delta^{T_2^*(s_i)})$$

This is because, V^{C+}, the value of the relationship between PF and CF, is increasing in $\sum \Pr(s_i|\overline{e})\delta^{T(s_i)}$, and $\sum\limits_{i=1}^{N} \Pr(s_i|\overline{e})_1\delta^{T_2^*(s_i)}) > \sum\limits_{i=1}^{N} \Pr(s_i|\overline{e})_2\delta^{T_2^*(s_i)}$.which implies the last two inequalities. The first inequality is implied by the fact that for C_1, the optimal punishment is $T_1^*(s_i)$. ■

Proof of Proposition 7: The optimal relational punishment $(T_P^{WE*}, T_{NP}^{WE*})$ is given by the following equality

$$\Psi^{WE}(T_P^{WE*}, T_{NP}^{WE*}, \alpha, \beta, \gamma) = c$$

Following the arguments of the proof of Proposition 1 we can rewrite this equality as follows:

$$\frac{\delta\left[\pi(1-U^*) + (\alpha-\beta)(\frac{\delta^{T_P^{WE*}}}{1-\alpha} - \frac{U^*}{1-\alpha})\right]}{1-\pi\delta-(1-\pi)\delta U^*} = \frac{c-d\gamma}{P-c-a\gamma}$$

Where, as in Proposition 1, $U^* = \alpha\delta^{T_P^{WE*}} + (1-\alpha)\delta^{T_{NP}^{WE*}}$ refers to the optimal relational punishment, and $d = (1-\beta)D - (1-\pi)(1-\alpha)D$ and $a = (1-\pi)(1-\alpha)D$ are two constants. From Proposition 1 we know that the left hand side of the equality is decreasing in U^*, then the lower is the right hand side, the higher is $U^* = \alpha\delta^{T_P^{WE*}} + (1-\alpha)\delta^{T_{NP}^{WE*}}$, and the lower is the optimal relational punishment.

If we derive $\frac{c-d\gamma}{P-c-a\gamma}$ with respect to γ

$$\frac{d}{d\gamma}\left(\frac{c-d\gamma}{P-c-a\gamma}\right) = \frac{-d(P-c-a\gamma)+a(c-d\gamma)}{(P-c-a\gamma)^2}$$

This derivative is negative:

$$-d(P-c)+ac<0 \Leftrightarrow \frac{c}{d} < \frac{(P-c)}{a}$$

This inequality is satisfied since we are assuming that i) $d = (1-\beta)D - (1-\pi)(1-\alpha)D > c \Rightarrow \frac{c}{d} < 1$ and ii) $a = (1-\pi)(1-\alpha)D < P-c \Rightarrow \frac{(P-c)}{a} > 1$. Then, the right hand side is decreasing in γ, and we can conclude that the optimal relational punishment $(T_P^{WE*}, T_{NP}^{WE*})$ is decreasing in degree of enforceability of the formal contract γ. ■

Proof of Proposition 8: As in the baseline model, the optimal relational contract T^* will be such that the incentive compatibility constraint is binding.

$$\pi\delta\frac{(1-\delta^{T^*})}{1-\pi\delta-(1-\pi)\delta^{T^*+1}} = \frac{c-(c(a|\underline{e})-c(a|\overline{e}))}{P-c-c(a|\overline{e})}.$$

The proof of Lemma 1 above shows that the left hand side of the incentive compatibility constraint is increasing in T^*. Then, as part i) of the Proposition states, the higher is the right hand side, the higher is T^*. Part ii) of the Proposition follows from the right hand side being decreasing in $c(a|\overline{e})$ and increasing in $c(a|\underline{e})$. ■

Proof of Proposition 9: The contracting problem is defined as follows

$$\lambda^* \in \arg\max V(\lambda) = \frac{P-c-\lambda}{1-\pi\delta-(1-\pi)IP(\lambda)}$$

We focus on increasing (decreasing) differences that it is a sufficient condition for supermodularity (submodularity) and then for comparative statics. Then

$$\frac{\partial V(\lambda, P-c)}{\partial\lambda\partial P-c} = \frac{(1-\pi)IP(\lambda)'}{(1-\pi\delta-(1-\pi)IP(\lambda))^2} \geq 0$$

As the cross derivative is positive, the value function $V(\lambda, P-c)$ is supermodular in λ and $P-c$, and the investment in contracting λ^* is increasing in $P-c$.

$$\frac{\partial V(\lambda,\pi)}{\partial\lambda\partial\pi}=\frac{(-1+\delta+(\delta-IP(\lambda))(1-\pi))}{(1-\pi\delta-(1-\pi)IP(\lambda))^3}IP(\lambda)'$$

The sign of the cross derivative depends on the expression $(-1+\delta+(\delta-IP(\lambda))(1-\pi))$ that may be positive or negative for some parameter values (since $\delta\in[0,1]$ and $\delta>IP(\lambda)$). Notice, however, that if δ is low enough, the whole expression and the cross derivative are negative and consequently, the value function $V(\lambda,\pi)$ is submodular in λ and π, and the investment in contracting λ^* is decreasing in π. ■

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