The Tail that Wags the Economy: Beliefs and Persistent Stagnation

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Abstract

The Great Recession was a deep downturn with long-lasting effects on credit, employment and output. While narratives about its causes abound, the persistence of GDP below pre-crisis trends remains puzzling. We propose a simple persistence mechanism that can be quantified and combined with existing models. Our key premise is that agents don't know the true distribution of shocks, but use data to estimate it non-parametrically. Then, transitory events, especially extreme ones, generate persistent changes in beliefs and macro outcomes. Embedding this mechanism in a neoclassical model, we find that it endogenously generates persistent drops in economic activity after tail events.

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The Great Recession was a deep downturn with long-lasting effects on credit markets, labor markets and output. Why did output remain below trend long after financial markets had calmed and uncertainty diminished? This recession missed the usual business cycle recovery. Such a persistent, downward shift in output (Figure 1) is not unique to the 2008 crisis. Financial crises, even in advanced economies, typically fail to produce the robust GDP rebound needed to restore output to its pre-crisis trend.[1]

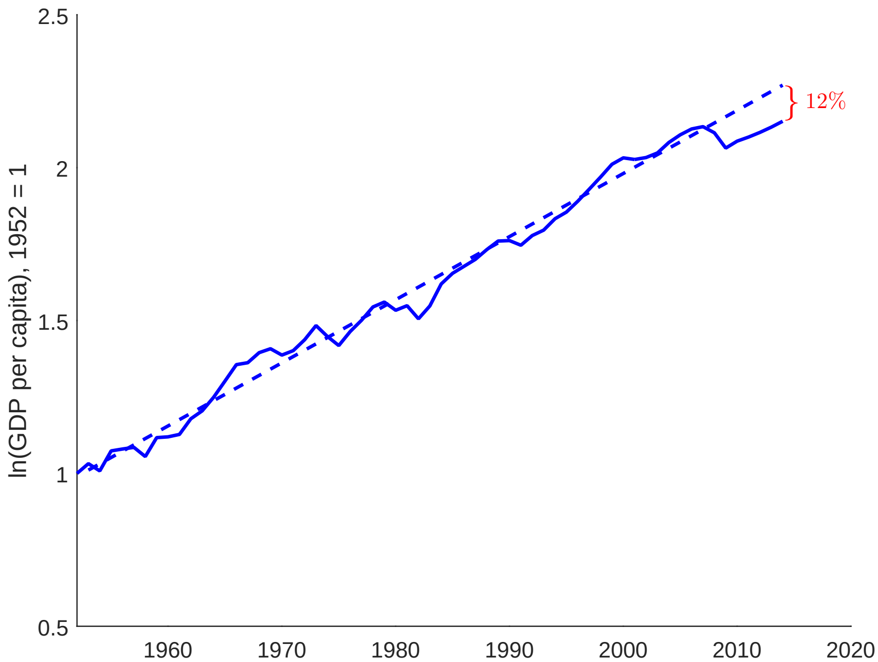


Figure 1: **Real GDP in the U.S. and its trend.**
Dashed line is a linear trend that fits data from 1950-2007. In 2014, real GDP was 0.12 *log points below trend.*

Our explanation is that crises produce persistent effects because they scar our beliefs. For example, in 2006, few people entertained the possibility of financial collapse. Today, the possibility of another run on the financial sector is raised frequently, even though the system today is probably much safer. Such persistent changes in the assessments of risk came from observing new data. We thought the U.S. financial system was stable. Economic outcomes taught us that the risks were greater than we thought. It is this new-found knowledge that is having long-lived effects on economic choices.

The contribution of the paper is a simple tool to capture and quantify this scarring effect, which produces more persistent responses from extreme shocks than from ordinary business cycle shocks. We start from a simple assumption: agents do not know the true distribution of shocks in the economy, but estimate the distribution using realtime data, exactly like an econometrician would. Data on extreme events tends to be scarce, makes new tail observations

[1] See Reinhart and Rogoff (2009), fig 10.4.

particularly informative. Therefore, tail events trigger larger belief revisions. Furthermore, because it will take many more observations of non-tail events to convince someone that the tail event really is unlikely, changes in tail risk beliefs are particularly persistent. To explore these changes in a meaningful way, we need to use an estimation procedure that does not unduly constrain the shape of the distribution's tail. Therefore, we assume that our agents adopt a non-parametric approach to learning about the distribution of aggregate shocks. Each period, they observe one more piece of data and update their estimates using a standard kernel density estimator. Section 1 shows that this process leads to long-lived responses of beliefs to transitory events, especially extreme, unlikely ones. The mathematical foundation for persistence is the martingale property of beliefs. The logic is that once observed, the event remains in agents' data set. Long after the direct effect of the shock has passed, the knowledge of that tail event continues to affect estimated beliefs and restrains the economic recovery.

To illustrate the economic importance of these belief dynamics, Section 2 applies our belief updating tool to an existing model of the great recession. The model in Gourio (2012, 2013) is well-suited to our exploration of the persistent real effects of financial crises because the underlying assumptions are carefully chosen to link tail events to macro outcomes, in a quantitatively plausible way. It features firms that are subject to bankruptcy risk from idiosyncratic profit shocks and aggregate capital quality shocks. This set of economic assumptions is not our contribution. It is simply a laboratory we employ to illustrate the persistent economic effects from observing extreme events. Section 3 describes the data we feed into the model to discipline our belief estimates. Section 4 combines model and data and uses the resulting predictions to show how belief updating quantitatively explains the persistently low level of output colloquially known as "secular stagnation." We compare our results to those from the same economic model, but with agents who have full knowledge of the distribution, to pinpoint belief updating as the source of the persistence.

Our main insight about why tail events have persistent effects does not depend on the specific structure of the Gourio (2012) model. To engage our persistence mechanism, three ingredients are needed. One is a shock process that captures the extreme, unusual aspects of the Great Recession. These were evident mainly in real estate and capital markets. Was this the first time we have ever seen such shocks? In our data set, which spans the post-WWII period in the US, yes. Total factor productivity, on the other hand, does not meet this criterion.[2] The capital quality shock specification is arguably the most direct one that does. Of course, similar extreme events have been observed before in global history – e.g. in other countries or during the Great Depression. Section 4.3 explores the effect of expanding the data set to include

[2]It begins to fall prior to the crisis and by an amount that was not particularly extreme. See Appendix D.5 for an analysis of TFP shocks.

additional infrequent crises and shows that it does temper persistence, but only modestly.

The second ingredient is a belief updating process that uses new data to estimate the distribution of shocks, or more precisely, the probability of extreme events. It is not crucial that the estimation is frequentist.[3] What is important is that the learning protocol does not rule out fat tails by assumption (e.g. by imposing a normal distribution).

The third necessary ingredient is an economic model that links the risk of extreme events to real output. The model in Gourio (2012, 2013) has the necessary curvature (non-linearity in policy functions) to deliver sizable output responses from modest changes in disaster risk. The assumptions about preferences and debt/bankruptcy, that make Gourio's model somewhat complex, are there to deliver that curvature. This also makes the economy more sensitive to disaster risk than extreme boom risk. Section 4.5 explores the role of these ingredients, by turning each on and off. That exercise shows that even though these assumptions deliver a large drop in output (though not in investment), they do not in any way guarantee the success of our objective, which is to generate persistent economic responses. In other words, when agents do not learn from new data, the same model succeeds in matching the size of the initial output drop, but fails to produce persistent stagnation.

Finally, we find that recent data on asset prices and debt are also consistent with an increase in tail risk. At first pass, one might think that financial market data are at odds with our story. For instance, Hall (2015a) objects to tail risk as a driver of the recent stagnation on the grounds that an increase in tail risk should show up as high credit spreads. In the data, credit spreads – the difference between the return on a risky loan and a riskless one – were only a few basis points higher in 2015 relative to their pre-crisis levels. Similarly, one would think a rise in tail risk should push down equity prices, when in fact, they too have recovered. Our model argues against both these conjectures – it shows that when tail risk rises, firms borrow less to avoid the risk of bankruptcy, lowering credit risk and increasing the value of their equity claims. Thus, low credit spreads and a rise in equity prices are not inconsistent with tail risk. Others point to low interest rates as a potential cause of stagnation. Our story complements this narrative by demonstrating how heightened tail risk makes safe assets more attractive, depressing riskless rates in a persistent fashion. In sum, none of these patterns disproves our theory about elevated tail risk, though, in fairness, they also do not distinguish it from others.

There are other asset market variables that speak more directly to tail risk, in particular options on the S&P 500. The SKEW index, published by the Chicago Board Options Exchange, uses these to back out the implied measure of skewness. Figure 2 shows that this index has stayed persistently high. In Section 4.4, where we review the asset pricing evidence, we use

[3]For an example of Bayesian estimation of tail risks in a setting without an economic model, see Orlik and Veldkamp (2014).

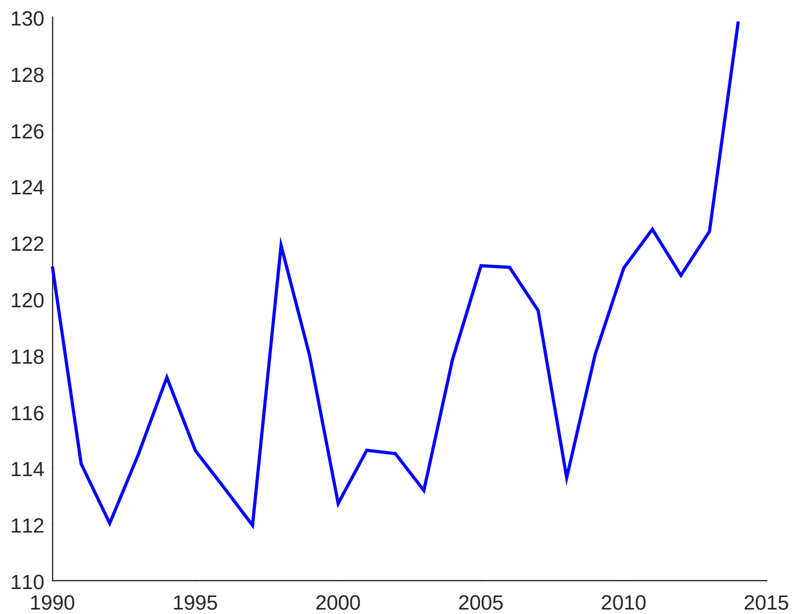


Figure 2: **The SKEW Index.**
An index of skewness in returns on the S&P 500, constructed using option prices. Source: Chicago Board Options Exchange (CBOE). 1990:2014.

this series construct measures of tail risk – e.g. third moment of equity returns and the implied probability of large negative returns – and show that the model's predictions for changes in these objects lines up quite well with the data. Finally, other rough proxies for beliefs also show signs of persistently higher tail risk today. Google searches for the terms "economic crisis," "financial crisis," or "systematic risk" all rose during the crisis and never returned to their pre-crisis levels (see Appendix D.1).

Comparison to the literature There are many theories now of the financial crisis and its consequences, many of which provide a more detailed account of its mechanics (e.g., Gertler et al. (2010), Gertler and Karadi (2011), Brunnermeier and Sannikov (2014) and Gourio (2012, 2013)). Our goal is not a new explanation for why the crisis arose, or a new theory of business cycles. Rather, we offer a velief-based mechanism that complements these theories by adding endogenous persistence. It helps explain why extreme events, like the recent crisis, lead to more persistent responses than milder downturns. In the process, we also develop a new tool for tying beliefs firmly to data that is compatible with modern, quantitative macro models.

A few uncertainty-based theories of business cycles also deliver persistent effects from transitory shocks. In Straub and Ulbricht (2013) and Van Nieuwerburgh and Veldkamp (2006), a negative shock to output raises uncertainty, which feeds back to lower output, which in turn creates more uncertainty. Fajgelbaum et al. (2014) combine this mechanism with an irreversible investment cost, a combination which can generate multiple steady-states. These uncertainty-based explanations leave two questions unanswered. First, why did the depressed level of economic activity continue long after measures of uncertainty (like the VIX index) had recovered? Our theory emphasizes tail risk. Unlike measures of uncertainty, tail risk has lingered (as Figure 2 reveals), making it a better candidate for explaining continued depressed output. Second, why were credit markets most persistently impaired after the crisis? Rises in tail risk hit credit markets because default risk is particularly sensitive to tail events.

Our belief formation process is similar to the parameter learning models by Johannes et al. (2015), Cogley and Sargent (2005) and Orlik and Veldkamp (2014) and is advocated by Hansen (2007). However, these papers focus on endowment economies and do not analyze the potential for persistent effects in production settings. Pintus and Suda (2015) embed parameter learning in a production economy, but feed in persistent leverage shocks and explore the potential for amplification when agents hold erroneous initial beliefs about persistence. In Moriera and Savov (2015), learning changes demand for shadow banking (debt) assets. But, again, agents are learning about a hidden two-state Markov process, which has a degree of persistence built in.[4] While this literature has taught us a lot about the mechanisms that triggered declines in lending and output, it often has to resort to exogenous persistence. We, on the other hand, have transitory shocks and focus on endogenous persistence. In addition, our non-parametric approach allows us to talk about tail risk.

Finally, our paper contributes to the recent literature on secular stagnation. Eggertsson and Mehrotra (2014) argue that a combination of low effective demand and the zero lower bound on nominal rates can generate a long-lived slump. In contrast, Gordon (2014), Anzoategui et al. (2015) and others attribute stagnation to a decline in productivity, education or shift in demographics. But, these longer-run trends may well be suppressing growth, they don't explain the level shift in output after with the financial crisis. Hall (2015a) surveys these and other theories. While all these alternatives may well be part of the explanation, our simple mechanism reconciles the recent stagnation with economic, financial and internet search evidence suggesting heightened tail risk.

The rest of the paper is organized as follows. Section 1 describes the belief-formation mechanism. Section 2 presents the economic model. Section 3 shows the measurement of shocks and calibration of the model. Section 4 analyzes the main results of the paper while Section 4.5 decomposes the key underlying economic forces. Finally, Section 5 concludes.

[4]Other learning papers in this vein include papers on news shocks, such as, Beaudry and Portier (2004), Lorenzoni (2009), Veldkamp and Wolfers (2007), uncertainty shocks, such as Jaimovich and Rebelo (2006), Bloom et al. (2014), Nimark (2014) and higher-order belief shocks, such as Angeletos and La'O (2013) or Huo and Takayama (2015).

1 Belief Formation

A key contribution of this paper is to explain why tail risk fluctuates and why such fluctuations are persistent. Before laying out the underlying economic environment, we begin by explaining the novel part – belief formation and the persistence of belief revisions.

In order to explore the idea that beliefs about tail risk changed, it is essential to depart from the assumption that agents know the true distribution of shocks to the economy. Specifically, we will assume that they estimate such distributions, updating beliefs as new data arrives. The first step is to choose a particular estimation procedure. A common approach is to assume a normal distribution and estimate its parameters (namely, mean and variance). While tractable, this has the disadvantage that the normal distribution, with its thin tails, is unsuited to thinking about changes in tail risk. We could choose a distribution with more flexibility in higher moments. However, this will raise obvious concerns about the sensitivity of results to the specific distributional assumption used. To minimize such concerns, we take a non-parametric approach and let the data inform the shape of the distribution.

Specifically, we employ a kernel density estimation procedure, one of most common approaches in non-parametric estimation. Essentially, it approximates the true distribution function with a smoothed version of a histogram constructed from the observed data. By using the widely-used normal kernel, we impose a lot of discipline on our learning problem but also allow for considerable flexibility. We also experimented with a handful of other kernel and Bayesian specifications, which yielded similar results (see Appendix C.12).

Setup Consider a shock ϕ_t whose true density g is unknown to agents in the economy. The agents do know that the shock ϕ_t is i.i.d. Their information set at time t, denoted $\mathcal{I}_t$, includes the history of all shocks ϕ_t observed up to and including t. They use this available data to construct an estimate $\hat{g}_t$ of the true density g. Formally, at every date, agents construct the following normal kernel density estimator of the pdf g

$$\hat{g}_t(\phi) = \frac{1}{n_t \kappa_t} \sum_{s=0}^{n_t - 1} \Omega\left(\frac{\phi - \phi_{t-s}}{\kappa_t}\right)$$

where $\Omega(\cdot)$ is the standard normal density function, n_t is the number of available observations at date t and κ_t is the smoothing or bandwidth parameter. We use the optimal bandwidth $\kappa_t = \hat{\sigma}(4/3n_t)^{1/5}$, where $\hat{\sigma}$ is an estimate of the standard deviation.[5] For example, using the 1950-2007 data, we obtain $\kappa_{2007} = 0.0056$. As new data arrives, agents add each new observation to their data set and update their estimates, generating a sequence of beliefs $\{\hat{g}_t\}$.

[5]The optimal bandwidth minimizes the expected squared error when the underlying density is normal. It is widely used and is the default option in MATLAB's *ksdensity* function.

The key mechanism in the paper is the persistence of belief changes induced by transitory ϕ_t shocks. This stems from the martingale property of beliefs: conditional on time-t information ($\mathcal{I}_t$), the estimated distribution is a martingale. Thus, on average, the agent expects her future belief to be the same as her current beliefs. This property holds exactly if the bandwidth parameter κ_t is set to zero.[6]

Kernels smooth the density but this smoothing creates a deviation from the martingale property. Numerically, deviations of beliefs from a martingale are minuscule, both for the illustrative example in this section and in our full model. In other words, the kernel density estimator with the optimal bandwidth is, approximately, a martingale $\mathbb{E}_t\left[\hat{g}_{t+j}\left(\phi\right)\middle|\,\mathcal{I}_t\right] \approx \hat{g}_t\left(\phi\right)$. As a result, any changes in beliefs induced by new information are, in expectation, permanent. This property, which also arises with parametric learning (Hansen and Sargent, 1999; Johannes et al., 2015), plays a central role in generating long-lived effects from transitory shocks.

We now illustrate how this belief formation mechanism works by applying the estimation procedure described above to a time series of U.S. capital quality shocks. Since our goal here is purely to illustrate the effects of outlier realizations on beliefs, we could have used any time series with an outlier. So we postpone this discussion of what these shocks are and how they are measured until Section 3. We proceed using this series as an almost-arbitrary time series to illustrate how belief formation works.

Estimated belief changes The second panel of Figure 3 takes all the data up to and including 2007 and shows the estimated probability distribution, based on that (pre-crisis) data. Then it takes all data up to and including 2009 (post-crisis) to plot the new probability distribution estimate. The two adverse realizations in '08 and '09 lead to an increase in the assessment of tail risk: The 2009 distribution, $\hat{g}_{2009}$ shows a pronounced hump in the density around the 2008 and 2009 realizations, relative to the pre-crisis one. Crucially, even though these negative realizations were short-lived, this increase in left tail risk persists. To see how persistent beliefs are, we ask the following question: What would be the estimated probability distribution in 2039? To answer this question, we need to simulate future data. Since our best estimate of the distribution of future data in 2009 is $\hat{g}_{2009}$, we draw many 30-year sequences of future data from this $\hat{g}_{2009}$ distribution. After each 30-year sequence, we re-estimate the distribution g, using all available data. The shaded area in the third panel of Figure 3 shows the results from this Monte Carlo exercise. Obviously, each simulated path gives rise to a different estimated

[6] When $\kappa_t = 0$, the kernel puts positive probability mass only on realizations seen before. In other words, an event that isn't exactly identical to one in the observed sample is assigned zero probability, even if there are other observations arbitrarily close to it in the sample. This is probably too extreme a specification for our purposes – since events are never identical in actual macro data, every observation will have zero probability before it occurs.

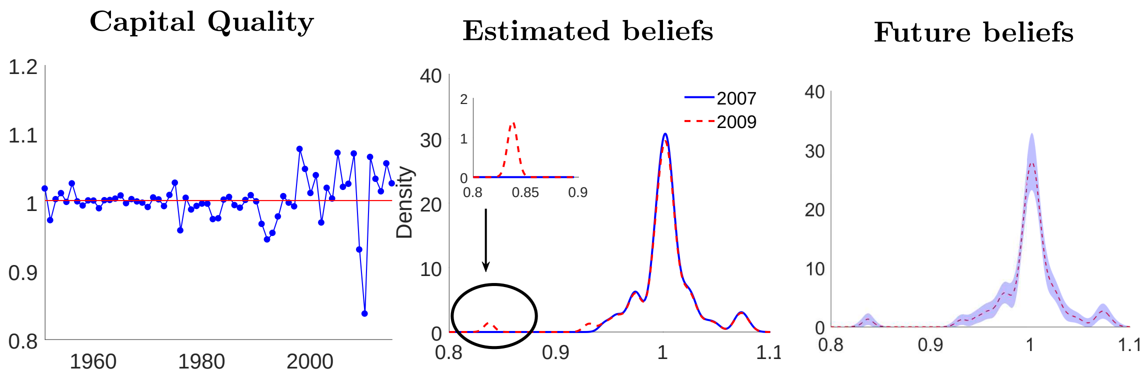


Figure 3: **Estimated Beliefs.**

The first panel shows the realizations of "capital quality" shocks, defined later in the paper in (15) and measured as described in Section 3. The second panel shows the kernel density, estimated from data available up to 2007 and up to 2009. The change in the left tail represents the effect of the Great Recession. The third panel shows the average estimate of the probability density (along with a 2 standard deviation band) in 2039. This is computed by simulating data for the period 2010-2039 and estimating a kernel on each simulated series. Future realizations are drawn from the estimated distribution in 2009.

distribution, but averaging across all those paths yields the 2009 distribution (dashed line). This simulation illustrates how tail risk induced by financial crisis may never go away. The left tail "hump" persists. Because we are drawing from the $\hat{g}_{2009}$ distribution, every once in a long while, another crisis is drawn, which keeps the left tail from disappearing. If we instead drew future data from a distribution without tail risk (e.g. $\hat{g}_{2007}$), the hump would still be very persistent, but not permanent (see Section 4).

Thus, every new shock, even a transitory one, has a persistent effect on beliefs. This pattern is reminiscent of the evidence of heightened tail risk from asset markets and other proxies presented in the Introduction. In the rest of the paper, we will use a specific economic model, which maps shocks and beliefs into investment, hiring and production decisions, in order to assess the implications of these belief changes for macroeconomic outcomes. However, it is worth noting that our approach and mechanism have broader relevance as simple tools to generate endogenous persistence in many economic environments.

2 Economic Model

To explore whether our belief formation mechanism can help explain the persistence of the recent stagnation, we need to embed it in an economic environment. To have a shot at quantitatively explaining the recent episode, our model needs two key features. First, since extreme shocks create the most persistence, we need a model with shocks can capture the extreme and unusual aspects of the 2008-'09 recession, in particular the unusually low returns to firms' (non-residential) capital. To generate large fluctuations in returns, we use a shock to capital quality. These shocks, which scale up or down the effective capital stock, are not to be interpreted literally. A decline in capital quality captures the idea that a Las Vegas hotel built in 2007 may deliver less economic value after the financial crisis, e.g. because it is consistently half-empty. This would be reflected in a lower market value, a feature we will exploit later in our measurement strategy. This specification is not intended as a deep explanation of what triggered the financial crisis. Instead, it is a summary statistic that stands in for many possible explanations and allows the model to speak to both financial and macro data.[7] This agnostic approach to the cause of the crisis also puts the spotlight on our contribution, which is the ability of learning to generate persistent responses to extreme events.

Second, we need a setting where economic activity is sensitive to the probability of extreme capital shocks. Gourio (2012, 2013) presents a model optimized for this purpose. Two key ingredients – namely, Epstein-Zin preferences and costly bankruptcy – combine to generate significant sensitivity to tail risk. Adding the assumption that labor is hired in advance with an uncontingent wage increases the effective leverage of firms and therefore, accentuates the sensitivity of investment and hiring decisions to tail risk. Similarly, preferences that shut down wealth effects on labor avoid a surge in hours in response to crises.

Thus, this combination of assumptions offers a laboratory to assess the quantitative potential of our belief revision mechanism. It is worth emphasizing that none of these ingredients guarantees persistence, our main focus. The capital quality shock has a direct effect on output upon impact but, absent belief revisions, does not change the long-run trajectory of the economy. Similarly, the non-linear responses induced by preferences and debt influence the size of the economic response, but by themselves do not generate any internal propagation. They influence the magnitude of the response, but persistence comes solely from the belief mechanism, which is capable of generating persistence even without these ingredients.

To this setting, we add a novel ingredient, namely our belief-formation mechanism. We model beliefs using the non-parametric estimation described in the previous section and show how to discipline this procedure with observable macro data.

[7] Capital quality shocks have been employed for a similar purpose in Gourio (2012), as well as in a number of recent papers on financial frictions, crises and the Great Recession (e.g., Gertler et al. (2010), Gertler and Karadi (2011), Brunnermeier and Sannikov (2014)). Their use in macroeconomics and finance, however, goes back at least to Merton (1973), who uses them to generate highly volatile asset returns.

2.1 Setup

Preferences and technology: An infinite horizon, discrete time economy has a representative household, with preferences over consumption (C_t) and labor supply (L_t):

$$U_t = \left[(1-\beta)\left(C_t - \frac{L_t^{1+\gamma}}{1+\gamma}\right)^{1-\psi} + \beta E_t \left(U_{t+1}^{1-\eta}\right)^{\frac{1-\psi}{1-\eta}}\right]^{\frac{1}{1-\psi}} \tag{1}$$

where ψ is the inverse of the intertemporal elasticity of substitution, η indexes risk-aversion, γ is inversely related to the elasticity of labor supply, and β represents time preference.[8]

The economy is also populated by a unit measure of firms, indexed by i and owned by the representative household. Firms produce output with capital and labor, according to a standard Cobb-Douglas production function $k_{it}^{\alpha} l_{it}^{1-\alpha}$. Firms are subject to an aggregate shock to capital quality ϕ_t. A firm that enters the period t with capital $\hat{k}_{it}$ has effective capital $k_{it} = \phi_t \hat{k}_{it}$. These capital quality shocks, i.i.d. over time and drawn from a distribution $g(\cdot)$, are the only aggregate disturbances in our economy. The i.i.d. assumption is made in order to avoid an additional, exogenous, source of persistence.[9]

Firms are also subject to an idiosyncratic shock v_{it}. These shocks scale up and down the total resources available to each firm (before paying debt, equity or labor)

$$\Pi_{it} = v_{it}\left[k_{it}^{\alpha} l_{it}^{1-\alpha} + (1-\delta)k_{it}\right] \tag{2}$$

where δ is the rate of capital depreciation. The shocks v_{it} are i.i.d. across time and firms and are drawn from a known distribution, F.[10] The mean of the idiosyncratic shock is normalized to be one: $\int v_{it} \quad di = 1$. The primary role of these shocks is to induce an interior default rate in equilibrium, allowing a more realistic calibration, particularly of credit spreads.

Labor, credit markets and default: We make two additional assumptions about labor markets. First, firms hire labor in advance, i.e. before observing the realizations of aggregate and idiosyncratic shocks. Second, wages are non-contingent – in other words, workers are promised a non-contingent payment and face default risk. These assumptions create an additional source of leverage.

[8]This utility function rules out wealth effects on labor, as in Greenwood et al. (1988). Appendix C.9 relaxes this assumption.

[9]The i.i.d. assumption also has empirical support. In the next section we use macro data to construct a time series for ϕ_t. We estimate an autocorrelation of 0.15, statistically insignificant. In Appendix C.10, we show that this generates almost no persistence in the economic response.

[10]This is a natural assumption - with a continuum of firms and a stationary shock process, firms can learn the complete distribution of any idiosyncratic shocks after one period.

Firms have access to a competitive non-contingent debt market, where lenders offer bond price (or equivalently, interest rate) schedules as a function of aggregate and idiosyncratic states, in the spirit of Eaton and Gersovitz (1981). A firm enters period $t+1$ with an obligation, b_{it+1} to bondholders and a promise of $w_{it+1}l_{it+1}$ to its workers. After workers exert labor effort, shocks are realized and the firm's shareholders decide whether to repay their obligations or default. Default is optimal for shareholders if, and only if,

$$\Pi_{it+1} - b_{it+1} - w_{it+1}l_{it+1} + \Gamma_{t+1} < 0$$

where Γ_{t+1} is the present value of continued operations. Thus, the default decision is a function of the resources available to the firm (Π_{it+1}) and the *total* obligations of the firm to both bondholders and workers ($b_{it+1} + w_{it+1}l_{it+1} \equiv B_{it+1}$). Let $r_{it+1} \in \{0,1\}$ denote the default policy of the firm.

In the event of default, equity holders get nothing. The productive resources of a defaulting firm are sold to a new firm at a discounted price, equal to a fraction $\theta < 1$ of the value of the defaulting firm. The proceeds are distributed *pro-rata* among the bondholders and workers.[11]

Let q_{it} denote the bond price schedule faced by firm i in period t. The lenders pay q_{it} at t in exchange for a promise of one unit of output at date $t+1$. Debt is assumed to carry a tax advantage, which creates incentives for firms to borrow. A firm which issues debt at price q_{it} and promises to repay b_{it+1} in the following period, receives a date-t payment of $\chi q_{it} b_{it+1}$, where $\chi > 1$. This subsidy to debt issuance, along with the cost of default, introduces a trade-off in the firm's capital structure decision, breaking the Modigliani-Miller theorem.[12]

For a firm that does not default, the dividend payout is its total available resources times output shock, minus its payments to debt and labor, minus the cost of building next period's capital stock (the undepreciated current capital stock is included in Π_{it}), plus the proceeds from issuing new debt, including its tax subsidy

$$d_{it} = \Pi_{it} - B_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1}. \tag{3}$$

Importantly, we do not restrict dividends to be positive, with negative dividends interpreted as (costless) equity issuance. Thus, firms are not financially constrained, ruling out another potential source of persistence.

[11]Default does not destroy resources - the penalty is purely private. This is not crucial - it is easy to relax this assumption and assume that all or part of the penalty represents physical destruction of resources.

[12]The subsidy is assumed to be paid by a government that finances it through a lump-sum tax on the representative household.

Timing and value functions:

1. Firms enter the period with capital $\hat{k}_{it}$, labor l_{it}, outstanding debt b_{it}, and a wage obligation $w_{it}l_{it}$.
2. The aggregate capital quality shock ϕ_t and the firm-specific profit shock v_{it} are realized. Production takes place.
3. The firm decides whether to default or repay ($r_{it} \in \{0, 1\}$) its bond and labor claims.
4. The firm makes capital $\hat{k}_{it+1}$ and debt b_{it+1} choices for the following period, along with wage/employment contracts w_{it+1} and l_{it+1}. Workers commit to next-period labor supply l_{it+1}. Note that all these choices are made concurrently.

In recursive form, the problem of the firm is

$$V\left(\Pi_{it}, B_{it}, S_t\right) = \max\left[0, \max_{d_{it}, \hat{k}_{it+1}, b_{it+1}, w_{it+1}, l_{it+1}} d_{it} + \mathbb{E}_t M_{t+1} V\left(\Pi_{it+1}, B_{it+1}, S_{t+1}\right)\right] \tag{4}$$

subject to

$$\begin{aligned}
\text{Dividends:} \quad & d_{it} \leq \Pi_{it} - B_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1} & (5)\\
\text{Discounted wages:} \quad & \mathcal{W}_t \leq w_{it+1} q_{it} & (6)\\
\text{Future obligations:} \quad & B_{it+1} = b_{it+1} + w_{it+1} l_{it+1} & (7)\\
\text{Resources:} \quad & \Pi_{it+1} = v_{it+1}\left[(\phi_{t+1}\hat{k}_{it+1})^{\alpha} l_{it+1}^{1-\alpha} + (1-\delta)\phi_{t+1}\hat{k}_{it+1}\right] & (8)\\
\text{Bond price:} \quad & q_{it} = \mathbb{E}_t M_{t+1}\left[r_{it+1} + (1 - r_{it+1})\frac{\theta \tilde{V}_{it+1}}{B_{it+1}}\right] & (9)
\end{aligned}$$

The first max operator in (4) captures the firm's option to default. The expectation $\mathbb{E}_t$ is taken over the idiosyncratic and aggregate shocks, given beliefs about the aggregate shock distribution. The value of a defaulting firm is simply the value of a firm with no external obligations, i.e. $\tilde{V}\left(\Pi_{it}, S_t\right) = V\left(\Pi_{it}, 0, S_t\right)$.

In (6), the firm's wage promise w_{it+1} is multiplied by bond price because wages are only paid if the firm does not default. It is as if workers are paid in bonds (but without the tax advantage of debt). The condition (6) requires the value of this promise be at least as large as $\mathcal{W}_t$, the representative household's marginal rate of substitution. This $\mathcal{W}_t$, along with the stochastic discount factor M_{t+1}, are defined using the representative household's utility function:

$$\mathcal{W}_t = \left(\frac{dU_t}{dC_t}\right)^{-1}\frac{dU_t}{dL_{t+1}} \qquad M_{t+1} = \left(\frac{dU_t}{dC_t}\right)^{-1}\frac{dU_t}{dC_{t+1}} \tag{10}$$

The aggregate state S_t consists of $(\Pi_t, L_t, \mathcal{I}_t)$ where $\Pi_t \equiv AK_t^\alpha L_t^{1-\alpha} + (1-\delta)K_t$ is the aggregate resources available, L_t is aggregate labor input (chosen in $t-1$) and $\mathcal{I}_t$ is the economy-wide information set. Equation (9) reveals that bond prices are a function of the firm's capital $\hat{k}_{it+1}$, labor l_{it+1} and debt B_{it+1}, as well as the aggregate state S_t. The firm takes the aggregate state and the function $q_{it} = q\left(\hat{k}_{it+1}, l_{it+1}, B_{it+1}, S_t\right)$ as given, while recognizing that its firm-specific choices affect its bond price.

Information and beliefs The set $\mathcal{I}_t$ includes the history of all shocks ϕ_t observed up to and including time-t. For now, we specify a general function, denoted Ψ, which maps $\mathcal{I}_t$ into an appropriate probability space. The expectation operator $\mathbb{E}_t$ is defined with respect to this space. Agents use the kernel density estimation procedure outlined in section 1 to map the information set into beliefs.

Equilibrium Definition. For a given belief function Ψ, a recursive equilibrium is a set of functions for (i) aggregate consumption and labor that maximize (1) subject to a budget constraint, (ii) firm value and policies that solve $(4-8)$, taking as given the bond price function (9) and the stochastic discount factor and aggregate MRS functions in (10) and are such that (iii) aggregate consumption and labor are consistent with individual choices.

2.2 Solving the Model

Here, we show the key equations characterizing the equilibrium, relegating detailed derivations to Appendix B. First, use the binding dividend and wage constraints (5) and (6) to substitute out for d_{it} and w_{it} in the firm's problem (4). This leaves 3 choice variables $(\hat{k}_{it+1}, l_{it+1}, b_{it+1})$ and a default decision. The latter is characterized by a threshold rule in the idiosyncratic output shock v_{it}:

$$r_{it} = \begin{cases} 0 & \text{if } v_{it} < \underline{v}_{it} \\ 1 & \text{if } v_{it} \geq \underline{v}_{it} \end{cases}$$

It turns out to be more convenient to cast the problem as a choice of $\hat{k}_{it+1}$, leverage, $lev_{it+1} \equiv \frac{B_{it+1}}{\hat{k}_{it+1}}$, and the labor-capital ratio, $\frac{l_{it+1}}{\hat{k}_{it+1}}$. Since all firms make symmetric choices, we can suppress the i subscript: $\hat{k}_{it+1} = \hat{K}_{t+1}$, $l_{it+1} = L_{t+1}$, $lev_{it+1} = lev_{t+1}$, $\underline{v}_{it+1} = \underline{v}_{t+1}$. The optimality condition for $\hat{K}_{t+1}$ can be written as:

$$1 + \chi \mathcal{W}_t \frac{L_{t+1}}{\hat{K}_{t+1}} = \mathbb{E}[M_{t+1}R^k_{t+1}] + (\chi - 1)lev_{t+1}q_t - (1-\theta)\mathbb{E}[M_{t+1}R^k_{t+1}h(\underline{v}_{t+1})] \tag{11}$$

$$\text{where} \qquad R^k_{t+1} = \frac{\phi^\alpha_{t+1}\hat{K}^\alpha_{t+1}L^{1-\alpha}_{t+1} + (1-\delta)\,\phi_{t+1}\hat{K}_{t+1}}{\hat{K}_{t+1}} \tag{12}$$

The term R^k_{t+1} is the average *ex-post* per-unit, pre-wage return on capital, while $h(\underline{v}) \equiv \int_{-\infty}^{\underline{v}} vf(v)dv$ is the expected value of the idiosyncratic shock in the default states.

The first term on the right hand side of (11) is the usual expected direct return from investing, weighted by the stochastic discount factor. The other two terms are related to debt. The second term reflects the indirect benefit from the tax advantage of debt – for each unit of capital, the firm raises $lev_{t+1}q_t$ from the bond market and earns a subsidy of $\chi - 1$ on the proceeds. The last term is the cost of this strategy – default-related losses, equal to a fraction $1 - \theta$ of available resources.

The optimal labor choice equates the expected marginal cost of labor, $\mathcal{W}_t$, with its expected marginal product, adjusted for the effect of additional wage promises on the cost of default:

$$\chi \mathcal{W}_t = \mathbb{E}_t \left[M_{t+1} (1-\alpha) \phi^{\alpha}_{t+1} \left(\frac{\hat{K}_{t+1}}{L_{t+1}} \right)^{\alpha} J^l(\underline{v}_{t+1}) \right] \tag{13}$$

where $J^l(\underline{v}) = 1 + h(\underline{v})(\theta\chi - 1) - \underline{v}^2 f(\underline{v}) \chi (\theta - 1)$ is the effect of labor being chosen in advance in exchange for a debt-like promise. Finally, the choice of leverage is governed by:

$$(1-\theta) \mathbb{E}_t \left[M_{t+1} \underline{v}_{t+1} f(\underline{v}_{t+1}) \right] = \left(\frac{\chi - 1}{\chi} \right) \mathbb{E}_t \left[M_{t+1} \left(1 - F(\underline{v}_{t+1}) \right) \right]. \tag{14}$$

The left hand side is the marginal cost of increasing leverage. Higher leverage shifts the default threshold $\underline{v}$, raising the expected losses from the default penalty (a fraction $1-\theta$ of the firm's value). The right hand side is the marginal benefit – higher leverage brings in more subsidy (the tax benefit times the value of debt issued).

The three firm optimality conditions, (11), (13), and (14) , along with those from the household side (10) and the economy-wide resource constraint, characterize the equilibrium.

3 Measurement, Calibration and Solution Method

This section describes how we use macro data to estimate beliefs and parameterize the model, as well as our computational approach. One of the key strengths of our theory is that we can use observable data to estimate beliefs at each date.

Measuring capital quality shocks Recall from Section 1 that the Great Recession saw unusually low returns to non-residential capital, stemming from unusually large declines in the market value of capital. To capture this, we need to map the model's aggregate shock, namely the capital quality shock, into market value changes. A helpful feature of capital quality shocks is that their mapping to available data is straightforward. A unit of capital installed in period

$t-1$ (i.e. as part of $\hat{K}_t$) is, in effective terms, worth ϕ_t units of consumption goods in period t. Thus, the change in its market value from $t-1$ to t is simply ϕ_t.

We apply this measurement strategy to annual data on non-residential capital held by US corporates. Specifically, we use two time series Non-residential assets from the Flow of Funds, one evaluated at market value and the second, at historical cost.[13] We denote the two series by NFA_t^{MV} and NFA_t^{HC} respectively. To see how these two series yield a time series for ϕ_t, note that, in line with the reasoning above, NFA_t^{MV} maps directly to effective capital in the model. Formally, letting P_t^k the nominal price of capital goods in t, we have $P_t^k K_t = NFA_t^{MV}$. Investment X_t can be recovered from the historical series, $P_{t-1}^k X_t = NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC}$. Combining, we can construct a series for $P_{t-1}^k \hat{K}_t$:

$$
\begin{aligned}
P_{t-1}^k \hat{K}_t &= (1-\delta) P_{t-1}^k K_{t-1} + P_{t-1}^k X_t \\
&= (1-\delta) NFA_{t-1}^{MV} + NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC}
\end{aligned}
$$

Finally, in order to obtain $\phi_t = \frac{K_t}{\hat{K}_t}$, we need to control for nominal price changes. To do this, we proxy changes in P_t^k using the price index for non-residential investment from the National Income and Product Accounts (denoted $PINDX_t$).[14] This yields:

$$
\begin{aligned}
\phi_t = \frac{K_t}{\hat{K}_t} &= \left(\frac{P_t^k K_t}{P_{t-1}^k \hat{K}_t}\right)\left(\frac{PINDX_{t-1}^k}{PINDX_t^k}\right) \\
&= \left[\frac{NFA_t^{MV}}{(1-\delta) NFA_{t-1}^{MV} + NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC}}\right]\left(\frac{PINDX_{t-1}^k}{PINDX_t^k}\right) \qquad (15)
\end{aligned}
$$

Using the measurement equation (15), we construct an annual time series for capital quality shocks for the US economy since 1950. The left panel of Figure 3 plots the resulting series. The mean and standard deviation of the series over the entire sample are 1 and 0.03 respectively. The autocorrelation is statistically insignificant at 0.15.

As Figure 3 shows, for most of the sample period, the shock realizations are in a relatively tight range around 1. However, we saw two large adverse realizations during the Great Recession: 0.93 in 2008 and 0.84 in 2009. These reflect the large drops in the market value of non-residential capital stock – in 2009, for example, the aggregate value of that stock fell by about 16%. What underlies these large fluctuations? The main contributor was a fall in the value of commercial real estate (which is the largest component of non-residential assets).[15]

[13] These are series FL102010005 and FL102010115 from Flow of Funds. See Appendix D.3.

[14] Our results are robust to alternative measures of nominal price changes, e.g. computed from the price index for GDP or Personal Consumption Expenditure, see Appendix C.1.

[15] One potential concern is that the fluctuations in the value of real estate stem mostly from land price movements. While the data in the Flow of Funds do not allow us to directly control for changes in the market

Through the lens of the model, these movements are mapped to a change in the economic value of capital – in the spirit of the hypothetical example of the Las Vegas hotel at the beginning of Section 2 whose market value falls.

Belief Estimation We then apply our kernel density estimation procedure to this time series to construct a sequence of beliefs. In other words, for each t, we construct $\{\hat{g}_t\}$ using the available time series until that point. The resulting estimates for two dates - 2007 and 2009 - are shown in the right panel of Figure 3. They show that the Great Recession induced a significant increase in the perceived likelihood of extreme negative shocks. The estimated density for 2007 implies almost zero mass below 0.90, while the one for 2009 attach a non-trivial (approximately 2.5%) probability to this region of the state space.

Calibration A period is interpreted as a year. We choose the discount factor β and depreciation δ to target a steady state capital-output ratio of 3.5 (this is taken from Cooley and Prescott (1995)) and an investment-output ratio of 0.12 (this is the average ratio of non-residential investment to output during 1950-2007 from NIPA accounts).[16] The share of capital in the production, α, is 0.40, which is also taken from Cooley and Prescott (1995). The recovery rate upon default, θ, is set to 0.70, following Gourio (2013). The distribution for the idiosyncratic shocks, v_{it} is assumed to be lognormal, i.e. $\ln v_{it} \sim N\left(-\frac{\hat{\sigma}^2}{2}, \hat{\sigma}^2\right)$ with $\hat{\sigma}^2$ chosen to target a default rate of 0.02.[17] The labor supply parameter, γ, is set to 0.5, in line with Midrigan and Philippon (2011), corresponding to a Frisch elasticity of 2.

For the parameters governing risk aversion and intertemporal elasticity of substitution, we use standard values from the asset pricing literature and set $\psi = 0.5$ (or equivalently, an IES of 2) and $\eta = 10$.[18] The tax advantage parameter χ is chosen to match a leverage target of 0.70, which is obtained by adding the wage bill (approximately 0.2 of the steady state capital stock) to financial leverage (the ratio of external debt to capital, about 0.5 in US data - from Gourio (2013)). Table 1 summarizes the resulting parameter choices.

value of land, they do suggest a limited role for land. Measured at historical cost, land accounts for less than 5% of total non-residential capital. The observed fluctuations in the value of these assets during 2008-09 are simply too large to be accounted for by land price movements, even if they are sizable.

[16] This leads to values for β and δ of 0.91 and 0.03 respectively. These are lower than other estimates in the literature. However, when we used an alternative calibration strategy with $\delta = 0.06$ (which is consistent with reported depreciation rates in the Flow of Funds data) and $\beta = 0.95$ (which leads to the same capital-output ratio), the resulting impulse responses were almost identical.

[17] This is in line with the target in Khan et al. (2014), though a bit higher than the one in Gourio (2013). We verified that our quantitative results are not sensitive to this target.

[18] In Appendix C.8, we examine the robustness of our main results to these parameter choices. See also the discussion in Gourio (2013).

Parameter	Value	Description
Preferences:		
β	0.91	Discount factor
η	10	Risk aversion
ψ	0.50	1/Intertemporal elasticity of substitution
γ	0.50	1/Frisch elasticity
Technology:		
α	0.40	Capital share
δ	0.03	Depreciation rate
$\hat{\sigma}$	0.25	Idiosyncratic volatility
Debt:		
χ	1.06	Tax advantage of debt
θ	0.70	Recovery rate

Table 1: **Parameters**

Numerical solution method Given the importance of curvature in policy functions for our results, we solve the non-linear system of equations (11) $-$ (14) using collocation methods. Appendix A describes the iterative procedure. In order to maintain tractability, we need to make one approximation. Policy functions at date-t depend both on the current estimated distribution, $\hat{g}_t(\phi)$, and the distribution $\mathcal{H}$ over next-period estimates, $\hat{g}_{t+1}(\phi)$. Keeping track of $\mathcal{H}(\hat{g}_{t+1}(\phi))$, (a distribution over a distribution, i.e. a compound lottery) as a state variable would render the analysis intractable. However, the approximate martingale property of $\hat{g}_t$ discussed in Section 1 offers an accurate and computationally efficient approximation to this problem. The martingale property implies that the average of the compound lottery is $\mathbb{E}_t[\hat{g}_{t+1}(\phi)] \approx \hat{g}_t(\phi)$, $\forall\phi$. Therefore, when computing policy functions, we approximate $\mathcal{H}(\hat{g}_{t+1}(\phi))$ with its mean $\hat{g}_t(\phi)$, the current estimate of the distribution. Appendix C.2 uses a numerical experiment to show that this approximation is quite accurate. Intuitively, future estimates $\hat{g}_{t+1}$ are tightly centered around $\hat{g}_t(\phi)$, i.e. $\mathcal{H}(\hat{g}_{t+1})$ has a relatively small variance. The shaded area in the third panel of Figure 3 reveals that even 30 periods out, $\hat{g}_{t+30}(\phi)$ is still quite close to its mean $\hat{g}_t(\phi)$. For 1-10 quarters ahead, where most of the utility weight is, this error is even smaller.

4 Main Results

In this section, we evaluate, quantitatively, the ability of the model of generate persistent responses from tail events and confront its predictions with data. The key model feature behind persistence is the assumption that people estimate the distribution of aggregate economic shocks from available data. To isolate its role, we compare results from our model to those from the same model where the distribution of shocks is assumed to be known with certainty. In this "no learning" economy, agents know the true probability of the tail event and so, observing such a realization does not change their beliefs. Next, we demonstrate how learning makes large, unusual recessions different from smaller, more normal ones by comparing the model's predictions for the response to the Great Recession to a counterfactual, much less extreme shock. Then, we explore an economy where agents have learned from earlier episodes such as the Great Depression. It shows that beliefs about tail risk are particularly persistent, not because tail events were never seen before, but because relevant data on tail events is observed infrequently. Finally, we show that incorporating learning delivers more realistic equity, bond and option price predictions.

4.1 Belief Updating and Persistence

Our first set of results compare the predictions of the learning and no-learning models for macro aggregates (GDP, investment and labor) since 2008-'09. They show that the model with learning does significantly better in terms of matching the observed, persistent behavior of macro variables. Then, to rule out the possibility that persistence comes primarily from the occurrence of future crises, we show that the economic responses are extremely persistent, even if no future crises occur.

To compute our benchmark results, we begin by estimating $\hat{g}_{2007}$ using the data on ϕ_t described above. We then compute the stochastic steady state by simulating the model for 1000 periods drawing from $\hat{g}_{2007}$. We discard the first 500 observations and time-average across the remaining periods. This corresponds to the average value of the variable of interest in the long run under the assumption that the true data generating process is $\hat{g}_{2007}$[19]. This steady state forms the starting point for our results. Subsequent results are in log deviations from this steady state level. Then, we subject the model economy to two adverse realizations - 0.93 and 0.84, which correspond to the shocks that we observed in 2008 and 2009. Using these two additional data points, we re-estimate the distribution, to get $\hat{g}_{2009}$. To see how persistent the responses are, we need to simulate a long future time series. We don't know what distribution future shocks will be drawn from. Given all the data available to us, our best estimate is also $\hat{g}_{2009}$. Therefore, we simulate future paths by drawing many sequences of future ϕ shocks from the $\hat{g}_{2009}$ distribution and we plot the mean future path of various aggregate variables.

As described in Section 3, we start the economy at the 2007 stochastic steady state, subject it to the 2008 and 2009 shocks, and then draw many future sequences of shocks from $\hat{g}_{2009}$. The top left panel of Figure 4 shows the the average of all simulated time paths for ϕ_t. Then we solve the model for each sequence of shocks, and average the results. In the remaining

[19]Under this assumption, the long-run behavior of the economy is described by an ergodic distribution.

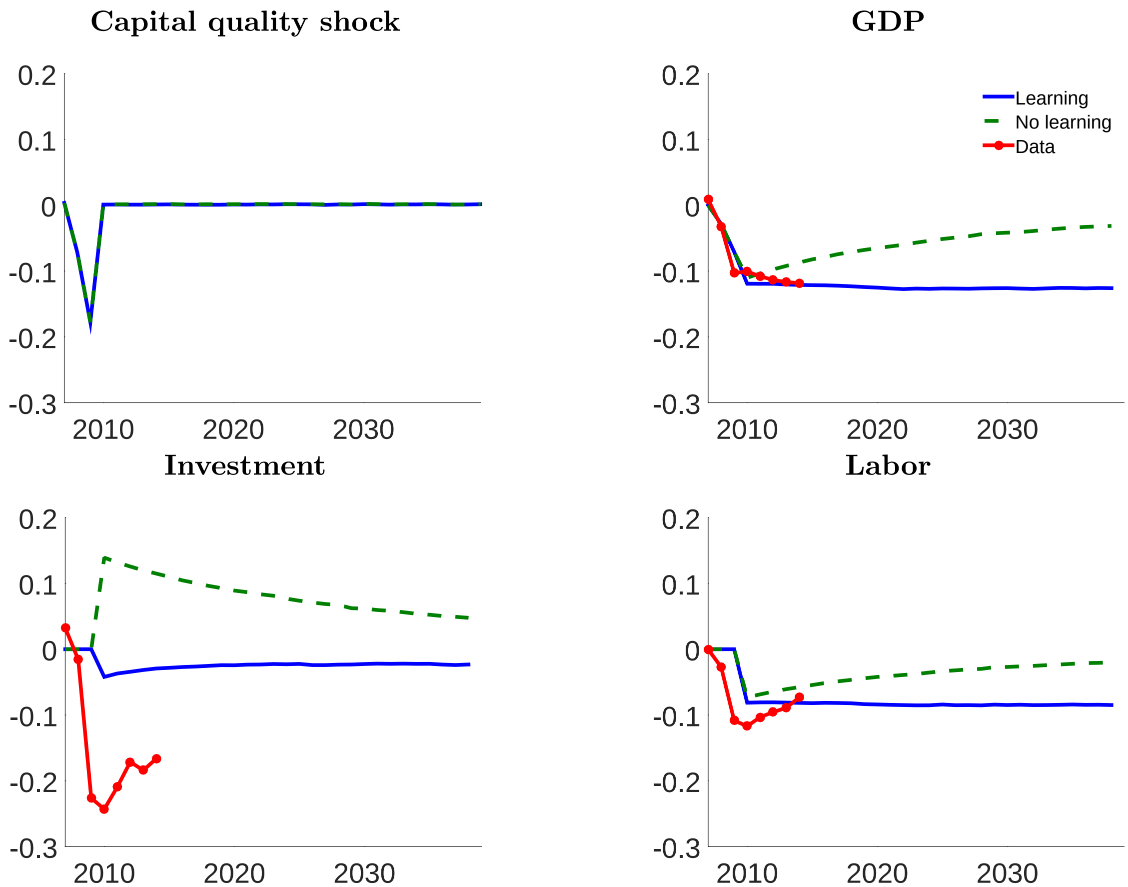


Figure 4: **Persistent responses in output, investment and labor.**
Solid line shows the change in aggregates (relative to the stochastic steady state associated with $\hat{g}_{2007}$). The circles show de-trended US data for the period 2008-2014. For the dashed line (no learning), agents believe that shocks are drawn from $\hat{g}_{2009}$ and never revise those beliefs.

panels, output, investment and employment show a pattern of prolonged stagnation, where the economy (on average) never recovers from the negative shocks in 2008-'09. Instead, all aggregate variables move towards the new, lower (stochastic) steady state. These results do not imply that stagnation will continue forever. The flat response tells us that, from the perspective of an agent with the 2009 information set, recovery is not expected.[20]

The solid line with circles in Figure 4 plots the actual data (in deviations from their respective 1950-2007 trends) for the US economy.[21] As the graph shows, the model's predictions for GDP and labor line up well with the recent data, though none of these series were used in

[20]Appendix C.13 shows that a large fraction of the persistence response is due to changes in beliefs rather than the fact that the shock hits the actual capital.

[21]Data on output and labor input are obtained from Fernald (2014). Data on investment comes from the series for non-residential investment from the NIPA published by the Bureau of Economic Analysis, adjusted for population and price changes. Each series is detrended using a log-linear trend estimated using data from 1950-2007, see Appendix D.4.

the calibration or measurement of the aggregate shock ϕ_t. The predicted path for employment lags and slightly underpredicts the actual changes, largely due to the assumption that labor is chosen in advance. Including shock realizations post-2009 does not materially change these findings (see Appendix C.3).[22]

For investment, the model performs poorly. It predicts less than half of the observed drop. However, without learning, the results are much worse. When agents do not learn, investment surges, instead of plummets. The reason is that the effective size of the capital stock was already diminished by the capital quality shock. The shock itself is like an enormous, exogenous disinvestment. We could solve this problem by adding more features and frictions.[23] But our main point is about learning and persistence. Despite only partially fixing the investment problem of the capital quality model, Figure 4 clearly demonstrates the quantitative potential of learning as a source of persistence.

Table 2 summarizes the long-run effects of the belief changes, by comparing averages in the stochastic steady states under $\hat{g}_{2007}$ and $\hat{g}_{2009}$. As mentioned earlier, these are the average levels that the economy ultimately converges to, under the assumption that the data-generating process (and therefore, long-run beliefs) is $\hat{g}_{2007}$ or $\hat{g}_{2009}$. Capital and labor are, on average, 17% and 8% lower under the post-crisis beliefs $\hat{g}_{2009}$, which translates into a 12% lower output and consumption level, in the long run. Investment falls by 7% [24] Thus, even though the ϕ_t shocks experienced during the Great Recession were transitory, the resulting changes in beliefs persistently reduce economic activity.

	Stochastic steady state levels		Change
	$\hat{g}_{2007}$	$\hat{g}_{2009}$	
Output	6.37	5.67	-12 %
Capital	27.52	22.80	-17%
Investment	0.71	0.66	- 7%
Labor	2.40	2.20	-8%
Consumption	5.66	5.01	-12%

Table 2: **Belief changes from 2008-'09 shocks lead to significant reductions in economic activity.**
Columns marked $\hat{g}_{2007}$ and $\hat{g}_{2009}$ represent average levels in the stochastic steady state of a model where shocks are drawn from $\hat{g}_{2007}$ or $\hat{g}_{2009}$ distributions respectively.

[22] Additional outcomes are reported in Appendix C.5.

[23] Financial frictions which impeded investment include those in Gertler and Karadi (2011) and Brunnermeier and Sannikov (2014). Alternative amplification mechanisms are studied in Adrian and Boyarchenko (2012); Jermann and Quadrini (2012); Khan et al. (2014); Zetlin-Jones and Shourideh (2014); Bigio (2015); Moriera and Savov (2015), among others.

[24] The fall in investment is smaller than that of capital because the shock distribution has also changed. For example, under $\hat{g}_{2009}$, the mean shock is slightly lower (relative to $\hat{g}_{2007}$). Intuitively, this acts like a slightly higher depreciation rate – so, even though capital is lower by 17%, the drop in investment is only 7%.

Turning off belief updating To demonstrate the role of learning, we plot average simulated outcomes from an otherwise identical economy where agents know the final distribution $\hat{g}_{2009}$ with certainty, from the very beginning (dashed line in Figure 4). Now, by assumption, agents do not revise their beliefs after the Great Recession. This corresponds to a standard rational expectations econometrics approach, where agents are assumed to know the true distribution of shocks hitting the economy and the econometrician estimates this distribution using all the available data. The post-2009 paths are simulated as follows: each economy is assumed to be at its stochastic steady state in 2007 and is subjected to the same sequence of shocks – two large negative ones in 2008 and 2009 and subsequently, sequences of shocks drawn from the estimated 2009 distribution.

Without belief revisions, the negative shocks lead to an investment boom, as the economy replenishes the lost effective capital. While the curvature in utility moderates the speed of this transition, the overall pattern of a steady recovery back to the original steady state is clear.[25] This shows that learning is key to generating persistent reductions in economic activity.

What if shocks are persistent? An alternative explanation for persistence is that there was no learning. Instead, the shocks simply had persistently bad realizations. In Appendix C.10, we show that allowing for a realistic amount of persistence in the ϕ_t shocks does not materially change the dynamics of aggregate variables. This is because the the observed autocorrelation of the ϕ_t process is too low to generate any meaningful persistence.

What if there are no more crises? In the results presented above, we put ourselves on the same footing as the agents in our model and draw future time paths of shocks using the updated beliefs $\hat{g}_{2009}$. One potential concern is that persistent stagnation comes not from belief changes *per se* but from the fact that future paths are drawn from a distribution where crises occur with non-trivial probability. This concern is not without merit. If we draw future shocks from the distribution, $\hat{g}_{2007}$, where the probability of a crisis is near zero, beliefs are not Martingales. In that world, beliefs change by the same amount on impact, but then converge back to their pre-crisis levels. Without the permanent effect on beliefs, persistence should fall.

However, Figure 5 shows that the persistence over a 30 year horizon is almost the same with and without future crises (solid and dashed lines). The reason belief effects are still so long-lived is that it takes many, many no-crisis draws to convince someone that the true probability of a crisis is less than an already-small probability. For example, if one observed 100 periods without a crisis, this would still not be compelling evidence that the odds are less than 1%. This

[25] Since the no-learning economy is endowed with the same end-of-sample beliefs as the learning model, they both ultimately converge to the same *levels*, even though they start at different points (normalized to 0 for each series).

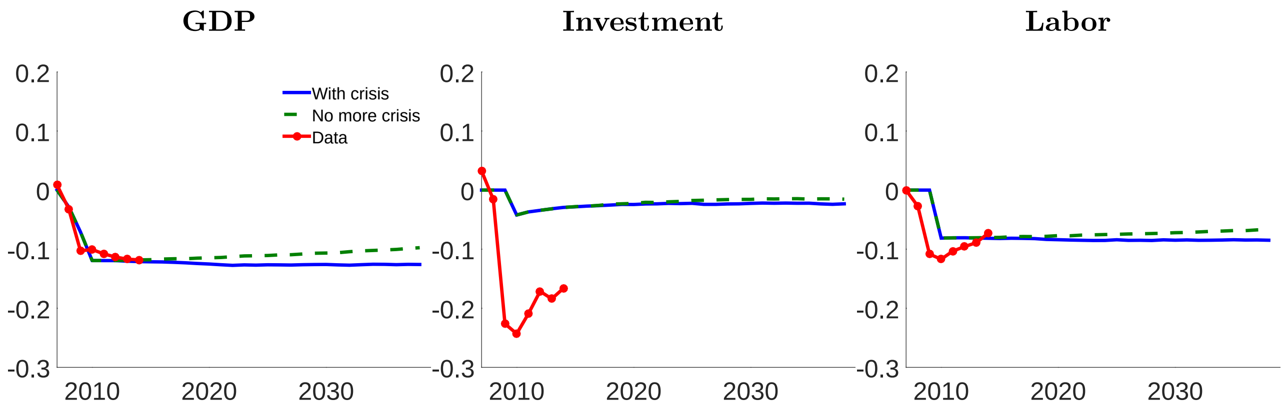


Figure 5: **What if there are no more crises?**
Solid (With crisis) line shows the change in aggregates when the data generating process is $\hat{g}_{2009}$ and agent updates beliefs. Dashed line (No more crisis) is an identical model in which future shocks are drawn from $\hat{g}_{2007}$. The circles show de-trended US data for the period 2008-2014.

highlights that beliefs about tail probabilities are persistent because tail-relevant data arrives infrequently.

The fact that most data is not relevant for inferring tail probabilities is a consequence of our non-parametric approach. If instead, we imposed a parametric form like a normal distribution, then probabilities (including those for tail events) would depend only on the mean and variance of the distribution. Since mean and variance are informed by all data, tail probability revisions are frequent and small. As a result, the effects of observing the '08 and '09 shocks are more transitory. See Appendix C.11 for more details.

4.2 Shock Size and Persistence

The secular stagnation puzzle is not about why all economic shocks are so persistent. The question is why this recession had more persistent effects than others. Assuming that shocks are persistent does not answer this question. Our model explains why persistent responses arise mainly after a tail event. Every decline in capital quality has a transitory direct effect (it lowers effective capital) and a persistent belief effect. Thus, the extent to which a shock generates persistent outcomes depends on the relative size of these two effects. Observing a tail event, one we did not expect, change beliefs a lot and generates a large persistent effect. A small shock has a negligible effect on beliefs and therefore, generates little persistence. This finding – that learning does not matter when 'normal' shocks hit – is also the reason why we focus on the Great Recession. We could use the model to explore regular business cycles, but the versions with and without learning would be almost observationally equivalent, yielding little insight into the role of learning.

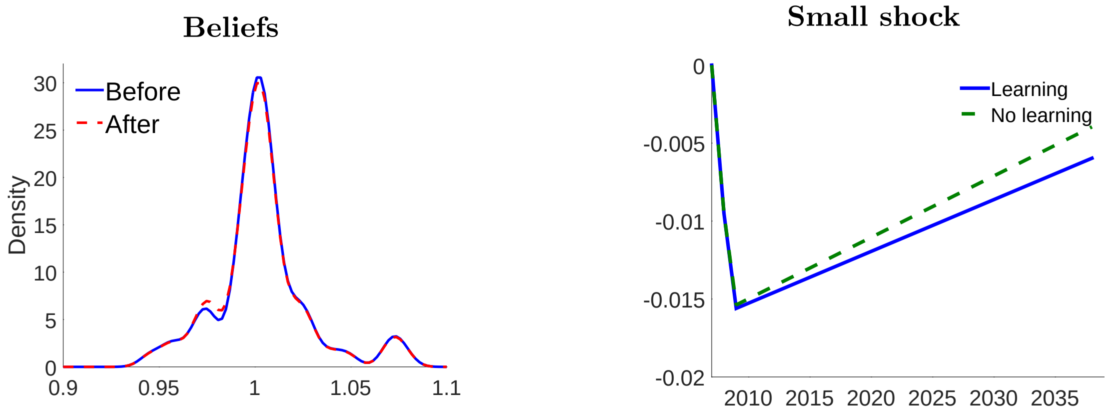


Figure 6: **Small shocks create negligible persistence.**
The first panel shows the estimated density before the shock (solid blue) and after a one standard deviation shock (dashed red). The second panel shows the response of output to the small shock under learning and no learning.

Figure 6 shows the effects on output of a small adverse shock (1 standard deviation below the mean[26]), again starting from the stochastic steady state associated with $\hat{g}_{2007}$. Obviously, the effects are smaller than the baseline model (note the scale on the y-axis). The smaller initial impact reflects the non-linearity of the model's policy functions.

More importantly for our mechanism, the effect of small shocks is transitory and nearly the same with or without learning. Learning is still a source of persistence, but quantitatively, it amounts to very little. The bulk of the persistence from the small shock just comes from agents gradually replenishing the capital stock, an effect that is there in the no-learning model as well.

The reason that persistence is so low for small shocks is that beliefs do not change much. This has more to do with the *likelihood* than about the size of the shock *per se*. The left panel of Figure 6 shows that the only change in beliefs after the small shock is a small deviation around 0.97. But, if large shocks were observed frequently and small ones infrequently, then small shocks would be surprising and would change beliefs by more with very persistent effects. Thus, our learning mechanism offers a novel explanation for why fluctuations triggered by rare events are particularly persistent.

4.3 Longer data sample and the Great Depression

Since our simulations start in 1950, the Great Depression is not in our agents' information set. This raises the question: How would access to more data, with large adverse shocks in it, affect the response of beliefs to the recent financial crisis? In the limit, as data accumulates, agents know the true distribution; new data ceases to affect beliefs. However, beliefs about tail events converge more slowly than those elsewhere in the distribution, because of infrequent

[26]This is roughly the magnitude of the shock observed during the 2001-'02 recession.

observations. In this section, we approximate data extending back to the 19th century and show that the belief changes induced by the 2008-09 experience continue to have a large, persistent effect on economic activity.

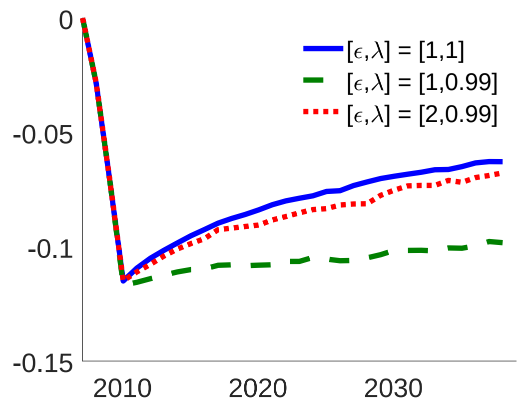


Figure 7: **Extending the data sample tempers persistence slightly.**
Each line shows the response of GDP to the 2008-09 shocks under a hypothetical information set, starting from 1890. To fill in the data for the period 1890-1949, we use the observed time series from 1950-2009, with $\{\phi_{1929}, \phi_{1930}\} = \{\phi^{\varepsilon}_{2008}, \phi^{\varepsilon}_{2009}\}$. *The parameter* λ *indexes the extent to which older observations are discounted where* $\lambda = 1$ *represents no discounting. The parameter* ε *represents the severity of the shocks in the Great Depression where* $\varepsilon = 1$ *represents that it was similar to the Great Recession.*

The difficulty with extending the data is that the non-financial asset data used in ϕ_t is available only for the post-WW II period. Other macro and financial series turn out to be unreliable proxies.[27] But our goal here is not to explain the Great Depression. It is to understand how having more data, especially previous crises, affects learning today. So we use the post-WW II sample to construct pre-WW II scenarios. Specifically, we assume that ϕ_t realizations for the period from 1890-1949 were identical to those in 1950-2009, with one adjustment: The Great Depression shocks were as bad or worse those in the Great Recession. To make this adjustment, we set $\{\phi_{1929}, \phi_{1930}\} = \{\phi^{\varepsilon}_{2008}, \phi^{\varepsilon}_{2009}\}$, with $\varepsilon \geq 1$. Recall that ϕ in 2008 and 2009 are less than one. So, raising them to a power greater than one makes them smaller.

We then repeat our analysis under the assumption that agents are endowed with this expanded 1890-2007 data series. Now, when the financial crisis hits, the effect on beliefs is moderated by the larger data sample, which contains a similar or worse previous crisis.

Once we include data from a different eras, the assumption that old and new data are treated as equally relevant becomes less realistic. We consider the possibility that agents discount older

[27]We projected the measured ϕ_t series post-1950 on a number of variables and used the estimated coefficients to impute values for ϕ_t pre-1950. However, this did not produce accurate estimates in-sample. Specifically, it missed crises. We explored a wide range of macro and asset pricing variables - including GDP, unemployment, S&P returns and the Case-Shiller index of home prices. We also experimented with lead-lag structures. Across specifications, the resulting projections for 1929-1930 showed only modestly adverse realizations.

observations. This could reflect the possibility of unobserved regime shifts or experiential learning with overlapping generations (Malmendier and Nagel, 2011).[28] To capture such discounting, we modify our kernel estimation procedure. Observation from s periods earlier are assigned a weight λ^s, where $\lambda \leq 1$ is a parameter. When $\lambda = 1$, there is no discounting.

Figure 7 reveals that, even without discounting ($\lambda = 1$), the difference between the model with and without Great Depression data is modest, as of 2016: There is a similar output drop on impact, with attenuated persistence from the additional data. When older data is discounted by 1% ($\lambda = 0.99$, the center panel), this attenuation almost completely disappears and the impulse responses replicate our baseline estimates.[29]

Perhaps the true magnitude of the Great Depression shocks is far larger than those seen in 2008-09. Suppose $\varepsilon = 2$, so that $(\phi_{1929}, \phi_{1930}) = (0.86, 0.70)$. These are very large shocks – 5 and 10 standard deviations below the mean. Taken together, they imply an erosion of almost 50% in the stock of effective capital. Figure 7 shows that, with 1% annual discounting ($\lambda = 0.99$), persistence is attenuated, but only modestly.

In sum, expanding the information set by adding more data does not drastically alter our main conclusions, especially once we assume that agents discount older data.

4.4 Evidence from Asset Markets

Our framework stays close to a standard neoclassical macro paradigm and therefore, inherits many of its limitations when it comes to asset prices. Our goal in this section is not to resolve these shortcomings but to show that the predictions of the model are broadly in line with the patterns in asset markets. As with macro aggregates, effects of learning are detectable only after tail events, so we focus on the period since the Great Recession. In Table 3, we compare the predictions of the model for various asset market variables, both pre- and post-crisis, with their empirical counterparts. We construct the latter by averaging over 1990-07 and 2010-15 respectively.[30] The model does not quite match levels, but our focus is on the changes induced by the 2008-09 experience. We find that while the model's predictions for the change in credit spreads and equity prices are broadly consistent with the data, these variables are not very sensitive to, and therefore, not very informative about tail risk. On the other hand, the model's predictions for option-implied tail risk, i.e. the probabilities of extreme events priced into

[28] This type of discounting is similar to Sargent (2001), Cho et al. (2002) and Evans and Honkapohja (2001).

[29] With stronger discounting, the decline in GDP becomes *bigger*. Intuitively, higher discounting increases the weight of recent observations relative to undiscounted case. For example, with $\lambda = 0.98$, the persistent drop in GDP is about 16%.

[30] The overall pattern of the model's performance is not sensitive to the time periods chosen. In an earlier, we used shorter samples for both pre- and post-crisis and the conclusions were very similar. We did not include 2008 and 2009 in order to avoid picking up outsized fluctuations in asset markets at the height of the crisis.

options, which is a much better indicator, line up quite well with the observed changes.

Credit spreads – the difference between the interest on a risky and a riskless loan – are commonly interpreted as a measure of risk. The first row of the table shows that by 2015 spreads are only slightly higher than their pre-crisis levels (they spiked at the height of the crisis but then came back down). Hall (2015a), for example, uses this observation to argue that a persistent increase tail risk is not a likely explanation for stagnation.

	Model			Data		
	$\hat{g}_{2007}$	$\hat{g}_{2009}$	Change	1990-2007	2010-2015	Change
Asset prices and debt						
Credit Spreads	0.75%	0.77%	0.02%	1.89%	2.23%	0.34%
Equity Premium	1.09%	2.51%	1.41%	3.29%	7.50%	4.21%
Equity/Assets	46.63%	47.27%	0.65%	55.28%	56.87%	1.59%
Risk free rate	9.57%	9.03%	-0.54%	1.42%	-1.32%	-2.74%
Debt			-17%	3%	-23%	-26%
Tail risk for equity						
Third moment ($\times 10^2$)	0.00	-0.26	-0.26	-1.34	-1.59	-0.25
Tail risk	0.0%	1.5%	1.5 %	9.3%	11.2%	1.9%

Table 3: **Changes in financial market variables, Model vs data.**
Model: shows average values in the stochastic steady state under $\hat{g}_{2007}$ and $\hat{g}_{2009}$. The equity/assets is the ratio of the market value of equity claims to capital K_t. Third moment is $E\left[\left(R^e - \bar{R}^e\right)^3\right]$, where R^e is the return on equity and Tail risk is $Prob\left(R^e - \bar{R}^e \leq -0.3\right)$, where both are computed under the risk-neutral measure. Without learning, all changes are zero.
Data: For credit spreads, we use the average spread on senior unsecured bonds issued by non-financial firms computed as in Gilchrist and Zakrajek (2012). For the equity premium, we follow Cochrane (2011) and Hall (2015b) and estimate the one-year ahead forecast for real returns on the S&P 500 from a regression using Price-Dividend and aggregate Consumption-GDP ratios. See footnote 34 for details. Equity/assets is the ratio of the market value of equities to value of non-financial assets from Table B.103 in the Flow of Funds. The risk free real interest rate is computed as the difference between nominal yield of 1-year US treasuries and inflation. Debt is measured as total liabilities of nonfinancial corporate business from the Flow of Funds (FL104190005, Table B.103), adjusted for population growth and inflation. The numbers reported are deviations from a log-linear 1952-2007 trend. The third moment and tail risk are computed from the VIX and SKEW indices published by CBOE. See footnote 36 and Appendix C.7 for details.

The results in Table 3 argue against this conclusion. The connection between spreads and tail risk is weak – our calibrated model predicts a negligible rise in spreads, about 2 basis points. Since the model without learning predicts no long-run changes in asset prices, this small change means that credit spreads are not a useful device for divining beliefs about tail risk. This is due, in part, to equilibrium effects: an increase in bankruptcy risk induces firms to issue less debt. Debt in the new steady state is about 17% lower.[31] In the data, total liabilities of non-

[31] The leverage ratio (debt and wage obligation divided by total assets) is also slightly lower, by about 0.5%.

financial corporations (relative to trend)[32] show a similar change – a drop of about 26%. This de-leveraging lowers default risk and therefore, offsets the rise in credit spreads. In the model, the net effect of higher tail risk and lower firm debt on spreads is nearly zero. Thus, the fact that spreads are almost back to their pre-crisis levels does not rule out tail risk – one of the surprising predictions of the model.

Similarly, one might think that equity should be worth less when risk is high. The fact that equity prices have surged recently and are higher than their pre-crisis levels thus appears inconsistent with a rise in tail risk. But again, this logic is incomplete – while higher tail risk does increase the risk premium, it also induces firms to cut debt, which mitigates the increase in risk (Modigliani and Miller, 1958). The net effect in the model is to slightly raise the market value of a dividend claim associated with a unit of capital under the post-crisis beliefs relative to the pre-crisis ones. In other words, the combined effect of the changes in tail risk and debt is mildly positive.[33] In the data, the ratio of the market capitalization of the non-financial corporate sector to their (non-financial) asset positions also shows an increase. While the magnitudes differ – we don't claim to solve all equity-related puzzles here – our point is simply that rising equity valuations are not evidence against tail risk.

Furthermore, the changes in equity premia (the difference between expected return on equity and the riskless rate) are in the right ballpark, even though the model underpredicts the level relative to the data (reflecting its limitations as an asset pricing model). The higher tail risk under the 2009 beliefs implies an rise of 1.5% in the equity premium, relative to that under the 2007 beliefs. To compute the analogous object in the data, we follow the methodology in Cochrane (2011) and Hall (2015b)[34], which estimates that equity premia in 2010-15 were about 4.21% higher than the pre-crisis average. In other words, tail risk can account for about a third of the recent rise in equity premia. Of course, our measure of equity premia, like all others, is noisy and volatile. We are not claiming that the model can explain all the fluctuations – no model can – but it doesn't seem to be at odds with recent trends in equity market variables.

The table also shows the model's predictions for riskless rates. Again, as with the equity premium, the model does not quite match the level[35], but does a better job predicting the change

[32] Total liabilities of nonfinancial corporate business is taken from series FL104190005 from Table B.103 in the Flow of Funds. As with the other macro series, we adjust for inflation and population growth and then detrend using a simple log-linear trendline. The numbers reported in the table are the (averages of the) deviations from a log-linear trend, computed from 1952-2007.

[33] The aggregate market capitalization in the model is obtained by simply multiplying the value of the dividend claim by the aggregate capital stock.

[34] We estimate one-year ahead forecast from a regression where the left-hand variable is the one-year real return on the S&P and the right hand variables are a constant, the log of the ratio of the S&P at the beginning of the period to its dividends averaged over the prior year, and the log of the ratio of real consumption to disposable income in the month prior to the beginning of the period.

[35] This is partly due to our choice of a relatively low value for the discount factor, β. As we discussed in

since the Great Recession. Higher tail risk increases the premium for safe assets, reducing the riskless rate. Under our calibration, the change in beliefs induced by the 2008-09 realizations leads to a 54 bp drop in the riskless rate. In the data, the real rate on Treasuries (computed as the difference between nominal yield of the 1-year Treasury and inflation) averaged -0.81 % between 2013-15, as against 0.61% during 2005-07, a drop of about 1.4%. Thus, the model underpredicts the drop. In ongoing work – Kozlowski et al. (2018) – we show how higher tail risk can interact with liquidity needs of firms, amplifying the drop in government bond yields.

In sum, none of these trends in asset markets is at odds with the tail risk story we are advancing. If credit spreads and equity premia are not clear indicators of tail risk, what is? For that, we need to turn to option prices, in particular options on the S&P 500, which can be used to isolate changes in perceived tail risk. A natural metric is the third moment of the distribution of equity returns. It is straightforward to compute this from the SKEW and VIX indices reported by the CBOE.[36] As Table 3 shows, the market-implied distribution has became more negatively skewed after the Great Recession. We compute the same risk-neutral third moment in the model (using the distribution for stock returns under the 2009 and 2007 beliefs). The model underpredicts the skewness in levels[37], but predicts a change (-0.0026) that lines up almost exactly with the data. To show how this change maps into probabilities of tail events, we also report the numbers for the implied (risk-neutral) odds of a return realization 30% less than the mean.[38] Again, the model-implied level is too low, but the predicted change (1.5%) is quite close to the corresponding object in the data (2.2%).[39]

4.5 Understanding the Economic Response to Belief Changes

What model ingredients are needed for belief to have substantial aggregate effects and why? To answer this, we perform a series of experiments, varying and turning off specific features of the model – learning about the mean vs higher order moments, curvature in utility and debt – one-by-one in order to isolate how much each one contributes. The bottom panel of Table 4 shows that removing any of these elements would eliminate between one-fourth and one-half of

footnote 16, an alternative calibration strategy with a higher value for β yields very similar results.

[36] Formally, the third central moment under the risk-neutral measure is given by

$$\mathbb{E}\left(R^e - \bar{R}^e\right)^3 = \frac{100 - SKEW_t}{10} \cdot VIX_t^3 \,.$$

For more information, see http://www.cboe.com/micro/skew/introduction.aspx.

[37] Fixing this would require additional shocks and/or amplification mechanisms.

[38] For details of the computation, see Appendix C.7.

[39] Including shock realizations post-2009 does not materially change these findings, see Appendix C.15.

our long-run effects. [40]

	2014	Long run
Data	-0.12	
Benchmark model	-0.12	-0.12
Counterfactuals:		
Constant mean	-0.09	-0.06
No curvature in utility	-0.07	-0.07
No debt	-0.12	-0.09

Table 4: **Change in GDP relative to 2007 steady state.**

Learning about the mean capital quality shock. We decompose the total effect of belief revisions into a component attributable to changes in the mean (average ϕ) and the remaining attributable to changes in higher moments. To do this, we adjust the estimated distribution in 2009 so that $\mathbb{E}_{2009}(\phi_t) = \mathbb{E}_{2007}(\phi_t)$. The change in the mean $\mathbb{E}_t[\phi_t]$ between 2007 and 2009 is relatively modest, only about 0.4%. Even with the mean change taken out, the long-run fall in GDP is about 6%, about half of the total effect in our baseline case (Figure 8, left panel). In Appendix C.4, we explore this high sensitivity using a deterministic version of the model without debt. This simplified model reveals, in closed form, the elasticity of long-run, steady-state capital to the mean capital quality:

$$\frac{d\ln k_{ss}}{d\ln \phi_{ss}} = \left(\frac{1+\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right) + \left(\frac{1}{1-\alpha}\frac{\alpha+\gamma}{\gamma}\right)\frac{(1-\delta)}{1/\beta-(1-\delta)\phi_{ss}} = 2+3(7.5) = 24.5.$$

Capital, and thus output, is highly sensitive to capital quality because it affects current returns (first term) and holdings gains (second term), which come from the undepreciated capital stock. This sensitivity, which is far greater than to total factor productivity, lies at the heart of the significant economic effects.

Role of curvature in utility Next, we explore the role of curvature in utility, by exploring an otherwise identical economy with quasilinear preferences. If $\psi = \eta = 0$, the utility function reduces to $C_t - \frac{L_t^{1+\gamma}}{1+\gamma}$. Because this eliminates risk aversion and the desire for consumption smoothing, the economy transitions immediately to any new steady state (second panel of Figure 8). However, belief revisions still have long-lived effects – long-run output is about 7% lower (compared to 12% in the baseline model). In other words, curvature in utility accounts for roughly 40% of long-run stagnation.

[40]These patterns are robust to drawing future time paths under the assumption that no future crises occur. For details, see Table 6 in Appendix C.6.

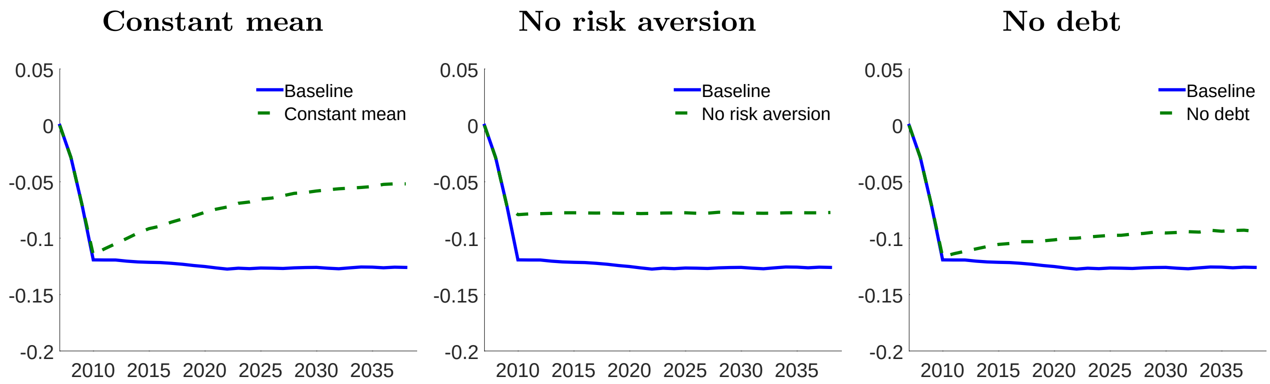


Figure 8: **Understanding the role of expected return, risk aversion and debt.**
Change in ln GDP under 3 different scenarios: (1) the mean capital quality shock $E[\phi_t]$ *is held fixed; (2) no risk aversion, and (3) no debt.*

Risk aversion matters because it introduces a risk premium for capital and labor. Tail risk raises this premium, further dampening incentives to invest and hire. Appendix C.8 shows that the effect of tail risk on macro aggregates is increasing, albeit modestly, in both risk aversion and the intertemporal elasticity of substitution. These results highlight the role of Epstein-Zin preferences. With CRRA preferences, high risk aversion implies low intertemporal elasticity, dampening the drop in long-run economic activity.

Role of debt When we set the tax advantage parameter χ to 1, all investment is financed through equity. Debt and leverage are 0. The third panel of Figure 8 shows that belief revisions trigger a 9% long-run reduction in output without debt, compared to 12% with debt. Thus, defaultable debt contributes about a fourth of the long run stagnation.

Debt also plays an important role in one of the main questions of the paper, namely why some shocks generate more persistent responses than others. The attractiveness of debt (and therefore, the incentives to borrow) is affected disproportionately by perceived tail risk - and since larger shocks changes belief further out in the tail, they are amplified by debt. Since tail risk is the source of persistence, by amplifying its effects increases persistence as well.

In Figure 9, we subjected our model economy to shocks ranging in size from 1 to 5 standard deviations and plotted the corresponding long-run GDP effect. The responsiveness to small shocks is almost the same with and without debt. Because debt adds aggregate non-linearity, larger shocks see significant amplification. Since the risk of a larger shock is what persists, debt makes the severity and persistence of unusual events differ from common downturns.

Figure 9: **Debt amplifies belief revisions from large shocks.**
Change in long-run GDP both with (solid line) and without debt (dashed line) in response to negative shocks of various sizes. The initial condition is the $\hat{g}_{2007}$ *steady state.*

4.6 Open Questions

While the analysis in the preceding sections demonstrates the quantitative potential for belief revisions, both for macro aggregates and financial market variables, it also points to shortcomings in our approach and raises new questions – both conceptual and empirical. These are areas where future research might be fruitful.

One obvious shortcoming of the model is its inability to reproduce the plunge in investment seen in the data. This is due to the nature of the shock – an negative shock reduces capital available for production, creating huge incentives to replenish that capital stock. This is a stark simplification – in reality, the events in 2008-09 probably reduced the economic value of capital without mechanically pushing up returns on new investments. Fixing this would requires some innovation on the modeling front: either a novel way of shocking capital returns – i.e. one which is capable of generating large adverse realizations, but does not reduce effective capital – or new mechanisms which limit incentives to invest following a tail event.

Another direction is an Bayesian approach to learning about tail risk. This would allow us to incorporate the risk of future belief changes into agents' current decisions, a channel we abstract from in our classical non-parametric approach. Bayesian parameter learning has been shown to improve asset pricing predictions in an endowment setting (Collin-Dufresne et al., 2016). Embedding this into a model with production could bring to light new implications for macro phenomena as well. We conjecture that tail events will have large, persistent effects even in a Bayesian setting, provided two conditions are satisfied. First, the specification is sufficiently flexible, e.g. one or more parameters governing tail risk. Second, the priors about these parameters reflects substantial uncertainty. Otherwise, there is not much scope for learning.

An open question is whether recent events reflect changes in beliefs or preferences. It is nearly impossible to disentangle the two with aggregate macroeconomic data. We took what we believe to be the most fruitful approach – hold preferences fixed and discipline beliefs with data. But, new approaches to preference formation could shed more light on this question and provide a deeper understanding on the role of tail events.

Finally, there are surely other surprising and extreme economic events. Agents who learn from those events might also exhibit persistent responses. Also, there were many outcomes in the great recession that were not extreme. Why did capital returns, in particular, reshape people's beliefs? Why didn't they focus on the less extreme GDP numbers instead? Pairing learning about tail events with limited attention might reveal that agents focused on investment returns because they are payoff relevant and because they were extreme, and thus highly-informative events.

5 Conclusion

Economists typically assume that agents in their models know the distribution of shocks. In this paper, we showed that relaxing this assumption has important implications for the response of the economy to tail events. The agents in our model behave like classical econometricians, re-estimating distributions as new data arrives. Under these conditions, observing a tail event like the 2008-09 Great Recession in the US, causes agents to assign larger weights to similar events in the future, depressing investment and output. Crucially, these effects last for a long time, even when the underlying shocks are transitory. The rarer the event that is observed, the larger and more persistent the revision in beliefs. The effects on economic activity are amplified when investments are financed with debt. This is because debt payoffs (and therefore, borrowing costs) are particularly sensitive to the probability of extreme negative outcomes.

When this mechanism is quantified using data for the US economy, the predictions of the model resemble observed macro and asset market outcomes in the wake of the Great Recession, suggesting that the persistent nature of the recent stagnation is due, at least partly, to the fact that the events of 2008-09 changed the way market participants think about tail risk.

References

Adrian, T. and N. Boyarchenko (2012). Intermediary leverage cycles and financial stability.

Angeletos, G.-M. and J. La'O (2013). Sentiments. *Econometrica 81*(2), 739–779.

Anzoategui, D., D. Comin, M. Gertler, and J. Martinez (2015). Endogenous technology adoption and r & d as sources of business cycle persistence. New York University working paper.

Backus, D. K., S. Foresi, and L. Wu (2008). Accounting for biases in black-scholes. *SSRN Working Paper Series*.

Bakshi, G., N. Kapadia, and D. Madan (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies 16*(1), 101–143.

Beaudry, P. and F. Portier (2004). An exploration into pigou's theory of cycles. *Journal of Monetary Economics 51*, 1183–1216.

Bigio, S. (2015). Endogenous liquidity and the business cycle. *American Economic Review*.

Bloom, N., M. Floetotto, N. Jaimovich, I. Sapora-Eksten, and S. Terry (2014). Really uncertain business cycles. NBER working paper 13385.

Brunnermeier, M. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review 104*(2).

Cho, I.-K., N. Williams, and T. J. Sargent (2002). Escaping nash inflation. *The Review of Economic Studies 69*(1), 1–40.

Cochrane, J. H. (2011). Presidential address: Discount rates. *The Journal of Finance 66*(4), 1047–1108.

Cogley, T. and T. Sargent (2005). The conquest of us inflation: Learning and robustness to model uncertainty. *Review of Economic Dynamics 8*, 528–563.

Collin-Dufresne, P., M. Johannes, and L. Lochstoer (2016). Parameter learning in general equilibrium: The asset pricing implications. *American Economic Review 106*(3), 664–98.

Cooley, T. and E. Prescott (1995). *Frontiers of Business Cycle Research*, Chapter Economic Growth and Business Cycles. Princeton University Press.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 289–309.

Eggertsson, G. and N. Mehrotra (2014). A model of secular stagnation. NBER Working Paper No. 20574.

Evans, G. W. and S. Honkapohja (2001). *Learning and expectations in macroeconomics*. Princeton University Press.

Fajgelbaum, P., E. Schaal, and M. Taschereau-Dumouchel (2014). Uncertainty traps. NBER Working Paper No. 19973.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. *Federal reserve bank of San Francisco working paper 19.*

Gertler, M. and P. Karadi (2011). A model of unconventional montary policy. *Journal of Monetary Economics 58*, 17–34.

Gertler, M., N. Kiyotaki, et al. (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of monetary economics 3*(3), 547–599.

Gilchrist, S. and E. Zakrajek (2012, June). Credit spreads and business cycle fluctuations. *American Economic Review 102*(4), 1692–1720.

Gomme, P., B. Ravikumar, and P. Rupert (2011). The return to capital and the business cycle. *Review of Economic Dynamics 14*(2), 262–278.

Gordon, R. (2014). The demise of u. s. economic growth: Restatement, rebuttal, and reflections. NBER working paper 19895.

Gourio, F. (2012). Disaster risk and business cycles. *American Economic Review 102(6)*, 2734–66.

Gourio, F. (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics 5(3)*, 1–34.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, 402–417.

Hall, R. (2015a). Macroeconomics of persistent slumps. *Handbook of Macroeconomics*.

Hall, R. E. (2015b, October). High discounts and high unemployment. Hoover Institution, Stanford University.

Hansen, B. E. (2015). *Econometrics.*

Hansen, L. (2007). Beliefs, doubts and learning: Valuing macroeconomic risk. *American Economic Review 97(2)*, 1–30.

Hansen, L. and T. Sargent (1999). *The Conquest of American Inflation.* Princeton University Press.

Huo, Z. and N. Takayama (2015). Higher order beliefs, confidence, and business cycles. University of Minnesota working paper.

Jaimovich, N. and S. Rebelo (2006). Can news about the future drive the business cycle? *American Economic Review 99(4)*, 1097–1118.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review 102*(1), 238–71.

Johannes, M., L. Lochstoer, and P. Collin-Dufresne (2015). The asset pricing implications of priced structural parameter uncertainty.

Johannes, M., L. Lochstoer, and Y. Mou (2015). Learning about consumption dynamics. *Journal of Finance forthcoming*.

Khan, A., T. Senga, and J. K. Thomas (2014). Default risk and aggregate fluctuations in an economy with production heterogeneity. Technical report, Ohio State University working paper.

Kozlowski, J., L. Veldkamp, and V. Venkateswaran (2018). The tail that keeps the riskless rate low. *National Bureau of Economic Research, Macroeconomics Annual forthcoming*.

Lorenzoni, G. (2009). A theory of demand shocks. *American Economic Review 99*(5), 2050–84.

Malmendier, U. and S. Nagel (2011). Depression babies: Do macroeconomic experiences affect risk taking?*. *The Quarterly Journal of Economics 126*(1), 373–416.

Merton, R. (1973). An intertemporal capital asset pricing model. *Econometrica 41*(5), 867–887.

Midrigan, V. and T. Philippon (2011). Household leverage and the recession.

Modigliani, F. and M. Miller (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review 48*(3), 261–297.

Moriera, A. and A. Savov (2015). The macroeconomics of shadow banking. New York University working paper.

Navarro, G. (2014). Financial crises and endogenous volatility. *manuscript, New York University*.

Nimark, K. (2014). Man-bites-dog business cycles. *American Economic Review 104(8)*, 2320–67.

Orlik, A. and L. Veldkamp (2014). Understanding uncertainty shocks and the role of the black swan. Working paper.

Pintus, P. and J. Suda (2015). Learning financial shocks and the great recession. Aix Marseille working paper.

Sargent, T. J. (2001). *The conquest of American inflation*. Princeton University Press.

Straub, L. and R. Ulbricht (2013). Credit crunches, information failures, and the persistence of pessimism. Toulouse School of Economics working paper.

Van Nieuwerburgh, S. and L. Veldkamp (2006). Learning asymmetries in real business cycles. *Journal of Monetary Economics 53(4)*, 753–772.

Veldkamp, L. and J. Wolfers (2007). Aggregate shocks or aggregate information? costly information and business cycle comovement. *Journal of Monetary Economics 54*, 37–55.

Zetlin-Jones, A. and A. Shourideh (2014). External financing and the role of financial frictions over the business cycle: Measurement and theory. Working paper.

A Appendix: Solution Method

The equilibrium is characterized by the following non-linear system:

$$1 + \chi \mathcal{W}_t \frac{L_{t+1}}{\hat{K}_{t+1}} = \mathbb{E}[M_{t+1} R^k_{t+1}] + (\chi - 1) lev_{t+1} q_t - (1-\theta)\mathbb{E}[M_{t+1} R^k_{t+1} h(\underline{v}_{t+1})] \tag{16}$$

$$\chi \mathcal{W}_t = \mathbb{E} M_{t+1} \left[(1-\alpha)\, \phi^\alpha \left(\frac{\hat{K}_{t+1}}{L_{t+1}} \right)^\alpha \right] J^l(\underline{v}_{t+1}) \tag{17}$$

$$(1-\theta)\, \mathbb{E}_t \left[M_{t+1} \underline{v}_{t+1} f\left(\underline{v}_{t+1}\right) \right] = \left(\frac{\chi - 1}{\chi} \right) \mathbb{E}_t \left[M_{t+1} \left(1 - F\left(\underline{v}_{t+1}\right) \right) \right] \tag{18}$$

$$M_{t+1} = \beta \left[\mathbb{E}\left(U_{t+1}^{1-\eta} \right) \right]^{\frac{\eta-\psi}{1-\eta}} U_{t+1}^{\psi-\eta} \left(\frac{u\left(C_{t+1}, L_{t+1}\right)}{u\left(C_t, L_t\right)} \right)^{-\psi} \tag{19}$$

$$\mathcal{W}_t = L^\gamma_{t+1} \mathbb{E} M_{t+1} \tag{20}$$

where

$$C_t = \phi_t^\alpha \hat{K}_t^\alpha L_t^{1-\alpha} + (1-\delta)\, \phi_t \hat{K}_t - \hat{K}_{t+1} \tag{21}$$

$$U_t = \left[(1-\beta) \left(u\left(C_t, L_t\right) \right)^{1-\psi} + \beta \mathbb{E} \left(U_{t+1}^{1-\eta} \right)^{\frac{1-\psi}{1-\eta}} \right]^{\frac{1}{1-\psi}} \tag{22}$$

$$R^k_{t+1} = \frac{\phi^\alpha_{t+1} \hat{K}^\alpha_{t+1} L^{1-\alpha}_{t+1} + (1-\delta)\, \phi_{t+1} \hat{K}_{t+1}}{\hat{K}_{t+1}} \tag{23}$$

$$\underline{v}_{t+1} = \frac{lev_{t+1}}{\phi^\alpha_{t+1} \left(\frac{\hat{K}_{t+1}}{L_{t+1}} \right)^{\alpha-1} + (1-\delta)\, \phi_{t+1}} \tag{24}$$

$$J^l(\underline{v}) = 1 + \underline{v}^2 f\left(\underline{v}\right) \chi \left(1-\theta\right) - \left(1 - \chi\theta\right) h\left(\underline{v}\right) \tag{25}$$

Solution Algorithm To solve the system described above at any given date t (i.e. after any observed history of ϕ_t), we recast it in recursive form with grids for the aggregate state (Π, L) and the shocks ϕ. We then use the following iterative procedure:

- Estimate $\hat{g}$ on the available history using the kernel density estimator.
- Start with a guess (in polynomial form) for $U(\Pi, L), C(\Pi, L)$.
- Solve (16)-(18) for $\hat{K}'(\Pi, L), L'(\Pi, L), lev'(\Pi, L)$ using a non-linear solver.
- Update the guess for U, C using (21)-(22) and iterate until convergence.

Online Appendix

This material is for a separate, on-line appendix and not intended to be printed with the paper.

B Model solution and derivations

Define

$$R^k_{it+1} \equiv {\phi_{t+1}}^{\alpha} \left(\frac{l_{it+1}}{\hat{k}_{it+1}}\right)^{1-\alpha} + (1-\delta)\phi_{t+1}.$$

Substituting for dividends and wages from (5) and (6), the firm's contiuation value can be expressed as the solution to the following maximization problem:

$$\Gamma_{it} = \max_{\hat{k}_{it+1},\ lev_{it+1}, \frac{l_{it+1}}{\hat{k}_{it+1}}} \hat{k}_{it+1} \left(-1 - \chi \mathcal{W}_t \frac{l_{it+1}}{\hat{k}_{it+1}} + \chi q_{it} lev_{it+1} \ + \mathbb{E} M_{t+1} r_{it+1} \left(v_{it} R^k_{it+1} - lev_{it+1} + \frac{\Gamma_{it+1}}{\hat{k}_{it+1}}\right)\right)$$

where

$$q_{it} = \mathbb{E} M_{t+1} \left[r_{it+1} + (1 - r_{it+1})\, \theta \frac{v_{it+i} R^k_{it+1} + \frac{\Gamma_{it+1}}{\hat{k}_{it+1}}}{lev_{it+1}} \right].$$

Leverage lev_{it+1} includes debt and the wage promises made to workers. However, wage promises (or operating leverage) is different from debt, in that it does not earn a tax advantage. Since the above formulation credits the firm with tax advantage χ on all leverage, the wage obligation $\mathcal{W}_t$ ends up being multiplied by χ, i.e. as if the firm pays back the tax advantage from labor payments. The net effect is that only external debt ends up accruing the advantage.

We guess (and later verify) that $\Gamma_{it+1} = 0$.[41] Using the threshold characterization of the default decision,

$$\begin{aligned} \Gamma_{it} &= \max \ \hat{k}_{it+1} \left(-1 - \chi \mathcal{W}_t \frac{l_{it+1}}{\hat{k}_{it+1}} + \chi q_{it} lev_{it+1} \ + \mathbb{E} M_{t+1} \int_{\underline{v}_{it+1}}^{\infty} \left(v R^k_{it+1} - lev_{it+1}\right) dF(v)\right) \\ &= \max \ \hat{k}_{it+1} \left(-1 - \chi \mathcal{W}_t \frac{l_{it+1}}{\hat{k}_{it+1}} + \chi q_{it} lev_{it+1} \ + \mathbb{E} M_{t+1} \left[\left(1 - h\left(\underline{v}_{it+1}\right)\right) R^k_{it+1} - lev_{it+1}\left(1 - F\left(\underline{v}_{it+1}\right)\right)\right]\right) \end{aligned}$$

where

$$\begin{aligned} q_{it} &= \mathbb{E} M_{t+1} \left[1 - F\left(\underline{v}_{it+1}\right) + h\left(\underline{v}_{it+1}\right) \theta \frac{R^k_{it+1}}{lev_{it+1}}\right] \\ \underline{v}_{it+1} &= \frac{lev_{it+1}}{R^k_{it+1}} \end{aligned}$$

Capital choice: Note that q_{it} is only a function of lev_{it+1} and R^k_{it+1} (which only depends on the labor-capital ratio $\frac{l_{it+1}}{\hat{k}_{it+1}}$). As a result, the objective function is linear in $\hat{k}_{it+1}$. At an

[41] Intuitively, given constant returns to scale, the firm's problem turns out to be linear in capital. In equilibrium, therefore, in order for the firm's value to be bounded, we must have $\Gamma_{it} = 0$. See Navarro (2014).

interior optimum, we must have:

$$\begin{aligned}
1+\chi\mathcal{W}_t\frac{l_{it+1}}{\hat{k}_{it+1}} &= \chi q_{it}lev_{it+1} \ +\mathbb{E}M_{t+1}\left[\left(1-h\left(\underline{v}_{it+1}\right)\right)R^k_{it+1}-lev_{it+1}\left(1-F\left(\underline{v}_{it+1}\right)\right)\right] \\
&= \mathbb{E}M_{t+1}R^k_{it+1}+\chi q_{it}lev_{it+1}-\mathbb{E}M_{t+1}\left[h\left(\underline{v}_{it+1}\right)R^k_{it+1}+lev_{it+1}\left(1-F\left(\underline{v}_{it+1}\right)\right)\right] \\
&= \mathbb{E}M_{t+1}R^k_{it+1}+\chi q_{it}lev_{it+1}-lev_{it+1}\mathbb{E}M_{t+1}\left[h\left(\underline{v}_{it+1}\right)\frac{R^k_{it+1}}{lev_{it+1}}+\left(1-F\left(\underline{v}_{it+1}\right)\right)\right] \\
&= \mathbb{E}M_{t+1}R^k_{it+1}+\chi q_{it}lev_{it+1}-lev_{it+1}\left[q_{it}+(1-\theta)\,\mathbb{E}M_{t+1}h\left(\underline{v}_{it+1}\right)\frac{R^k_{it+1}}{lev_{it+1}}\right] \\
&= \mathbb{E}M_{t+1}R^k_{it+1}+(\chi-1)\,q_{it}lev_{it+1}-(1-\theta)\,\mathbb{E}M_{t+1}h\left(\underline{v}_{it+1}\right)R^k_{it+1}\ . \qquad (26)
\end{aligned}$$

Note that this verifies our guess that $\Gamma_{it+1}=0$. Labor and leverage choice are the solutions to

$$\max_{lev_{it+1},\frac{l_{it+1}}{\hat{k}_{it+1}}} -1-\chi\mathcal{W}_t\frac{l_{it+1}}{\hat{k}_{it+1}}+\chi q_{it}lev_{it+1} \ +\mathbb{E}M_{t+1}\left[\left(1-h\left(\underline{v}_{it+1}\right)\right)R^k_{t+1}-lev_{it+1}\left(1-F\left(\underline{v}_{it+1}\right)\right)\right]$$

which, after substituting for q_{it}, becomes

$$\max_{lev_{it+1},\frac{l_{it+1}}{\hat{k}_{it+1}}} \qquad -1-\chi\mathcal{W}_t\frac{l_{it+1}}{\hat{k}_{it+1}}+\mathbb{E}M_{t+1}R^k_{it+1}J^k(\underline{v}_{it+1})$$

$$\text{where}\quad J^k(\underline{v})=1+(\chi-1)\,\underline{v}\,(1-F\left(\underline{v}\right))+(\chi\theta-1)\,h\left(\underline{v}\right)$$

Labor choice: The first order condition with respect to $\frac{l_{it+1}}{\hat{k}_{it+1}}$ is

$$\chi\mathcal{W}_t=\mathbb{E}M_{t+1}R^k_{it+1}\frac{\partial J^k(\underline{v}_{it+1})}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}}+\mathbb{E}M_{t+1}\frac{\partial R^k}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}}J^k(\underline{v}_{it+1}),$$

Now,

$$\begin{aligned}
R^k_{it+1}\frac{\partial J^k(\underline{v}_{it+1})}{\partial\frac{l_{t+1}}{\hat{k}_{it+1}}} &= R^k_{it+1}\frac{\partial\underline{v}_{it+1}}{\partial\frac{l_{t+1}}{\hat{k}_{it+1}}}\left((\chi-1)\left(1-F\left(\underline{v}_{it+1}\right)\right)-\underline{v}_{it+1}\,(\chi-1)\,f\left(\underline{v}_{it+1}\right)+(\chi\theta-1)\,\frac{\partial h\left(\underline{v}_{it+1}\right)}{\partial\underline{v}_{it+1}}\right) \\
\frac{\partial\underline{v}_{it+1}}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}} &= -\frac{lev_{it+1}}{\left(R^k_{it+1}\right)^2}\frac{\partial R^k_{it+1}}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}}=-\frac{\underline{v}^2_{it+1}}{lev_{it+1}}\frac{\partial R^k_{it+1}}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}} \\
\frac{dh\left(\underline{v}_{it+1}\right)}{d\underline{v}_{it+1}} &= \underline{v}_{it+1}f\left(\underline{v}_{it+1}\right) \\
\frac{\partial R^k_{it+1}}{\partial\frac{l_{it+1}}{\hat{k}_{it+1}}} &= (1-\alpha)\,{\phi_{t+1}}^{\alpha}\left(\frac{\hat{k}_{it+1}}{l_{it+1}}\right)^{\alpha}.
\end{aligned}$$

Substituting and rearranging terms yields

$$\chi \mathcal{W}_t = \mathbb{E}\left[M_{t+1}\left(1-\alpha\right)\phi_{t+1}{}^{\alpha}\left(\frac{\hat{k}_{it+1}}{l_{it+1}}\right)^{\alpha} J^l(\underline{v}_{it+1})\right] \tag{27}$$
$$J^l(\underline{v}) = 1 + h\left(\underline{v}\right)\left(\chi\theta - 1\right) - \underline{v}^2 f\left(\underline{v}\right)\chi\left(\theta-1\right),$$

Leverage choice: The first order condition with respect to leverage is

$$\mathbb{E}M_{t+1}R^k_{it+1}\frac{\partial J^k(\underline{v}_{it+1})}{\partial lev_{it+1}} = 0,$$

where

$$\begin{aligned}\frac{\partial J^k(\underline{v}_{it+1})}{\partial lev_{it+1}} &= \frac{\partial \underline{v}_{it+1}}{\partial lev_{it+1}}\left(\left(\chi-1\right)\left(1-F\left(\underline{v}_{it+1}\right)\right) - \left(\chi-1\right)\underline{v}_{it+1}f\left(\underline{v}_{it+1}\right) + \left(\chi\theta-1\right)\underline{v}_{it+1}f\left(\underline{v}_{it+1}\right)\right)\\ &= \frac{1}{R^k_{it+1}}\left(\left(\chi-1\right)\left(1-F\left(\underline{v}_{it+1}\right)\right) - \chi\left(1-\theta\right)\underline{v}_{it+1}f\left(\underline{v}_{it+1}\right)\right)\ .\end{aligned}$$

Substituting and re-arranging,

$$\left(1-\theta\right)\mathbb{E}_t\left[M_{t+1}\underline{v}_{it+1}f\left(\underline{v}_{it+1}\right)\right] = \left(\frac{\chi-1}{\chi}\right)\mathbb{E}_t\left[M_{t+1}\left(1-F\left(\underline{v}_{it+1}\right)\right)\right]. \tag{28}$$

Finally, since all firms make symmetric choices, we can suppress the i subscript, so

$$\hat{k}_{it+1} = \hat{K}_{t+1} \qquad l_{it+1} = L_{t+1} \qquad lev_{it+1} = lev_{t+1} \qquad \underline{v}_{it+1} = \underline{v}_{t+1}\ .$$

Using this, equations (26) – (28) become (11) – (14) in the main text.

C Additional Results

C.1 Measurement of ϕ_t: Alternative price indices

Figure 10 shows that the measurement of capital quality shocks is unaffected when we use the price index for GDP or Personal Consumption Expenditure to control for nominal price changes.

C.2 Numerical accuracy of solution method

To test the numerical accuracy of our solution method, we perform the following exercise. Starting from the steady state of $\hat{g}_{2007}$, we simulate time paths for two different economies. In

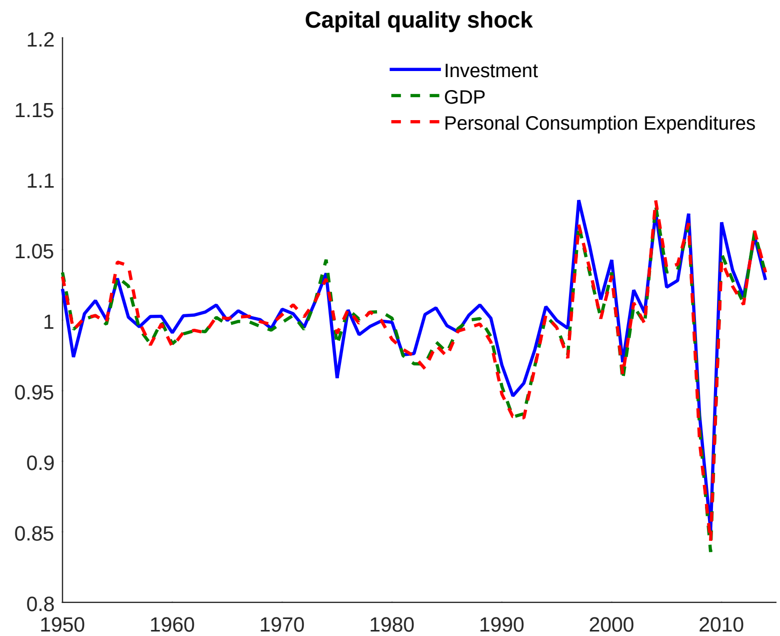


Figure 10: **Time series of shocks ϕ_t using different indices to control for nominal price changes.**

Model I, as new data arrives, we update beliefs and policy functions at each date and history. In Model II, beliefs and policy functions are fixed at $\hat{g}_{2007}$. In our solution, we essentially assume that agents use Model II as an approximation for Model I, while evaluating continuation values. Table 5 shows the sample mean and coefficient of variation for output at different horizons for these two versions.[42] It is easy to see that aggregates (or at least, the first two moments thereof) are very well-approximated by replacing the sequence of future distributions with their conditional mean. Recall that this numerical procedure works reasonable well thanks to the martingale property of beliefs.

C.3 Effect of 2010-2014 shocks

Here, we subject our baseline calibrated model to the full sequence of shocks, from 2008 through 2014. Agents' decisions in each year are a function of the estimated distribution at that date. The resulting time paths are plotted in Figure 11, along with the de-trended data. We note that the patterns implied by the model are quite close to the observed ones.

[42] These are averages over 4000 paths. Other aggregate variables, e.g. capital and labor, show similar patterns.

	Horizon			
	$s = 1$	$s = 5$	$s = 10$	$s = 15$
$\mathbb{E}_t\left[y_{t+s}\right]$				
Model I:	6.378	6.385	6.393	6.397
Model II:	6.378	6.385	6.394	6.398
$\mathbb{CV}_t\left[y_{t+s}\right]$				
Model I:	0.010	0.032	0.042	0.046
Model II:	0.010	0.031	0.040	0.044

Table 5: **Numerical accuarcy.**

The rows labeled Model I show the actual moments under the assumption that beliefs $\hat{g}_{2007+s}$ are re-estimated at each date. Model II corresponds to the assumption underlying our solution method, where future beliefs are replaced by $\hat{g}_{2007}$.

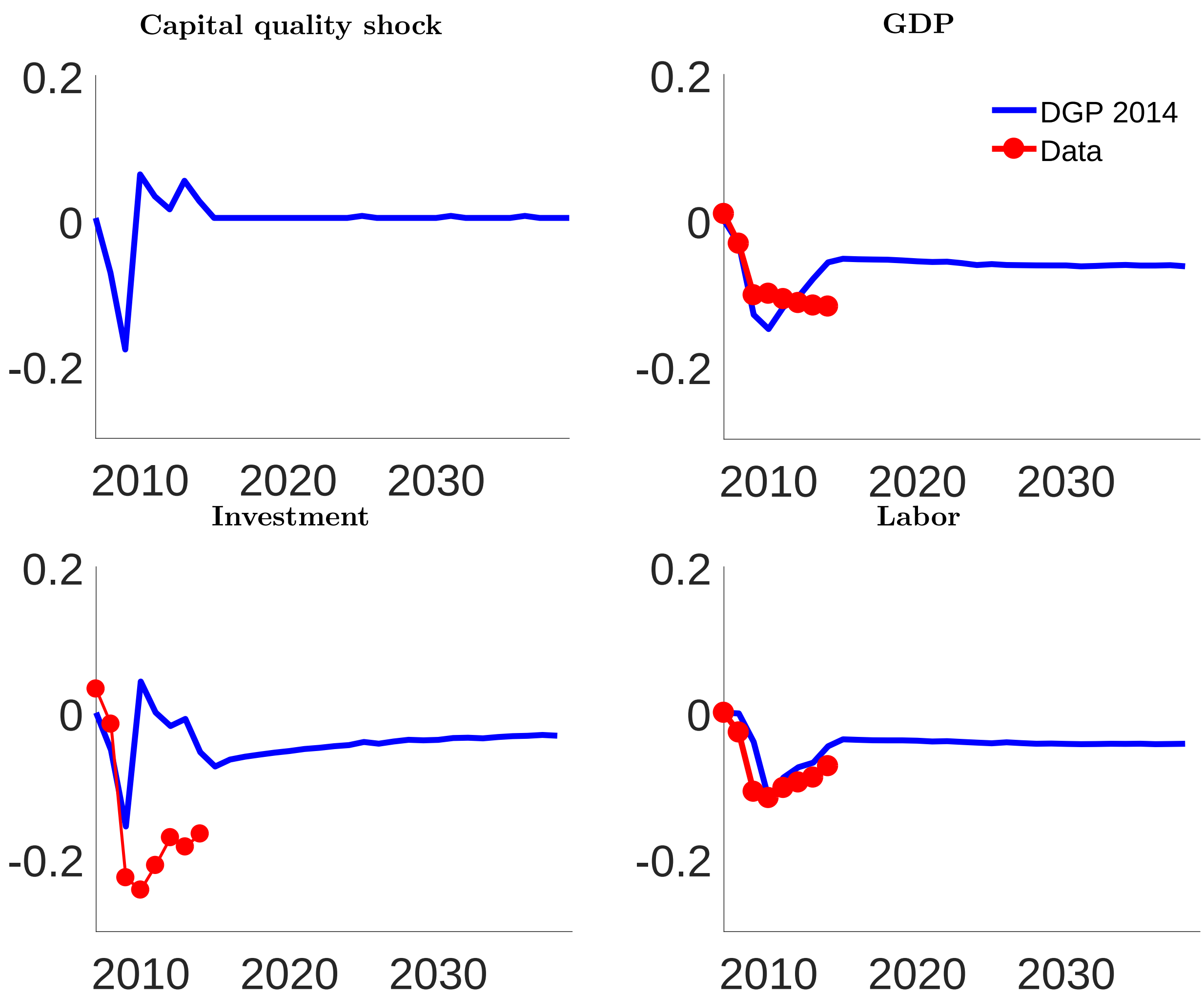


Figure 11: **Macro Aggregates: Model vs data, 2008-2014.**

Solid line is the baseline model subjected to the observed sequence of shocks from 2008-2014. The red circles are US data, in deviations from their pre-crisis trends.

C.4 Steady State Analysis

To dig a little deeper into why long-run outcomes are so sensitive to ϕ, we turn to a special case - a deterministic version of our economy without debt. The level of steady state capital is given by the following equation[43]

$$\ln k_{ss} = \text{Const.} + \left(\frac{1+\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right)\ln\phi_{ss} - \left(\frac{1}{1-\alpha}\frac{\alpha+\gamma}{\gamma}\right)\ln\left(\frac{1}{\beta} - (1-\delta)\,\phi_{ss}\right). \tag{29}$$

Hence, the effect of the mean shock on steady sate capital is given by

$$\frac{d\ln k_{ss}}{d\ln\phi_{ss}} = \left(\frac{1+\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right) + \left(\frac{1}{1-\alpha}\frac{\alpha+\gamma}{\gamma}\right)\frac{(1-\delta)}{1/\beta-(1-\delta)\phi_{ss}}.$$

Under our parameterization,

$$\frac{1+\gamma}{\gamma}\frac{\alpha}{1-\alpha} = 2, \qquad \frac{1}{1-\alpha}\frac{\alpha+\gamma}{\gamma} = 3, \qquad \left(\frac{(1-\delta)}{1/\beta-(1-\delta)\phi_{ss}}\right)_{\phi_{ss}=1} = 7.5$$

which implies $\frac{d\ln k_{ss}}{d\ln\phi_{ss}} = 2 + 3(7.5) = 24.5$. This simple calculation shows the source of the high sensitivity - the fact that capital quality shock affects not just the current return component but also the portion that comes from the undepreciated stock.

[43]In steady state, $M_t = 1$ and the intertemporal Euler equation and labor optimality conditions reduce to

$$\begin{aligned} 1 &= \beta\left(\alpha\phi_{ss}^{\alpha}k_{ss}^{\alpha-1}l_{ss}^{1-\alpha} + \phi_{ss}(1-\delta)\right) \\ l_{ss}^{\gamma} &= W_{ss} = (1-\alpha)\,\phi_{ss}^{\alpha}k_{ss}^{\alpha}l_{ss}^{-\alpha}. \end{aligned}$$

Substituting for l_{ss} from the second into the first and re-arranging yields the expression (29).

C.5 Behavior of Consumption

Figure 12 shows that the behavior of consumption, as predicted by the model and the corresponding pattern in the data. The model overpredicts the drop in consumption in the years immediately following impact – the flip side of its inability to match the full extent of the drop in investment during that time – but over a longer horizon, the predicted drop lines up quite well with the data.

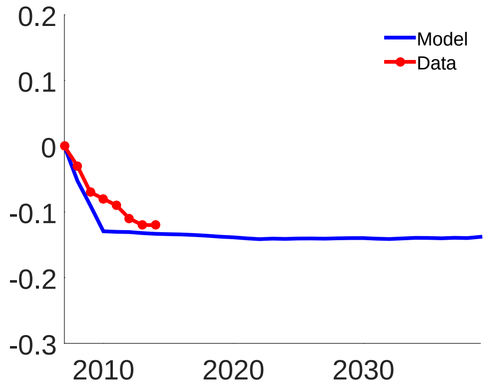


Figure 12: **Response of consumption.**

C.6 What if there are no more crises?

The bottom panel of Table 6 reports the change in GDP under the assumption that a crisis never occurs again, i.e. future time paths are drawn from $\hat{g}_{2007}$, under various assumptions about learning and the presence of debt. For comparison, the middle panel reproduces the corresponding numbers for the baseline version (where time paths are drawn $\hat{g}_{2009}$). Both cases are remarkably similar over a 30 year horizon, underscoring the persistent nature of belief revisions, even in the absence of crises.

	2014	2039
Data	-0.12	
Benchmark: Draws from $\hat{g}_{2009}$		
Learning, debt	-0.12	-0.12
Learning, no debt	-0.11	-0.09
No learning, debt	-0.9	0.00
No learning, no debt	-0.08	0.00
No more crises: Draws from $\hat{g}_{2007}$		
Learning, debt	-0.12	-0.10
Learning, no debt	-0.10	-0.06
No learning, debt	-0.08	0.00
No learning, no debt	-0.07	0.00

Table 6: Changes in GDP (relative to 2007 steady state)

C.7 Computing option-implied tail probabilities

To compute tail probabilities, we follow Backus et al. (2008) and use a Gram-Charlier expansion of the distribution function.[44] This yields an approximate density function for the standardized random variable, $\omega = \frac{x-\mu}{\sigma}$:

$$f(\omega) = \varphi(\omega)\left[1 - \gamma\frac{(3\omega - \omega^3)}{6}\right] \qquad \text{where} \qquad \gamma = E\left[\frac{x-\mu}{\sigma}\right]^3$$

where $\varphi(\omega)$ is the standard normal density and γ is the skewness.[45] The VIX and the SKEW indices provide the standard deviation and the skewness of the implied risk-neutral distribution of the returns on the S&P 500. The numbers reported for tail probabilities in Table 3 are computed using this distribution.

[44] The CBOE also follows this method in their white paper on the SKEW Index to compute implied probabilities.

[45] The Gram-Charlier expansion also includes a term for the excess kurtosis, but is omitted from the expansion because, as shown by Bakshi et al. (2003), it is empirically not significant.

C.8 Role of Risk Aversion, Intertemporal Elasticity of Substitution

Parameters	Epstein-Zin			CRRA	
Risk Aversion (η)	10	10	5	0.5	2
IES ($1/\psi$)	2	1	2	2	0.5
Change in					
GDP	-0.12	-0.09	-0.09	-0.08	-0.07
Labor	-0.18	-0.15	-0.15	-0.12	-0.11
Investment	-0.06	-0.03	-0.03	-0.01	0.01

Table 7: Role of risk aversion and intertemporal elasticity of substitution. The second panel reports the difference in the value of the variable in the new (post-crisis) and old (pre-crisis) stochastic steady states.

Risk aversion, IES and debt all play a role in determining the magnitude of the effects of increased tail risk. In order to show how much, here we compare our baseline results to a number of alternative parameterizations/assumptions. The results for the role of recursive preferences and assumptions are collected here in Table 7. The first column reproduces our benchmark results, which sets risk aversion = 10 and $IES = 2$. The next two columns vary, respectively, risk aversion holding IES constant and IES holding risk aversion constant. The last 2 columns show results under CRRA utility, with a risk aversion coefficient of 2 and 0.5 respectively.

Our estimate for the IES is drawn from the macro and asset-pricing literature – see, e.g., Bansal and Yaron (2004), Barro (2009), Baron et. al. (2014). In order to assess the robustness of our results to this parameter, we ran the model with an IES of 1 – the results are presented in Column 2. Under this parameterization, the model predicts a slightly lower, but importantly just as persistent, drop in GDP (9% vs 12% in the benchmark). This is due to a precautionary channel – agents dislike intertemporal fluctuations in consumption, so faced with the increased likelihood of a tail event, they have an incentive to hold *more* capital to mitigate the potential consumption drop. This channel is stronger, the lower is IES. In fact, as the IES approaches 0, this channel becomes so powerful that it can overwhelm the disincentives to invest and can lead agents to *increase* investment in response to higher tail risk. However, in the region that the macro and asset-pricing literature typically focuses on, the effects of varying IES are relatively modest.

Analogously, column 3 reveals that the size of the drop in economic activity from increased tail risk is lower when agents are less risk averse. This is intuitive – the extent to which agents dislike the increased riskiness of investment depends on their aversion to risk. However, as with IES, the magnitude of our effects is not particularly sensitive to this parameter.

The previous two exercises show that the magnitude of effects of increased tail risk on the macro economy are increasing, albeit modestly, in both risk aversion and IES. Under CRRA utility, of course, the two are tightly (and negatively) linked – a high risk aversion necessarily implies a low IES and vice-versa. For example, in Column 4 of Table 7, we show results for a CRRA specification with the same IES as the benchmark parameterization. However, this now comes with a much lower risk aversion (0.5 vs 10), which attenuates the long-run drop in GDP (from 11% to 8%). Finally, Column 5 shows results for a CRRA specification with an IES of 0.5 (or equivalently, risk aversion of 2). Now, GDP in the new steady state is lower by about 7%.

C.9 Role of GHH preferences

The GHH specification of utility has criticized as being inconsistent with the facts on long run growth, specifically the observation that labor input is more or less constant (or maybe, slightly declining) in most advanced economies. One resolution is the following specification proposed by Jaimovich and Rebelo (2006):

$$u(C_t, L_t) = C_t - X_t \frac{L_t^{1+\gamma}}{1+\gamma} \qquad\qquad X_t = X_{t-1}^{1-\varrho} C_t^{\varrho}$$

Now, on the balanced growth path, the state variable X_t grows at the same rate as wages, ensuring labor stays constant. The parameter ϱ governs the strength of wealth effects on labor supply away from the long run. The lower value of ϱ, the closer the behavior of the economy is to the GHH specification in the short-to-medium run. In their baseline calibration, Jaimovich and Rebelo use $\varrho = 0.001$ at a quarterly frequency.

Solving this version of our model with learning involves an additional state variable and considerable computational complexity. However, a simple back-of-the-envelope calculation suggests that the drop in GDP and consumption over a 30 year horizon would only be slightly lower than our baseline (GHH) specification (about 10% instead of 12%). To see why, a 10% drop in consumption, along with $\varrho = 0.001$, implies a change in X_t over 30 years of approximately $0.1(1 - 0.999^{120}) = 0.011$. Assuming that wages change by about the same as in the baseline, the optimality conditions for labor and capital imply that the drops in L_t and K_t are about 2% lower than under GHH (6% instead of 8% and 15% instead of 17%, respectively), which are consistent with the conjectured 10% drop in GDP and consumption. Over shorter horizons, e.g. the 7 years or so for which we actually have data, the two specifications would be virtually indistinguishable. [46]

[46] As an additional robustness exercise, we repeated the steady-state exercise in Appendix C.4 with Cobb-Douglas preferences: $u(C_t, L_t) = C_t^{\varkappa}(1 - L_t)^{1-\varkappa}$. The responsiveness of capital and output to a change in the steady-state level of ϕ is about 70% of the elasticity in the baseline case. In other words, even with wealth effects on labor supply, the effects of increased tail risk in the long run are quite significant.

C.10 Exogenous persistence in ϕ_t

In this section, we show that the observed degree of persistence in the data is just not enough to explain the prolonged stagnation since 2008-'09 in the absence of learning. To do this, we solved a rational expectations (i.e. no learning) version of our model where the ϕ_t shocks are no longer iid, but follow an AR(1) process (computationally, this requires an additional state variable). Recall from Section 3 that the autocorrelation of the observed ϕ_t series was 0.15. In Figure 13, we plot the impulse responses from the large negative realizations observed during 2008-09 in this version of the model, with persistence set to 0.15 . As the graph shows, the implications are quite similar to the iid, no-learning case – investment surges and the economy slowly but steadily recovers back to the pre-crisis level. Even if we used a shock process that was twice as persistent ($\rho = 0.30$) as the data, the results do not change significantly, as we see in Figure 14. From these results, it seems reasonable to conclude that persistence of the shock itself is an unlikely explanation for the last 8 years.

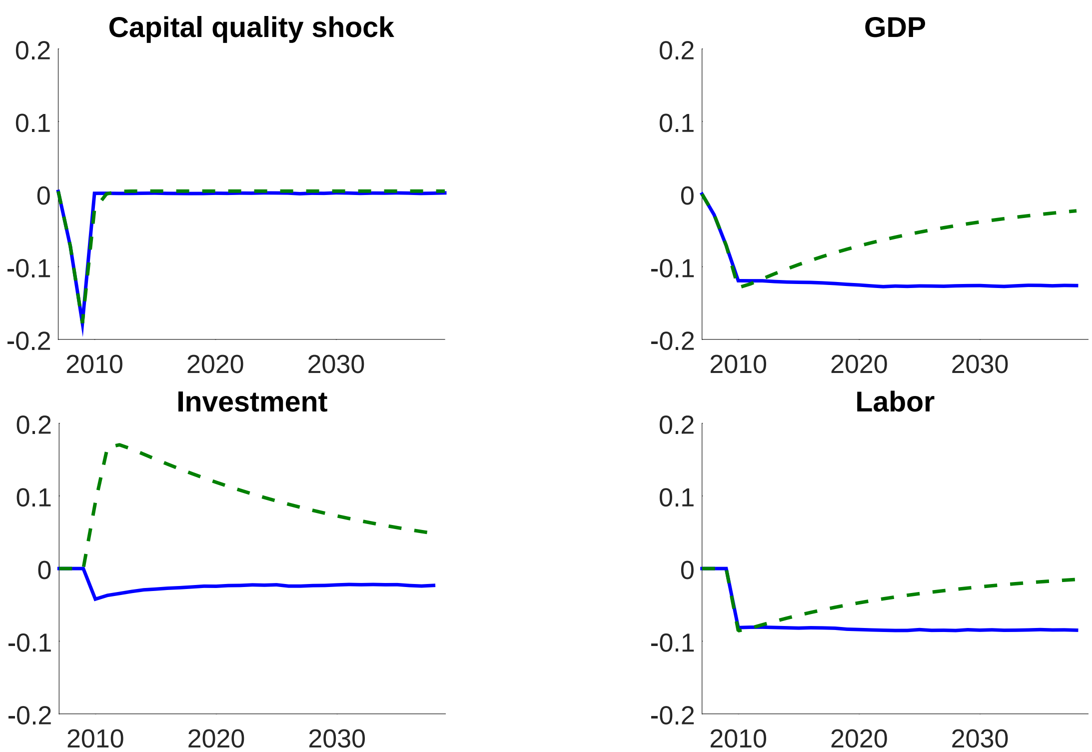


Figure 13: No learning model, with persistent shocks (dashed line, $\rho = 0.15$) vs. learning model with iid shocks.

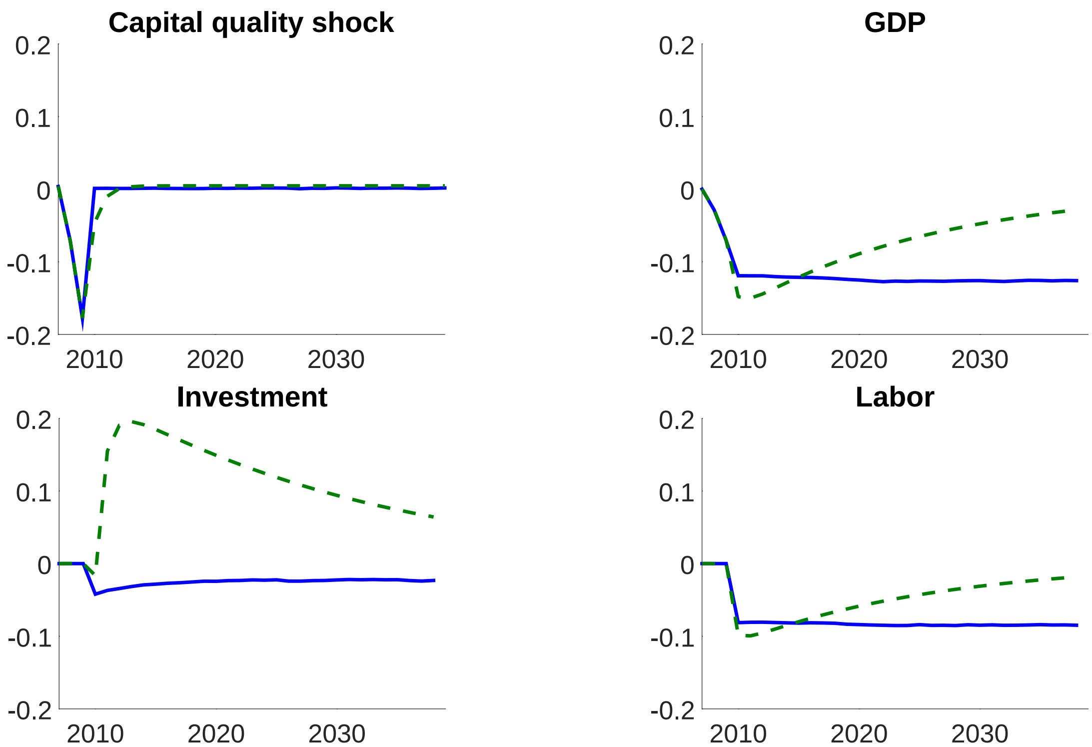


Figure 14: No learning model, with 2× estimated persistence (dashed line, $\rho = 0.30$) vs. learning model with iid shocks (solid line).

C.11 Learning with a Normal distribution

Here, we repeat our analysis under the assumption that agents fit a normal distribution to the available data. The resulting beliefs revisions are shown in the second panel of figure 15 (the first panel reproduces the baseline kernel density estimates). The large, negative tail realizations in 2008-09 lowers the mean and increases the variance of the estimated normal distribution. Qualitatively, these belief revisions are also long-lived, for the same reason as those under the kernel density estimation. The economic implications are also sizable and similar to our baseline, especially in the short run. This is partly the result of the direct impact of the shock itself and partly from the fact that changes in the first two moments have an substantial effect in this highly non-linear setting.

However, the two procedures imply different time paths for beliefs and economic activity. This is seen most clearly in the exercise where we simulate the economy by drawing time paths from the pre-crisis distribution. The third panel compares the average path for GDP when agents estimate a lognormal distribution to the baseline (kernel density) case. The graph shows faster recovery for macro variables under the former. This is because realizations anywhere in the support contain information about the mean and variance of the normal distribution. The kernel estimate of the distribution at a particular point in the support, on the other hand, places relatively more weight on the observed history close to it, making learning more 'local'. The non-parametric procedure captures the idea that tail events are harder to learn about, because they are, by definition, rare. Imposing a parametric form on the distribution essentially allows the agent to learn about the probability of disasters from more normal times, and therefore, ties learning about tail risk much more closely to learning about the rest of the distribution. Obviously, if the parametric form of the distribution was known, this is the efficient thing to do, but this exercise illustrates how the assumption can have a significant effect.

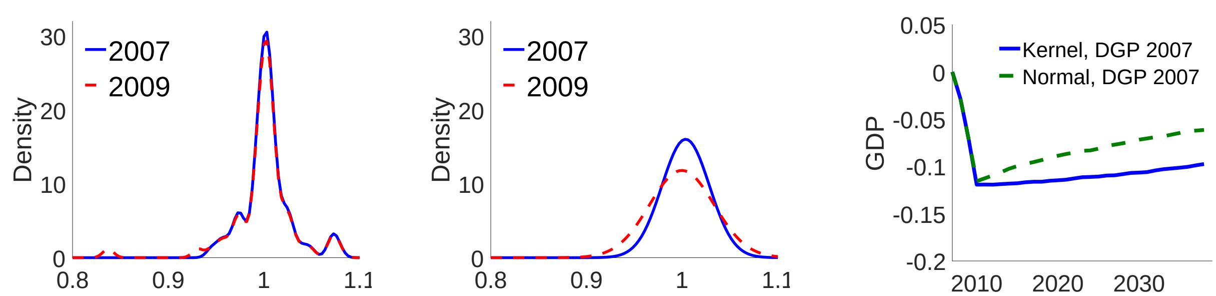


Figure 15: **Learning with a Normal distribution.**
Beliefs under our baseline non-parametric proecudure (first panel) and assuming a normal distribution (second panel). The third panel shows the exercise where we simulate the economy by drawing time paths from the pre-crisis distribution.

C.12 Alternative kernels

We estimated our belief process using alternative kernel densities. In the benchmark solution $\Omega(\cdot)$ is the normal density. This appendix consider another two common kernel densities: (i) the Epanechnikov, $\Omega(x) = \frac{3}{4}(1 - x^2)$, and (ii) the box or uniform density, $\Omega(x) = \frac{1}{2}$.[47] Figure 16 shows that these approaches yielded similar changes in tail probabilities and therefore, similar predictions for economic outcomes. A Bayesian approach is conceptually similar – posterior beliefs exhibit the martingale property, the key source of persistence. However, the departure from normality needed to capture tail risk, requires particle filtering techniques, making it difficult to integrate it into any but the simplest economic environments. For a detailed discussion of nonparametric estimation, see Hansen (2015).

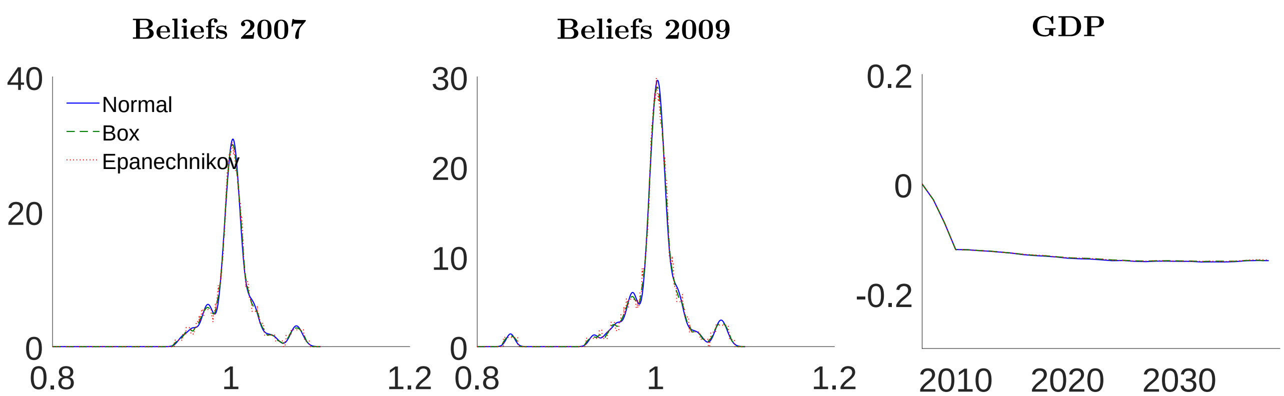


Figure 16: **Alternative Kernel densities**

The solution of the model is very similar under normal, box or epanechnikov kernel densities.

[47] We find similar results under further additional distributions: (i) the Champernowne transformation (which is designed to better capture tail risk), (ii) semi-parametric estimators, e.g. with Pareto tails and (iii) the g-and-h family of distributions which allows for a flexible specification of tail risk using various transformations of the normal distribution.

C.13 Tail Shocks vs Beliefs

The time path of aggregate variables in our baseline model – e.g. in Figure 4 – reflect the combined effects of the tail realizations in 2008-09 as well as the belief changes they induce. Here, we perform an exercise intended to highlight the role of belief changes. A large fraction of the persistence response is due to changes in beliefs rather than the fact that the shock hits the actual capital In Figure 17, we plot the time path of GDP under the assumption that beliefs (and therefore, policy functions) jump in 2009 to $\hat{g}_{2009}$ from $\hat{g}_{2007}$ but the realizations in 2008 and 2009 are not tail events (instead, we simulate shocks for those two years by drawing from $\hat{g}_{2009}$ and average over time paths). As a result, output does not drop immediately but the economy steadily reduces its capital (by investing less) and converges to the new steady state. Importantly, even without the large negative shocks, we obtain a significant and persistent drop in economic activity, underscoring the key role played by belief changes in driving our results.

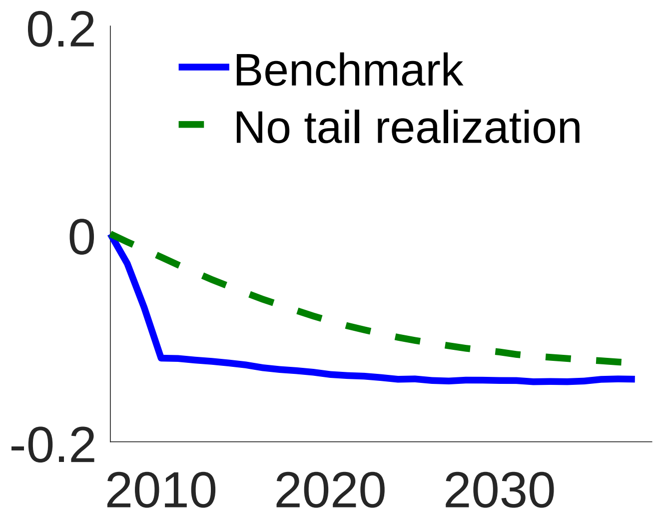


Figure 17: **Decomposition: Beliefs vs Shocks**
Response of GDP under the counterfactual in which beliefs change in 2009 but there is no actual realization of tail events ϕ_t that hit the capital stock.

C.14 Asymmetry: Right vs left tail events

Figure 18 compares the response of the economy after left and right tail events (i.e., $\phi_{2008} = 1.07$ and $\phi_{2009} = 1.16$, instead of $\phi_{2008} = 0.93$ and $\phi_{2009} = 0.84$). The long-run changes are smaller when the economy is hit by a positive tail event than under negative ones. This asymmetry is the result of non-linearities in the model – from risk aversion and the presence of debt.

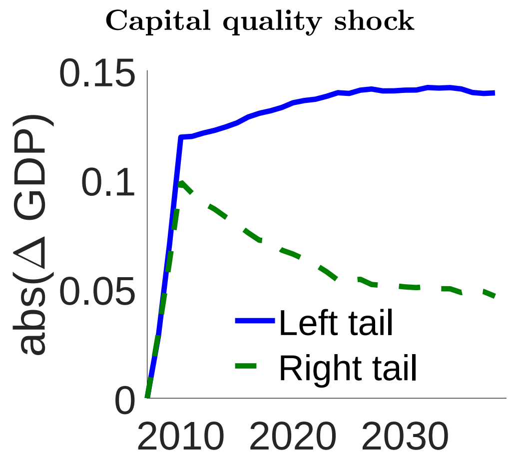


Figure 18: **Response of GDP to left and right tail shocks.**
Solid (dashed) line shows the absolute change in GDP after a left (right) tail event.

C.15 Response of asset prices to 2010-2014 shocks

Figure 19 compares the response of the economy when we use the full sequence of realized ϕ_t shocks from 2008 through 2014 to the benchmark exercise which uses the realized shocks upto 2009. The behavior of tail risk (the third moment of equity returns as detailed in Table 3) and credit spreads is almost identical in both cases. Debt (the third panel) shows some recovery – essentially, the positive shocks between 2010 and 2014 generates a transitory rebound in capital (and therefore, in debt) – but remains persistently below the pre-crisis level.

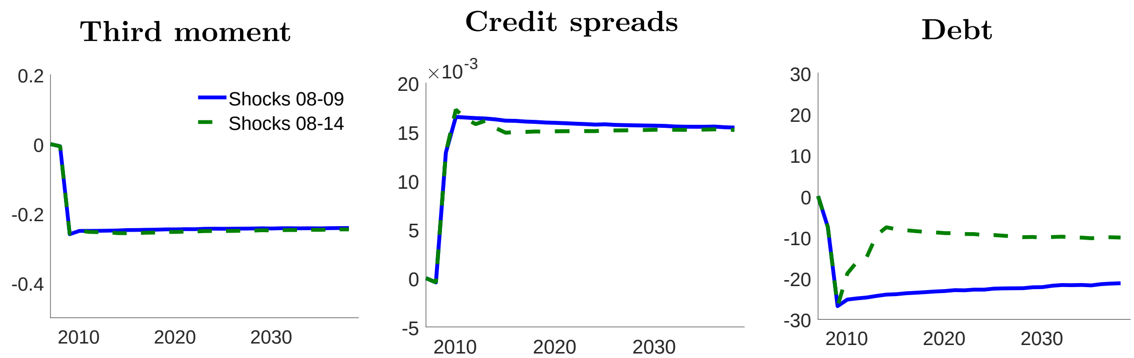


Figure 19: **Persistent responses in tail risk, credit spreads, and debt.**
The solid line shows the change in the third moment, credit spreads and debt in the benchmark impulse response (using the realized shocks in 2008 and 2009). For the dashed line (model shocks 2008-2014), we use the full sequence of shocks, from 2008 through 2014.

D Other evidence

D.1 Internet search behavior

Data on internet search behavior lends support to the idea that assessments of tail risk are persistently higher after the financial crisis. Figure 20 shows that the frequency of searches for the terms "financial crisis," "economic crisis," and "systemic risk" spiked during the crisis and then came back down. But this search frequency did not return to its pre-crisis level. In each case, there was some sustained interest in crises at a higher level than pre-2007. We find similar results for searches on the terms "economic collapse," "financial collapse," and "tail risk" yielded similar results.

D.2 Stock market

One question that often arises is whether other unusual events, such as the large stock market drop in 2008, might trigger a persistent economic response. Here, we illustrate what belief

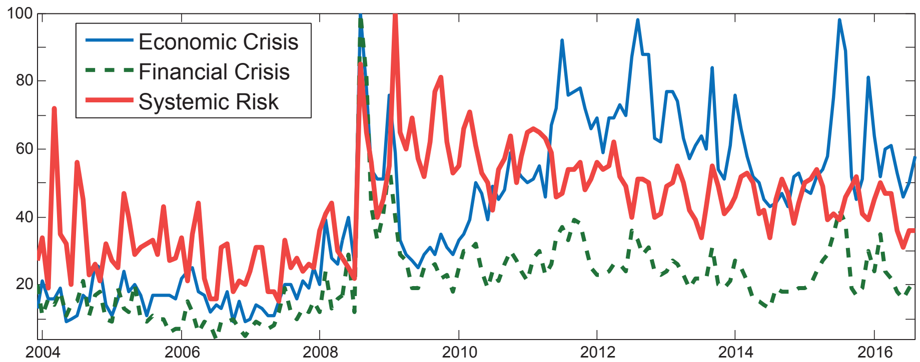


Figure 20: **Tail risk-related Google searches rose permanently after 2008.**
Search frequency for the terms 'financial crisis,' 'economic crisis,' and 'systemic risk' world-wide, from December 2003 - September 2016. Each series is normalized so that the highest intensity month is set to 100. Source: Google trends.

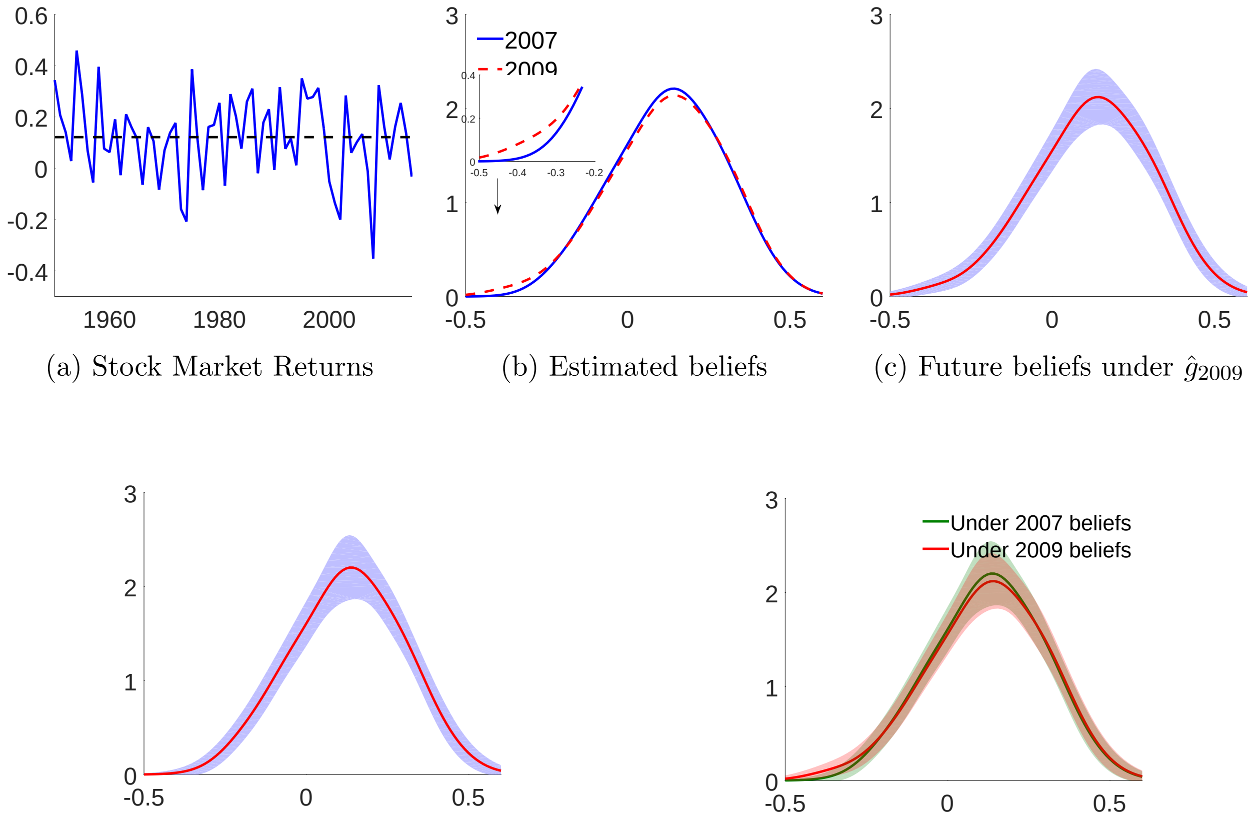
revisions would look like for agents learning about the distribution of stock returns. Of course, we acknowledge that this is not the driving force in our model. It is only intended to further illustrate possible future applications of our persistence mechanism.

Figure 21 shows the belief revision after observing 2008-09 equity returns, and the distribution of future beliefs under two different assumptions about the true distribution of shocks. Annual returns 1950-2009 come from Robert Shiller's website.

What we see is that large negative equity returns during 2008-09 are not all that unusual. The stock market has plunged many times. Seeing one more drop, while not very common, was not so unusual as to change beliefs by much. We conclude that while stock returns can also generate some persistence through belief updating, this force is not a likely candidate for the recent stagnation, relative to the capital quality shock, because the downturn in stock prices was less unusual.

D.3 Returns during the Great Recession

Not all authors agree that the Great Recession was an unusual event. For example, Gomme et al. (2011) present a series for returns on capital that show adverse realizations for 2008-09 that are not as extreme as our measures. The difference stems from their measurement strategy. To compute capital gains, they use data from the NIPA, which values non-residential capital (structures, equipment and software) at *replacement* cost. During 2008-09, we saw massive declines in the *market value* (particularly, for commercial real estate), even though the replacement cost of structures fell only modestly. While appropriate for their purposes, these NIPA measures miss one of the unusual aspects of the Great Recession – large declines in the market value of business capital, notably commercial real estate.



(a) Stock Market Returns (b) Estimated beliefs (c) Future beliefs under $\hat{g}_{2009}$

(d) Future beliefs under $\hat{g}_{2007}$ (e) Future beliefs under $\hat{g}_{2007}$ and $\hat{g}_{2009}$

Figure 21: **Estimated beliefs about stock market returns.**
The first panel shows the realized returns on the stock market. The second panel shows the estimated kernel density for 2007 and 2009. The third panel shows the mean belief (along with a 2 standard deviation band) in 2039 (computed by simulating data for the period 2010-2039 using the estimated distribution in 2009).

D.4 De-trending

Our learning mechanism generates persistent movements in aggregate variables after extreme events. Therefore, in order to make a meaningful comparison with the data, the choice of the right de-trending procedure for the data is very important. We use a log-linear trend, which removes only the lowest-frequency (permanent) part of the series. A common approach in business cycle analysis is to non-linear filters (like the Hodrick-Prescott filter), which take out more of the persistent movements in the series. By design, what is left will not have much persistence left. In figure 22, we illustrate this using aggregate non-residential investment (other aggregate series show very similar patterns). As the graph reveals, the trend component of the HP filter (smoothing parameter 100) picks up some of the deviation from the linear trend. Given that our focus is on low-frequency or persistent components, a linear detrending procedure seems most appropriate.

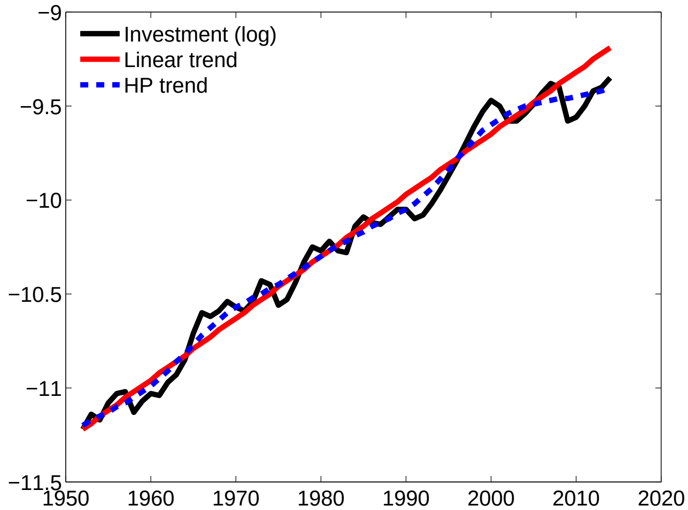


Figure 22: Non-residential Investment, with log-linear and HP trends.

D.5 Productivity

While a productivity slowdown may have contributed to low output, it does not explain the timing or the rise in tail risk indicators. Figure 23 shows the time series of raw total factor productivity, constructed as $dtfpt = dY_t - \alpha_t dk_t - (1 - \alpha_t)(dhours_t + dLQ_t)$ from Fernald (2014). When we examine instead utilization-adjusted TFP, we find a slight decline during the recession, but a decline that is will within two-standard deviation bands of the distribution of TFP changes. Productivity did not have a precipitous decline that could be considered a tail event.

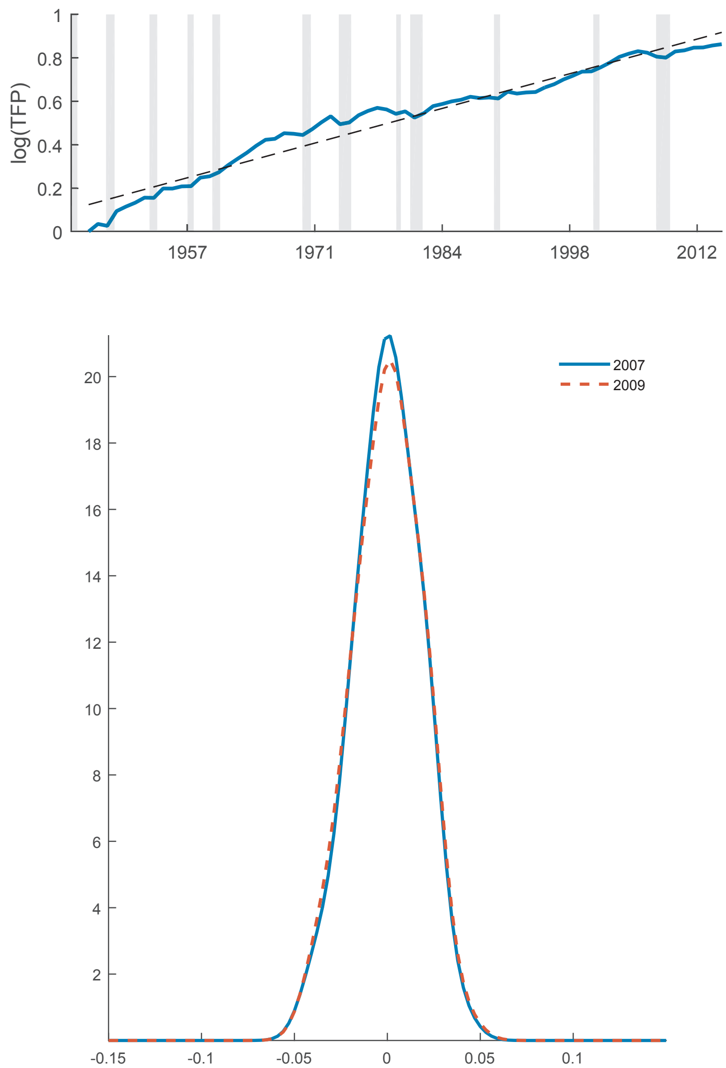


Figure 23: **Productivity.**