

MUSSA PUZZLE REDUX*

Oleg Itskhoki

itskhoki@princeton.edu

Dmitry Mukhin

dmitry.mukhin@yale.edu

February 20, 2019

UNDER CONSTRUCTION

DOWNLOAD THE MOST UP-TO-DATE VERSION FROM:

<http://www.princeton.edu/~itskhoki/papers/Mussa.pdf>

Abstract

The [Mussa \(1986\)](#) puzzle — a sharp and simultaneous increase in the volatility of both nominal and real exchange rates after the end of the [Bretton Woods System](#) of pegged exchange rates in early 1970s — is commonly viewed as a central piece of evidence in favor of monetary non-neutrality. Indeed, a change in the monetary regime has caused a dramatic change in the equilibrium behavior of a real variable — the real exchange rate. The Mussa fact is further interpreted as direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data, as there was no simultaneous change in the properties of the other macro variables — neither nominal like inflation, nor real like consumption, output or net exports. We show that the extended set of Mussa facts equally falsifies both flexible-price RBC models and sticky-price New Keynesian models. We present a resolution to this broader puzzle based on a model of segmented financial market — a particular type of financial friction by which the bulk of the *nominal* exchange rate risk is held by a small group of financial intermediaries and not shared smoothly throughout the economy. We argue that rather than discriminating between models with sticky versus flexible prices, and monetary versus productivity shocks, the Mussa puzzle provides sharp evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation*. Sticky prices are neither necessary, nor sufficient for the qualitative success of the model.

*We thank Andy Atkeson and Jón Steinsson for stimulating discussions, Mark Aguiar and Chris Sims for useful comments, and Gordon Li for outstanding research assistance.

1 Introduction

The [Mussa \(1986\)](#) puzzle is the fact that the end of the [Bretton Woods System](#) and the change in the monetary policy regime in the early 1970s away from pegged towards floating exchange rates had naturally increased the volatility of the nominal exchange rates (by an order of magnitude), but had also instantaneously increased the volatility of the *real* exchange rate almost by the same proportion (see [Figure 1](#)). This fact is commonly viewed by economists as a central piece of evidence in favor of monetary non-neutrality, since a change in the monetary regime has caused a dramatic change in the equilibrium behavior of a real variable — the real exchange rate.¹ Indeed, in models with complete monetary neutrality, the property of the real exchange rate should not be affected by the change in the monetary rule, absent other contemporaneous changes.² However, the Mussa fact is further interpreted as the direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data and provide an alternative explanation to the puzzle.

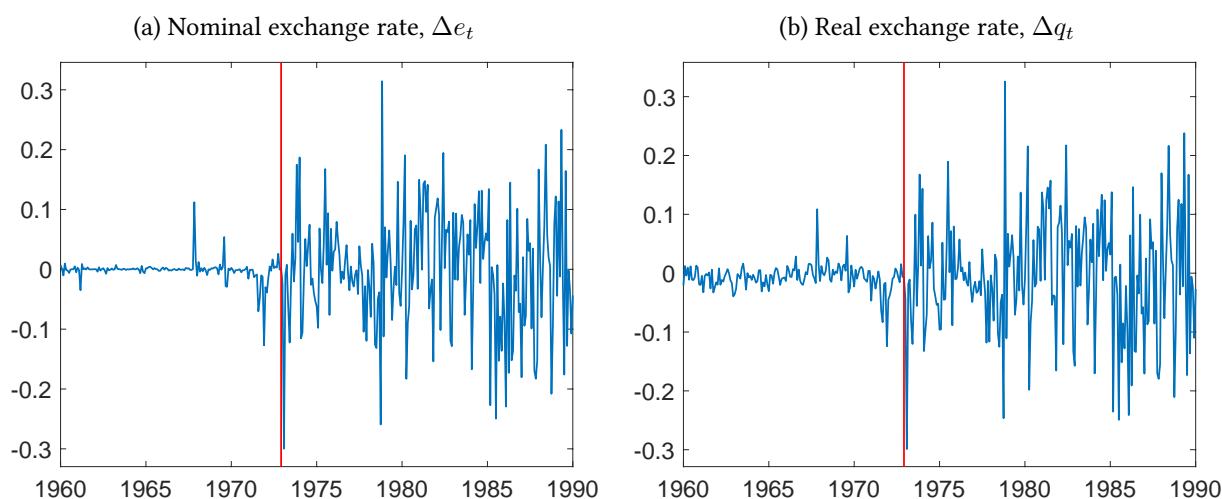


Figure 1: Nominal and real exchange rates, log changes

Note: US vs the rest of the world (defined as G7 countries except Canada plus Spain), monthly data from IFM IFS database. See [Appendix Figure A1](#) for the comparison of volatilities and the correlation of the two exchange rate series over time.

We start by documenting empirically that while there was a change in the properties of the real exchange rate, there was no change in the properties of other macro variables — neither nominal like inflation, nor real like consumption, output or net exports (see [Figure 2](#), which exhibits no evident structural break). One could interpret this as an extreme form of *neutrality*, where a major shift in the monetary regime, which increased the volatility of the nominal exchange rate by an order of magnitude,

¹When [Nakamura and Steinsson \(2018, pp.69–70\)](#) surveyed “prominent macroeconomists [on what is the most convincing evidence for monetary nonneutrality], the three most common answers have been: the evidence presented in [Friedman and Schwartz \(1963\)](#) regarding the role of monetary policy in the severity of the Great Depression; the [Volcker disinflation](#) of the early 1980s and accompanying twin recession; and the sharp break in the volatility of the US real exchange rate accompanying the breakdown of the [Bretton Woods System](#) of fixed exchange rates in 1973.”

²The argument here relies on the timing and the sharp discontinuity in the behavior of the exchange rates (see [Figure 1](#)), absent other immediate major changes in the environment.

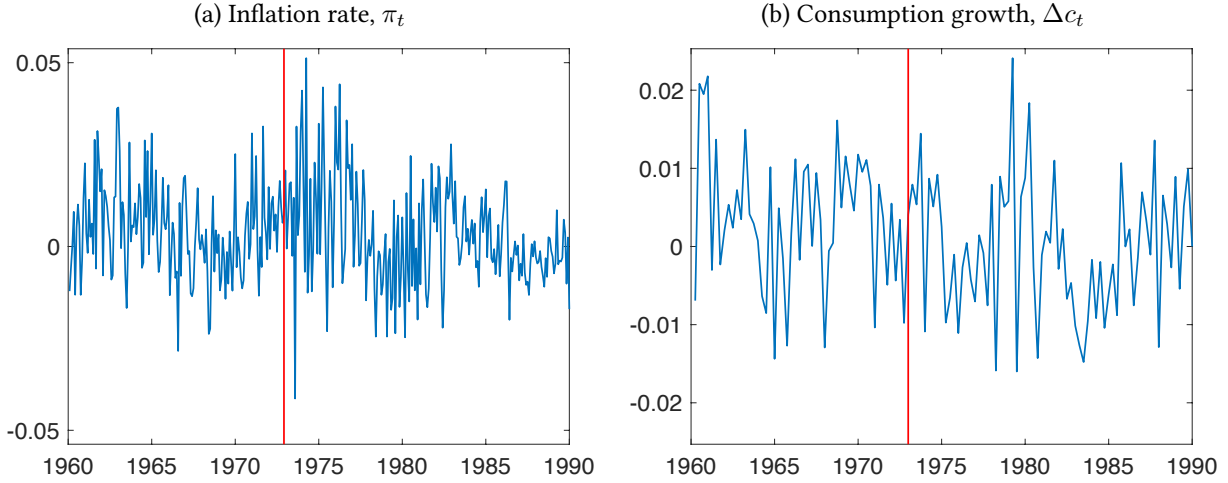


Figure 2: Inflation and consumption growth

Note: average inflation rates (monthly) and consumption growth rates (quarterly) for G7 countries except Canada plus Spain.

fails to affect the equilibrium properties of any macro variables, apart from the real exchange rate. In fact, this is a considerably more puzzling part of the larger set of “Mussa facts”: while the lack of change in the volatility of nominal variables, like inflation, is inconsistent with models of monetary neutrality, the lack of change in the volatility of real variables, like consumption and output, is inconsistent with sticky-price models. Therefore, if we take the combined evidence, it does not seem to favor one type of models over the other, but rather rejects both types.

To provide immediate intuition for this logic, consider two equilibrium conditions. The first is simply the definition of the real exchange rate (in logs):

$$q_t = e_t + p_t^* - p_t, \quad (1)$$

where p_t and p_t^* are consumer price levels at home and abroad, and e_t and q_t are the nominal and real exchange rates respectively. In models with monetary neutrality (e.g., international RBC), a change to the monetary policy rule should *not* affect the process for q_t , and therefore (1) necessary implies that the volatility of $\pi_t - \pi_t^* \equiv \Delta p_t - \Delta p_t^*$ must change along with the volatility of Δe_t . In the data, the volatility of Δq_t and Δe_t increased simultaneously, while the volatility of $\pi_t - \pi_t^*$ remained stable and low (see Figure 3 and Table 1). This pattern can, however, be consistent with the conventional sticky-price models (see e.g. Monacelli 2004). This observation is at the core of the traditional interpretation of the Mussa puzzle, suggesting that sticky price models (NOEM) beat RBC models, and monetary policy must have real effects due to nominal rigidities.

This interpretation, however, misses the second half of the picture. Equilibrium dynamics in a general class of models satisfies the following cointegration property between relative consumption (with the rest of the world) and real exchange rate:

$$\sigma(c_t - c_t^*) = q_t + \zeta_t, \quad (2)$$

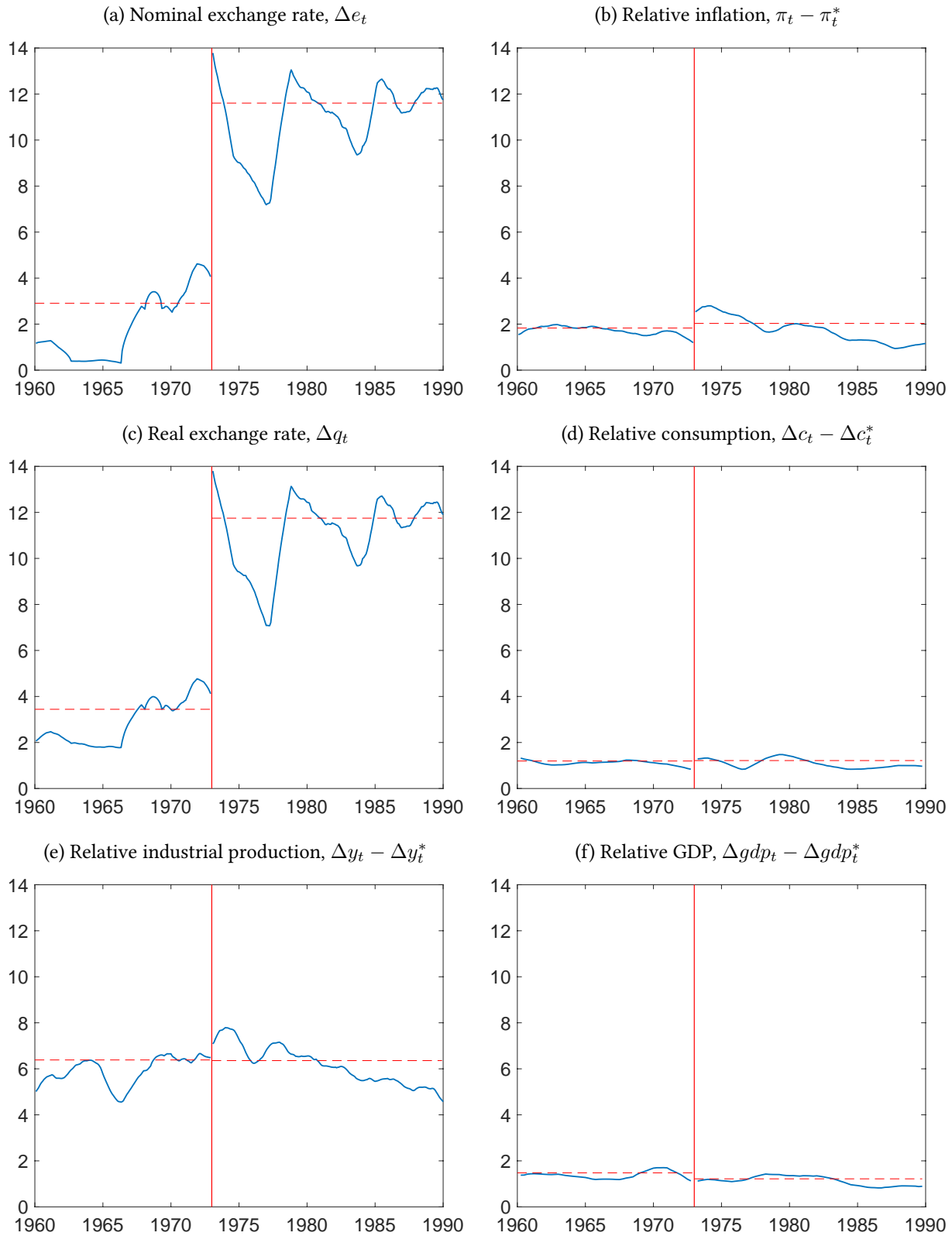


Figure 3: Volatility of macroeconomic variables over time

Note: All panels plot annualized standard deviations (of the log changes), estimated as triangular moving averages with a window over 18 months (or 10 quarters for quarterly data) before and after, treating 1973:01 as the end point for the two regimes; the dashed lines correspond to standard deviations measured over the entire subsamples (before and after 1973). US vs the rest of the world (as in Figure 1); monthly data from IFM IFS for panels a, b and e, and quarterly data from OECD in panels c, d and f. Appendix Figure A2 zooms in on the range of variation in panels b, d, e and f; Table 1 provides further details.

where $\sigma > 0$ and ζ_t can be interpreted as the equilibrium departure from the optimal international risk sharing.³ Indeed, equation (2) with $\zeta_t \equiv 0$ corresponds to the classic [Backus and Smith \(1993\)](#) condition under separable utility with constant relative risk aversion σ . We show that equation (2) is considerably more general and emerges as an equilibrium cointegration relationship independently of asset market completeness and other features of the model. Furthermore, we show that in a large class of conventional models – including both IRBC and NOEM – the residual term ζ_t is independent of the monetary policy regime. Therefore, a shift in the monetary policy regime, which changes dramatically the volatility of Δq_t , should necessarily change the volatility of $\Delta c_t - \Delta c_t^*$. In the data, however, the volatility of relative consumption growth, just like that of inflation, remained both stable and small (see Figure 3).

To summarize, the models of monetary neutrality are consistent with the observed lack of change in the volatility of consumption, but for the wrong reason – as they fail to predict the change in the volatility of the real exchange rate. In contrast, models with nominal rigidities can explain the changing behavior of the real exchange rate, but have the counterfactual implications for the missing change in the volatility of the real variables. Therefore, the extended Mussa facts falsify the conventional RBC and New Keynesian models alike.

We then present a new resolution to the Mussa puzzle, which is simultaneously consistent with all the empirical facts. In particular, we show that in a model with segmented financial markets and limits to arbitrage developed in [Itskhoki and Mukhin \(2019\)](#), shifts in monetary policy regime affect the volatility of both nominal and real exchange rates, *even* when prices are fully flexible. Intuitively, the unpredictable movements in *nominal* exchange rate are the main source of uncertainty for financial intermediaries, who as a result are less aggressive under free floating exchange rates in taking large currency positions to ensure that uncovered interest parity (UIP) holds. By consequence, the equilibrium UIP violations are larger under the floating exchange rate regime, consistent with the data (see [Kollmann 2005](#)). A pegged exchange rate, in contrast, decreases uncertainty and stimulates arbitrageurs to take larger positions. As a result, the real exchange rate is less sensitive to shocks in the financial market and has lower volatility. At the same time, the financial shocks do not constitute the main source of volatility in the other macro variables under *either* monetary regime, and thus the model is consistent with nearly no change in the macroeconomic volatility, apart from the exchange rates.⁴

This logic is summarized in the *modified UIP* condition, which holds in equilibrium of our economy with a segmented financial market:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \quad (3)$$

where the left-hand side is the expected carry trade return, ψ_t is a demand shock for foreign-currency

³Note the parallel between ζ_t and q_t , which can be viewed as defined by identifies (2) and (1) respectively: just like q_t is the departure from *parity* in the goods market (namely, the purchasing power parity), ζ_t can be viewed as the departure from “parity” in the financial market (namely, the optimal risk sharing).

⁴This additionally requires that economies are sufficiently closed to international trade – an important feature of the world as argued by [Obstfeld and Rogoff \(2001\)](#) – so that real exchange rate volatility does not translate into a large volatility increase in the price level, production and consumption. This is consistent with the *exchange rate disconnect* mechanism under the floating (Taylor rule) regime developed in [Itskhoki and Mukhin \(2019\)](#).

bonds and b_{t+1} is the net foreign assets of the home country. Crucially, the coefficients on the right-hand side of (3) are endogenous to the monetary policy regime, and in particular

$$\chi_1, \chi_2 \propto \sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1}), \quad (4)$$

that is increase proportionally with the unexpected exchange rate volatility. Equation (3) with endogenous coefficients (4) is the only unconventional equilibrium condition in an otherwise standard international DSGE model. A change in the monetary regime has a direct effect on the equilibrium in the financial market via (3), and this is what allows the model to be consistent with the umbrella of Mussa facts, independently of the presence of nominal rigidities and the source of the other shocks, as long as ψ_t is an important contributor to the exchange rate volatility under a floating regime.⁵

We conclude that rather than discriminating between models with sticky versus flexible prices, and monetary versus productivity shocks, the Mussa puzzle provides a strong evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation* – a particular type of financial friction by which the bulk of the *nominal exchange rate risk* is held by a small group of financial intermediaries and not shared smoothly throughout the economy. Sticky prices are neither necessary, nor sufficient ingredient for the qualitative success of this model. Nonetheless, realistic price and wage stickiness can improve the model’s quantitative fit. Our analysis emphasizes that monetary non-neutrality is *not exclusive* to nominal rigidities in price setting, as changes in equilibrium properties of the nominal variables – such as the nominal exchange rate – can change the degree of financial market (in)completeness, and hence have real consequences for the real equilibrium outcomes.

Overidentifying moments Backus-Smith correlation, Fama regression coefficient, Balassa-Samuelson...

Related literature Baxter and Stockman (1989), Flood and Rose (1995), Jeanne and Rose (2002), Kollmann (2005), Alvarez, Atkeson, and Kehoe (2007), Colacito and Croce (2013),...Engel et al. (2018), Devereux-Hnatkovska, Kollmann, Frenkel-Levich...

2 Empirical Facts

Data We start by briefly describing the construction of our dataset, and provide further details in Appendix A.2. All monthly data (for nominal exchange rate, consumer prices and production index) come from the [IFM IFS database](#), while all quarterly data (for GDP, consumption, imports and exports) are from the [OECD database](#). All quantity variables (GDP, consumption, imports and exports) are real and seasonally-adjusted. Production index is also seasonally-adjusted, while nominal exchange rates and consumer price indexes are not. The *net export* variable is defined as the ratio of exports minus

⁵Itskhoki and Mukhin (2019) (and 2019b) show that financial shocks ψ_t are essential for a successful model of exchange rate disconnect under a Taylor rule regime, and resolve a variety of exchange rate puzzles, including Meese-Rogoff, PPP, Backus-Smith and UIP puzzles. Their presence *per se* is not sufficient to resolve the Mussa puzzle, which also requires the endogeneity of the coefficient in (3). We further show the relationship between (2) and (3), and how (4) implies the endogeneity of ζ_t to the exchange rate regime. Importantly, (3) does not rely on the assumption that an Euler equation holds for a representative consumer.

imports to the sum of exports and imports, in order to counter a mechanical increase in the volatility of net exports to GDP due to higher openness of the economies in later periods. All data are annualized to make volatilities (standard deviations) comparable across series.

There is ambiguity associated with identifying the exact end of the Bretton Woods System. In particular, during the Bretton Woods period, there are already large devaluations in the U.K. and Spain in November 1967, a devaluation in France and an appreciation in Germany in August–October 1969. While all countries officially allowed their exchange rates to float in February 1973, most of them were already adjusting their exchange rates since the “Nixon shock” in August 1971, which limited the direct convertibility of dollar to gold. Therefore, we label the period from 1960:01–1971:07 as “peg” and the period from 1973:01–1989:12 as “float”, as used in tables and scatter plots below (which exclude the intermediate period 1971:08–1972:12).⁶ The “regression discontinuity” graphs are done for two alternative break points – 1973:01 in the main figures and 1971:08 in the robustness figures in the appendix.

The rest of the world for the U.S. is constructed as a weighted average of percent changes in series across France, Germany, Italy, Japan, Spain and the U.K. (G7 countries except Canada plus Spain). Average GDP shares during the sample period are used to construct country weights.

Macroeconomic volatility Figure 3 displays the main empirical results of the paper. In the spirit of the regression discontinuity design (RDD), we estimate standard deviations of the variables using a rolling window that starts at 1973:01 and goes either forward or backward. In line with the seminal [Mussa \(1986\)](#) paper, the end of the Bretton Woods System is associated with a dramatic change in the volatility of both nominal and real exchange rates, from around 2% to 10% (more precisely, in units of annualized standard deviations of log changes). What makes this fact much more puzzling, however, is the absence of any comparable change in the volatility of the other variables – either nominal like inflation, or real like production and consumption.⁷ Thus, while under the peg regime, the volatility of the real exchange rate is of the same order of magnitude as for the other macroeconomics variables, there is a clear “disconnect” between the real exchange rate and macroeconomic fundamentals in the floating regime. We emphasize the relative magnitudes of volatilities across different variables and regimes by keeping the same scale for standard deviations of all variables in Figure 3, while Appendix Figure A2 zooms in on the range of variation of individual macroeconomic variables to see (the lack of) the discontinuity in the behavior of inflation, consumption and output.⁸

The rest of the pictures and tables expand on this finding and provide some additional details. Figure A3 provides a robustness check using 1971:08 as the alternative break point. There is no evidence of changing volatility for macroeconomic variables in this case either. The missing change in the volatil-

⁶In Canada, the two exchange rate regimes occurred over different periods with free floating before 1962:06 and after 1970:05, and a peg in between. This is why we exclude Canada from the construction of the “rest of the world” in the figures.

⁷Figure 3 presents the *relative* volatilities of macroeconomic variables between the US and the rest of the world, as these are the relevant objects emphasized by the theory in Section 3. Table 1 also present the results for individual country variables across a range of countries, which exhibit similar patterns as the relative variables for the US vs the ROW.

⁸Note a slight increase in the volatility of consumer price inflation in the brief period after the break up of the Bretton Woods System, which quickly comes back down so that the average relative inflation volatility before and after 1973 is about the same. This increase in the volatility of inflation in the second half of 1970s is likely a response to the two large oil price shocks. There is also a slight increase in the volatility of consumption briefly after 1973, due to the 1974 recession in Japan.

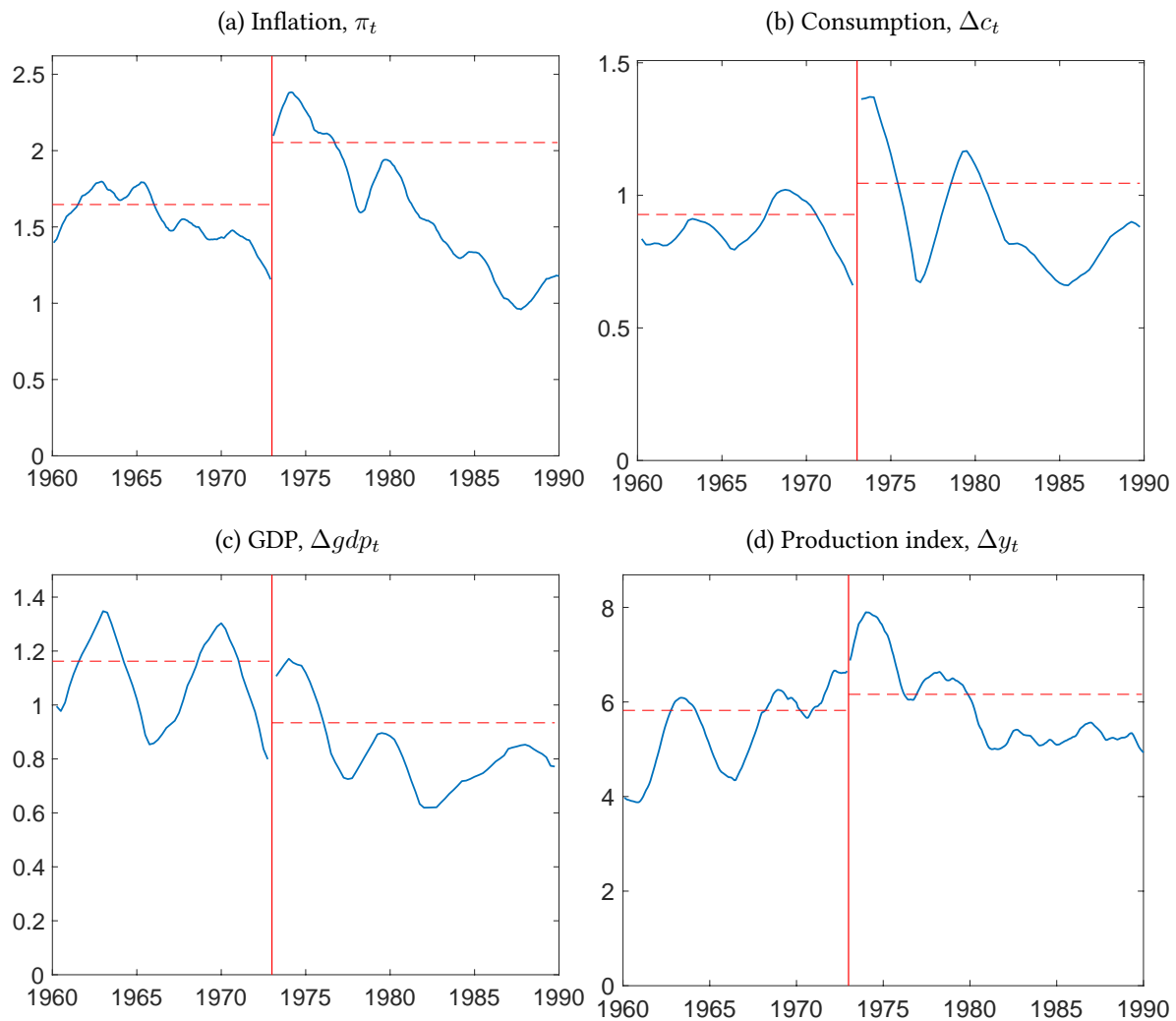


Figure 4: Macroeconomic volatility over time: country-level variables

Note: Triangular moving averages of the standard deviations of macro variables, treating 1973:01 as the break point; average across countries (G7 except Canada plus Spain).

ity is true not only for the cross-country differences of variables, but also for the fundamentals at the country level. In particular, from Figure 4, there is almost no differences in the volatilities of macro variables.

We now unpack the rest of the world (RoW) into separate countries and show that the main results hold in the panel as well. Table 1 summarizes the standard deviation of various variables for each country in our sample across the two monetary regimes, as well as provides a formal test of the equality of the variances of variables under the two regimes. We confirm that the change in the volatility of the exchange rates was large and highly significant in every country, while changes in the volatility of the other variables were small and generally insignificant. Note that rather than emphasizing the lack of any change in other variables, we emphasize the difference in the order of magnitude. Table 1 shows that while nominal and real exchange rate volatility increased on average by about 8 and 6 times respectively, the volatility of the other variables changed in different direction across countries

Table 1: Empirical moments: standard deviations

	Δe_t			Δq_t			$\pi_t - \pi_t^*$			$\Delta c_t - \Delta c_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	0.8	4.4	5.7*	1.5	4.7	3.0*	1.3	1.4	1.1	0.8	1.1	0.9
France	3.4	11.8	3.5*	3.7	11.8	3.2*	1.3	1.3	1.0	1.2	0.9	0.7*
Germany	2.4	12.3	5.0*	2.7	12.5	4.7*	1.4	1.3	0.9	1.3	1.2	0.9
Italy	0.5	10.4	18.8*	1.5	10.4	6.9*	1.4	1.9	1.3*	1.0	1.1	1.0
Japan	0.8	11.7	13.8*	2.7	11.9	4.4*	2.7	2.8	1.0	1.1	1.3	1.2
Spain	4.4	10.8	2.5*	4.7	10.8	2.3*	2.7	2.6	0.9	1.2	1.0	0.8
U.K.	4.1	11.5	2.8*	4.4	12.0	2.7*	1.7	2.5	1.5*	1.4	1.5	1.1
RoW	1.2	9.8	8.0*	1.8	9.9	5.6*	1.3	1.4	1.1	0.9	0.9	1.0
	$\Delta gdp_t - \Delta gdp_t^*$			$\Delta y_t - \Delta y_t^*$			Δnx_t			$\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.0	1.0	1.0	3.8	4.9	1.3	1.7	1.6	0.9	2.4	4.5	1.9*
France	1.2	1.0	0.8	5.3	5.6	1.1	1.5	1.4	0.9	4.4	12.2	2.7*
Germany	1.8	1.2	0.7*	6.7	6.0	0.9	1.8	1.7	0.9	3.9	13.7	3.5*
Italy	1.5	1.3	0.8	8.1	9.7	1.2	2.5	2.2	0.9	2.8	11.4	4.1*
Japan	1.5	1.3	0.8	5.5	5.0	0.9	2.4	2.2	0.9	2.8	13.1	4.7*
Spain	1.6	1.2	0.7*	10.1	10.4	1.0	5.4	2.1	0.4*	5.8	11.4	2.0*
U.K.	1.4	1.4	0.9	3.9	6.0	1.5*	2.2	1.9	0.9	5.2	11.8	2.2*
RoW	1.1	1.0	0.8	3.9	3.5	0.9	1.1	1.0	0.9	2.5	10.7	4.3*
	π_t			Δc_t			Δgdp_t			Δy_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.3	1.4	1.1	0.8	0.9	1.1	0.9	0.9	1.0	4.1	5.1	1.2
France	1.1	1.3	1.2*	0.9	0.8	0.9	0.9	0.6	0.6*	4.2	5.4	1.3*
Germany	1.2	1.1	0.9	1.0	1.0	1.0	1.5	1.0	0.7	6.2	5.7	0.9
Italy	1.0	2.1	2.0*	0.7	0.8	1.2	1.3	1.0	0.8	7.5	9.5	1.3*
Japan	2.6	2.9	1.1	1.0	1.3	1.3	1.1	1.1	0.9	4.6	4.9	1.1
Spain	2.5	2.5	1.0	1.0	0.7	0.7	1.4	0.7	0.5*	10.1	10.1	1.0
U.K.	1.6	2.6	1.6*	1.2	1.4	1.2	1.0	1.3	1.2	3.5	5.9	1.7*
U.S.	0.9	1.3	1.5*	0.7	0.8	1.1	0.9	1.0	1.1	2.9	2.9	1.0

Average

Note: "peg" corresponds to the period from 1960:01-1971:07 (except for Canada where it is from 1962:04-1970:01); "float" is from 1973:08-1989:12. The RoW corresponds to France, Germany, Italy, Japan, Spain, and the U.K., aggregated using 2010 GDP as weights. nx_t is the ratio of export minus imports to the sum of imports and exports. We use $\sigma = 2$ in panel eight. * indicates significance of the difference between peg and float at the 5% level (robvar test in Stata).

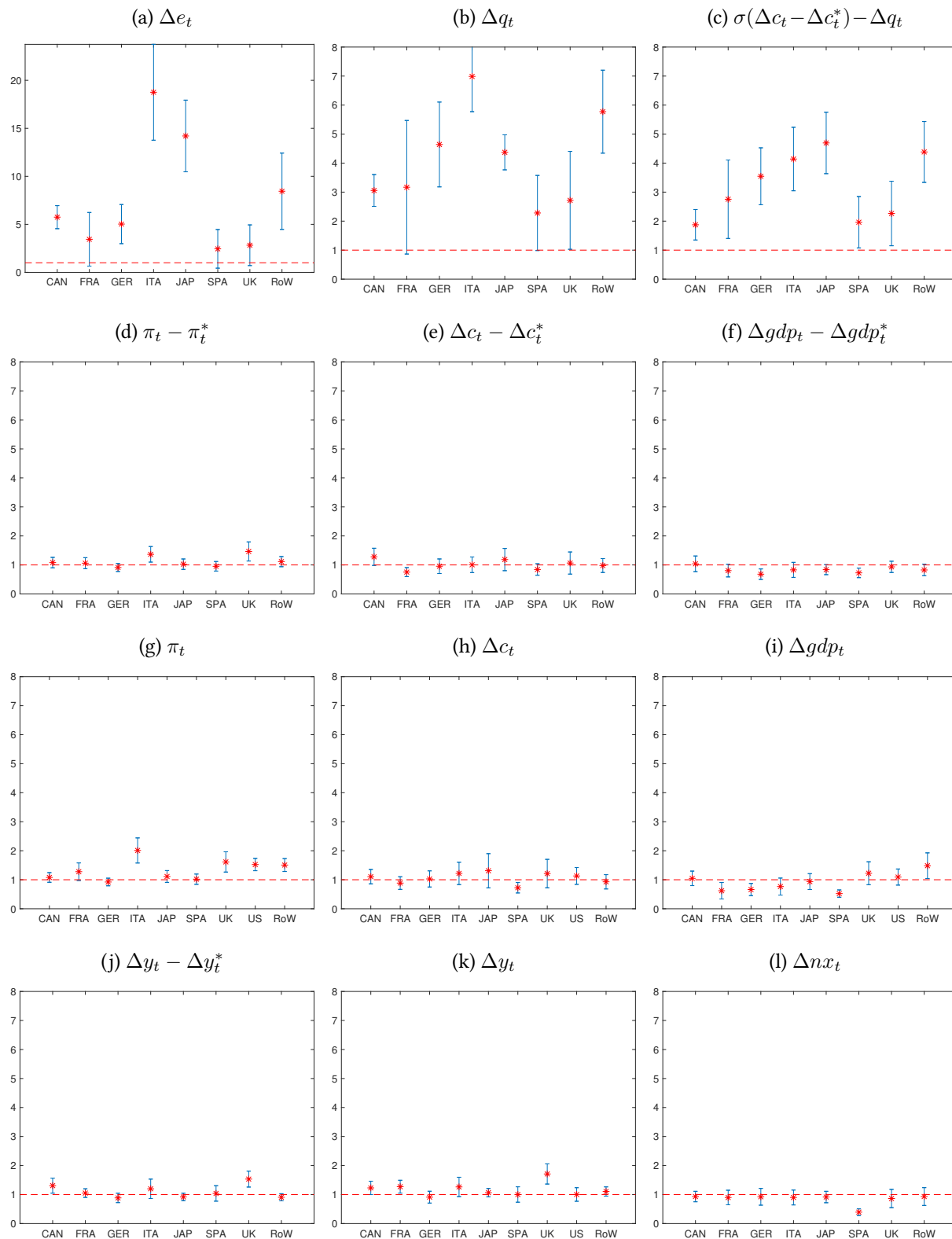


Figure 5: Volatility ratio float/peg, across variables and countries

Note: plots show the ratios of standard deviations under floating and peg regimes across individual countries with 90% confidence intervals estimated using Newey-West (HAC) standard errors. y -axis has the same scale for all plots except Δe_t .

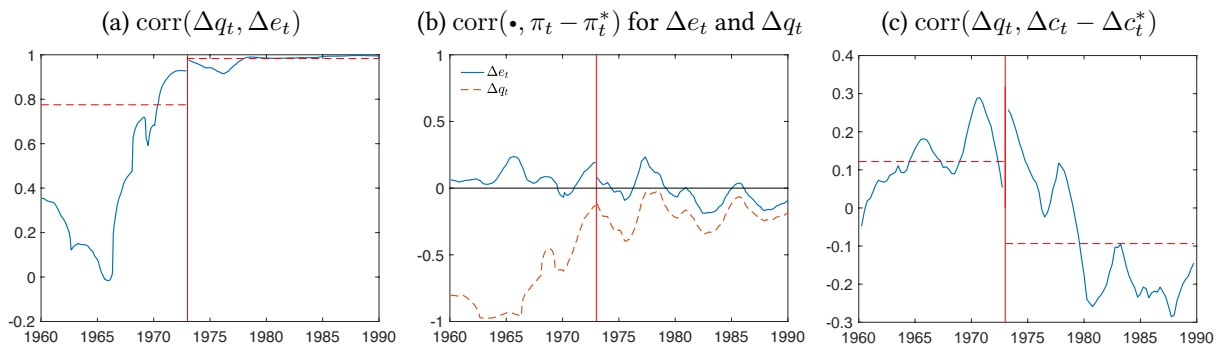


Figure 6: Correlations of exchange rates over time

Note: Triangular moving average correlations, treating 1973:01 as the end point for the two regimes.

and by an order of magnitude of about 10% — a stark difference. We further illustrate this point in Figure 5 which plots the ratio of the volatilities under the two regimes for each variable and country, on a common scale for comparability, with a Newey-West (HAC) robust 90% confidence interval (see also Appendix Figures A5 and A6).

Perhaps most surprisingly, there is no increase in the volatility of net exports, despite a large increase in the volatility of the real exchange rate (see Figures A2(e) and 5(l) and Table 1). Systematic data on terms of trade is not available for this period, however, in the FRED data we see that the volatility of the US terms of trade increased only twofold, while the volatility of the US real exchange rate increased six times. Therefore, we conjecture that the lack of the increased volatility in net exports is in part due to a much muted response of the behavior of the terms of trade and in part due to muted response of net exports themselves to international relative prices.

Correlations Figure 6 plots the correlations between the nominal and real exchange rates, as well as their correlations with macro variables — relative inflation and relative consumption growth — over time, calculated as triangular moving average with a break point at 1973:01. The first two panels identify two clear shifts with the break of the fixed exchange rate regime. In particular, the correlation between nominal and real exchange rates is positive but not very strong during the peg, where nominal exchange rates barely moved before 1967, and it becomes virtually perfect (0.98) after the early 1970s. The latter correlation is around 0.7 and does not change significantly with the end of the Bretton Woods System. In contrast, the real exchange rate is tightly correlated with the relative inflation during the peg, yet it quickly becomes nearly orthogonal with relative inflation as soon as nominal exchange rates begin to float. At the same time, the nominal exchange rate is orthogonal with the relative inflation both before and after the 1973. While the pattern of correlation during the peg is mechanical — since $\Delta e_t \approx 0$ and thus $\Delta q_t \approx -(\pi_t - \pi_t^*)$ — the comovement of variables during the float is rather puzzling, as nominal depreciations if anything are negatively correlated with relative domestic inflation.

The last panel of Figure 6 shows that the correlation between real exchange rate and relative consumption growth, while somewhat positive under the peg, has become noticeably negative under the

Table 2: Empirical moments: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta n x_t$		$\Delta g d p_t, \Delta g d p_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta g d p_t$	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Canada	0.77	0.92	0.03	-0.07	0.01	0.05	0.31	0.47	0.40	0.25	0.28	0.57
France	0.96	0.99	0.05	-0.08	0.23	0.12	0.09	0.30	-0.24	0.29	0.51	0.48
Germany	0.87	0.99	0.04	-0.19	-0.06	0.00	-0.01	0.28	-0.11	0.11	0.57	0.58
Italy	0.54	0.97	0.07	-0.13	0.02	-0.01	0.04	0.17	-0.18	0.13	0.64	0.45
Japan	0.76	0.98	0.21	-0.00	0.03	0.21	-0.08	0.24	0.11	0.23	0.70	0.71
Spain	0.83	0.96	-0.09	-0.18	-0.06	0.16	0.05	0.09	-0.06	0.05	0.56	0.63
U.K.	0.94	0.96	0.09	-0.10	-0.39	-0.16	-0.11	0.30	-0.02	0.22	0.59	0.71
RoW	0.80	0.98	0.05	-0.19	-0.20	0.21	-0.03	0.39	-0.11	0.31	0.68	0.72

Note: see notes to Table 1. Cross-country correlation are with the U.S. as the foreign counterpart indicated with a star.

float, suggesting that the Backus-Smith puzzle became more pronounced with floating exchange rates.⁹ While the correlations are not very large, this pattern is observed robustly across the countries, as we document in Table 2.

Table 2 reports correlations between various variables under the two exchange rate regimes, while Appendix Figure A4 plots the evolution of these correlation over time. Table 2 identifies another robust correlation pattern – between consumption and GDP growth, which is stable around 0.7 and does not change at all with the end of the Bretton Woods System. The other correlations, including that between RER and net exports, as well as for GDP (consumption) growth between countries, are generally not very strong and not particularly stable over time, suggesting only weak patterns of change across the two monetary regimes. The correlation between the real exchange rate and net exports switches from negative under the peg to positive under the float, but in both cases it is close to zero for most countries in our sample.

Finally, both GDP growth and consumption growth are uncorrelated or mildly negatively correlated across countries (with the exception of Canada) before 1970s and since then become sizably positively correlated, especially the GDP growth rates – a surprising pattern emphasized by Kollmann (2005). Figure A4 reveals that this is, however, largely driven by the high correlation of growth rates across countries in the late 1970s, a period of large global oil price shocks.

Fama regression coefficient and volatility of relative interest rates and stock market..

⁹This observation is consistent with the findings in Colacito and Croce (2013) that both Backus-Smith and UIP conditions held better under the pegged exchange rates, as well as in Devereux and Hnatkovska (2014) that the Backus-Smith condition holds better across regions within countries, in contrast with its cross-country violations. Another pattern emphasized by Berka, Devereux, and Engel (2012) is a substantially greater role of the non-tradable (Balassa-Samuelson) component in the RER variation under a nominal peg.

3 Conventional Models

Conventional models are defined by the following property: if prices are flexible, a switch in the monetary regime does not affect the behavior of real variables, including the real exchange rate q_t , consumption c_t and output y_t . Therefore, we only consider the case of sticky prices in this section, as flexible-price version of these models is falsified by the changing property of the real exchange rate (Figure 1). We first describe the model setup and then prove a negative result that even the sticky-price version of these models is falsified by the extended set of Mussa facts documented above.

3.1 Model setup

We build on a standard New Keynesian open-economy model (NOEM) with productivity shocks, PCP Calvo price stickiness, flexible wages and no capital. In our quantitative exploration in Section 5, we generalize this environment by introducing intermediate goods, capital, sticky wages and considering alternative variants of price stickiness including LCP and DCP, as well as monetary shocks. The model features home bias in consumption and exogeneous shocks to international risk sharing. We allow for various degrees of financial market (in)completeness, including complete markets, bonds-only and financial autarky.

There are two mostly symmetric countries – home (Europe) and foreign (US, denoted with a *). Each country has its nominal unit of account in which the local prices are quoted: for example, the home wage rate is W_t euros and the foreign wage rate is W_t^* dollars. The nominal exchange rate \mathcal{E}_t is the price of dollars in terms of euros, hence an increase in \mathcal{E}_t signifies a nominal devaluation of the euro (the home currency).

The monetary policy is conducted according to a conventional Taylor rule targeting inflation or nominal exchange rate – depending on the monetary regime. In particular, the foreign country (US) always targets inflation, while the home country (Europe) switches from an exchange rate peg (‘peg’) to an inflation targeting (‘float’). We consider the limiting case where the monetary authorities have the ability to fully stabilize prices or the exchange rate, depending on the regime.

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right), \quad (5)$$

where σ is the relative risk aversion parameter and φ is the inverse Frisch elasticity of labor supply. The flow budget constraint is given by:

$$P_t C_t + \sum_{j \in J_t} \Theta_t^j B_{t+1}^j \leq W_t L_t + \sum_{j \in J_{t-1}} e^{-\zeta_t^j} D_t^j B_t^j + \Pi_t + T_t, \quad (6)$$

where P_t is the consumer price index and W_t is the nominal wage rate, Π_t are profits of home firms, T_t are lump-sum transfers from the government, and B_{t+1}^j is quantity of asset $j \in J_t$ purchased at time t

at price Θ_t^j and paying out a state-contingent dividend D_{t+1}^j at $t + 1$ taxed at rate ζ_t^j (which we think of as [Chari, Kehoe, and McGrattan 2007](#) wedges).¹⁰

Household optimization results in standard optimality conditions for labor supply:

$$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t}, \quad (7)$$

Euler equations for state-contingent bonds:

$$\Theta_t^j = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} e^{-\zeta_{t+1}^j} D_{t+1}^j \right\} \quad \text{for all } j \in J_t. \quad (8)$$

The foreign households are symmetric, having access to a set J_t^* of state contingent assets with dividends taxed at country-specific tax rate ζ_t^{j*} . The assets $j \in J_t \cap J_t^*$ can be purchased by households of both countries at a common price Θ_t^j in units of home currency.¹¹

Expenditure and demand The domestic households allocate their within-period consumption expenditure between home and foreign varieties of the goods, $P_t C_t = \int_0^1 [P_{Ht}(i)C_{Ht}(i) + P_{Ft}(i)C_{Ft}(i)] di$ to maximize the CES consumption aggregator:

$$C_t = \left[\int_0^1 \left([(1-\gamma)e^{-\gamma\xi_t}]^{1/\theta} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + [\gamma e^{(1-\gamma)\xi_t}]^{1/\theta} C_{Ft}(i)^{\frac{\theta-1}{\theta}} \right) di \right]^{\frac{\theta}{\theta-1}}, \quad (9)$$

with parameter $\gamma \in [0, 1/2)$ capturing the level of the *home bias*, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see [Obstfeld and Rogoff 2001](#)) and ξ_t denoting the relative demand shock for the foreign good or other sources of time-varying home bias (see [Pavlova and Rigobon 2007](#)).¹² The solution to the optimal expenditure allocation results in the conventional constant-elasticity demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t} \left(\frac{P_{Ht}(i)}{P_t} \right)^{-\theta} C_t \quad \text{and} \quad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t} \left(\frac{P_{Ft}(j)}{P_t} \right)^{-\theta} C_t, \quad (10)$$

where the price index is given by $P_t = \left[\int_0^1 \left((1-\gamma)e^{-\gamma\xi_t} P_{Ht}(i)^{1-\theta} + \gamma e^{(1-\gamma)\xi_t} P_{Ft}(i)^{1-\theta} \right) di \right]^{1/(1-\theta)}$.

¹⁰When set J_t contains a home-currency risk-free bond B_{t+1}^f , its price is one over gross nominal interest rate $\Theta_t^f = 1/R_t$ and it pays out one unit of home currency in every state of the world next period, $D_{t+1}^f \equiv 1$; when it contains a foreign-currency risk-free bond B_{t+1}^{f*} , its price is \mathcal{E}_t/R_t^* and its dividend is $D_{t+1}^{f*} = \mathcal{E}_{t+1}$.

¹¹From the point of view of foreign households, the foreign currency price of asset j is Θ_t^j/\mathcal{E}_t in units of foreign currency, and the Euler equation is correspondingly $\frac{\Theta_t^j}{\mathcal{E}_t} = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} e^{-\zeta_{t+1}^{j*}} \frac{D_{t+1}^j}{\mathcal{E}_{t+1}} \right\}$ for $j \in J_t^*$.

¹²The particular way in which we introduce the foreign-good demand shock ξ_t in (9) ensures that changes in ξ_t shift the relative demand between home and foreign goods without having a first-order effect on the price level. The aggregate implications of the model do not depend on whether the home bias emerges on the extensive margin due to non-tradable goods or on the intensive margin due to trade costs and preferences, and therefore we do not explicitly introduce the non-tradables.

The expenditure allocation of the foreign households is symmetrically given by:

$$C_{Ht}^*(i) = \gamma e^{(1-\gamma)\xi_t^*} \left(\frac{P_{Ht}^*(i)}{P_t^*} \right)^{-\theta} C_t^* \quad \text{and} \quad C_{Ft}^*(j) = (1-\gamma) e^{-\gamma\xi_t^*} \left(\frac{P_{Ft}^*(j)}{P_t^*} \right)^{-\theta} C_t^*, \quad (11)$$

where ξ_t^* is the foreign demand shock for home goods, $P_{Ht}^*(i)$ and $P_{Ft}^*(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and P_t^* is the foreign price level. The *real exchange rate* is the relative consumer price level in the two countries:

$$Q_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}, \quad (12)$$

with an increase in Q_t corresponding to a real depreciation, that is a decrease in the relative price of the home consumption basket (note that (1) is the log version of (12)).

Production and price setting Home output is produced by a given pool of symmetric firms according to a linear technology in labor:

$$Y_t(i) = e^{a_t} L_t(i), \quad (13)$$

where a_t is the aggregate productivity shock, which implies that the marginal cost of production is:

$$MC_t = e^{-a_t} W_t, \quad (14)$$

identical for all firms.

The firms adjust prices infrequently à la Calvo with a constant per-period hazard rate λ of price nonadjustment, that is $P_{Ht}(i) = P_{H,t-1}(i)$ with probability λ and with complementary probability $(1-\lambda)$ the firm resets its price to $\bar{P}_{Ht}(i)$. For concreteness, we assume producer-currency pricing (PCP), which implies that the law of one price holds:

$$P_{Ht}(i)^* = P_{Ht}(i)/\mathcal{E}_t. \quad (15)$$

The per-period profit of the firm is given by

$$\Pi_t(i) = (P_{Ht}(i) - MC_t)(C_{Ht}(i) + C_{Ht}^*(i)) \quad (16)$$

where consumer demand $C_{Ht}(i)$ and $C_{Ht}^*(i)$ satisfies (10) and (11). The aggregate profits of the domestic firms, $\Pi_t = \int_0^1 \Pi_t(i) di$, are distributed to the domestic households.

The firms set prices $\bar{P}_{Ht}(i)$ as the solution of the following optimization problem:

$$\bar{P}_{Ht}(i) = \arg \max \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\lambda)^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \Pi_{t+k}(i), \quad (17)$$

where future profits are discounted using the SDF of the households and the probability of the reset price $\bar{P}_{Ht}(i)$ staying in effect k periods after it has been set. Since firms are symmetric (conditional on the last date they adjusted prices), they all set the price at the same reset level, $\bar{P}_{Ht}(i) = \bar{P}_{Ht}$ for all i .

Government The fiscal authority collects taxes on financial dividends and returns them lump-sum to the households:

$$T_t = \sum_{j \in J_{t-1}} (1 - e^{-\zeta_t^j}) D_t^j B_t^j. \quad (18)$$

The monetary policy is implemented by means of a Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m) [\phi_\pi \pi_t + \phi_e (e_t - \bar{e})] + \sigma_m \varepsilon_t^m, \quad (19)$$

where $i_t = \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, $\varepsilon_t^m \sim iid(0, 1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$, and the parameter ρ_m characterizes the persistence of the monetary policy rule. The coefficients $\phi_\pi > 1$ and ϕ_e are the Taylor rule parameters which weight the two nominal objectives of the monetary policy – inflation and exchange rate stabilization. We assume that the monetary authority can fully achieve the chosen goal (by increasing ϕ_π or ϕ_e unboundedly) – that is, either $\pi_t \equiv 0$ or $e_t \equiv \bar{e}$, depending on the regime. We further assume that the foreign country (the US) only has the inflation objective, so that $\phi_e^* = 0$, and $\pi_t^* \equiv 0$. We study the differential behavior of the macro variables across the two monetary regimes of the home country.

Equilibrium conditions The labor market clearing requires that the labor supplied by households according to (7) equals the aggregate labor demand of the home firm, $L_t = e^{-a_t} \int_0^1 Y_t(i) di$, where we used the production function (13). The goods market clearing for each firm i requires $Y_t(i) = C_{Ht}(i) + C_{Ht}^*(i)$, defined in (10)–(11), and given prices $P_{Ht}(i)$. Symmetric market clearing conditions hold in the foreign country. All assets B_{t+1}^j are in zero net supply, and for $j \in J_t \cap J_t^*$, we have $B_{t+1}^j + B_{t+1}^{j*} = 0$ given a common home-currency price Θ_t^j .

We focus here on two equilibrium conditions – the country budget constraint and the equilibrium in the financial market. The latter set of conditions can be written using (8) and their foreign counterparts as follows:

$$\mathbb{E}_t \left\{ e^{-\zeta_{t+1}^j} D_{t+1}^j \frac{P_t}{P_{t+1}} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} - \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{Q_t}{Q_{t+1}} e^{\tilde{\zeta}_{t+1}^j} \right] \right\} = 0, \quad \forall j \in J_t \cap J_t^*, \quad (20)$$

where by convention we denote $\tilde{\zeta}_{t+1}^j \equiv \zeta_{t+1}^j - \zeta_{t+1}^{j*}$ the relative wedge across countries. This condition simply states that home and foreign households agree on the price Θ_t^j of all assets j that they can mutually trade. This allows the household to equalize the stochastic discount factors across the two countries in the best possible way given the available set of internationally-traded assets.

The country budget constraint derives from substituting firm profits (16) and government transfers (18) into the household budget constraint (6):

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = NX_t, \quad (21)$$

where the right-hand side is net exports $NX_t = \int_0^1 P_{Ht}(i) C_{Ht}^*(i) di - \mathcal{E}_t \int_0^1 P_{Ft}^*(i) C_{Ft}(i) di$ and the left hand-side is the evolution of net foreign assets $\mathcal{B}_{t+1} \equiv \sum_{j \in J_t} \Theta_t^j B_{t+1}^j$ with the cumulative realized

return $\mathcal{R}_t \equiv \frac{\sum_{j \in J_{t-1}} D_t^j B_t^j}{\sum_{j \in J_{t-1}} \Theta_{t-1}^j B_t^j}$. The foreign budget constraint is redundant by Walras Law. Substituting the demand schedules (10)–(11) into the expression for net exports and combining prices into CES price indexes of exports P_{Ht} and imports P_{Ft}^* , we can rewrite the country budget constraint (21) as:

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = \gamma e^{(1-\gamma)\xi_t} (\mathcal{E}_t P_{Ft}^*)^{1-\theta} P_t^\theta C_t \left[e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{S}_t^{\theta-1} \mathcal{Q}_t^\theta \frac{C_t^*}{C_t} - 1 \right], \quad (22)$$

where again $\tilde{\xi}_t \equiv \xi_t - \xi_t^*$, \mathcal{Q}_t is the real exchange rate, and $\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_{Ft}^*}{P_{Ht}}$ is the terms of trade, which in light of PCP Calvo price setting and the implied law of one price (15) satisfies:

$$\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_{Ft}^*}{P_{Ht}} = \mathcal{Q}_t \left[\frac{(1-\gamma)e^{-\gamma\xi_t^*} + \gamma e^{(1-\gamma)\xi_t^*} \mathcal{S}_t^{-(1-\theta)}}{(1-\gamma)e^{-\gamma\xi_t} + \gamma e^{(1-\gamma)\xi_t} \mathcal{S}_t^{1-\theta}} \right]^{\frac{1}{1-\theta}} \Rightarrow \mathcal{S}_t \approx \mathcal{Q}_t^{\frac{1}{1-2\gamma}}, \quad (23)$$

where the approximation is a log-linearization around symmetric equilibrium with $\mathcal{Q}_t = \mathcal{S}_t = 1$ and $\xi_t = \xi_t^* = 0$. Conditions (20) and (22) allow to study a variety of cases with different degree of financial openness of the economies.

3.2 Cointegration relationship between consumption and exchange rate

We now show that under a variety of circumstances, there exists a constant ς such that the equilibrium process for $\varsigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ is independent of the monetary regime. In other words, a change in the statistical process for the real exchange rate Δq_t should result in a change in the statistical process for the relative consumption growth $\Delta c_t - \Delta c_t^*$.

3.2.1 Limiting cases: financial autarky and complete markets

We start with the two extreme cases of international financial integration – financial autarky and complete markets. We show that our main result obtains immediately in these two cases, independently of the structure of the rest of the model. We then proceed with the more interesting intermediate case.

Financial autarky Consider first the special case when $J_t \cap J_t^* = \emptyset$ at all t , so that net foreign assets $\mathcal{B}_t \equiv 0$, and therefore $NX_t \equiv 0$ in every state and period. Equation (22) then implies, $C_t/C_t^* = e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{S}_t^{\theta-1} \mathcal{Q}_t^\theta$, and using (23), we have:

$$c_t - c_t^* = -(1-\gamma)\tilde{\xi}_t + \left[\theta + \frac{\theta-1}{1-2\gamma} \right] q_t. \quad (24)$$

If the process for taste shocks (home bias) for goods $\tilde{\xi}_t$ is independent from the monetary regime, it must be that the process for $\varsigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ with $\varsigma = \frac{1-2\gamma}{2(1-\gamma)\theta-1}$ is also independent from the monetary regime. Note that for $\theta > 1$, $\varsigma < 1$ and is independent from relative risk aversion σ .

Complete markets Now consider a situation where there exists a $j \in J_t \cap J_t^*$ for every state of the world (Arrow securities), so that (20) holds as equality not just in expectation, but also state-by-state,

$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \equiv \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{Q_t}{Q_{t+1}} e^{\tilde{\zeta}_{t+1}}$, where $\tilde{\zeta}_{t+1}$ is the state-specific realization of the relative wedge, or equivalently:

$$\sigma(\Delta c_t - \Delta c_t^*) = \Delta q_t + \tilde{\zeta}_t. \quad (25)$$

This corresponds to the first difference of (2) in the introduction. Assuming that wedges $\tilde{\zeta}_t$ do not change with the monetary regime, it must be that $\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ does not change as well. Of course, this nests the case of perfect international risk-sharing with $\tilde{\zeta}_t \equiv 0$.

Cole and Obstfeld (1991) case This is a special case with $\sigma = \theta = 1$, which is commonly used in the literature, as in this case the equilibrium allocation does not depend on the degree of asset market completeness. Assume additionally that $\tilde{\zeta}_t^j = \tilde{\xi}_t \equiv 0$. Under these circumstances, balanced trade $NX_t \equiv 0$ implies $C_t/C_t^* = Q_t$, which ensures financial market equilibrium (20) independently of what assets j are in the set $J_t \cap J_t^*$. Therefore, this case is equivalent to the two cases considered above, but the logic of the Cole-Obstfeld case is more general, as we show below.

3.2.2 General asset market structure

Consider a case where $J_t \cap J_t^*$ contains at least one asset, namely the foreign-currency (US dollar) risk-free bond. Considering the risk-sharing condition (20) for this bond:

$$\mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Q_{t+1}}{Q_t} e^{-\tilde{\zeta}_{t+1}} - \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \right\} = 0, \quad (26)$$

where we made use of the assumption that foreign monetary policy ensures $\pi_{t+1}^* \equiv 0$, and we additionally assumed for concreteness $\zeta_{t+1}^* \equiv 0$, that is foreigners do not face a wedge when trading the foreign-currency bond.

To make further progress, we rely on a log-linear solution to the equilibrium system. We do it primarily for tractability reasons, but this is also justified on the following grounds: (a) the log-linear solution concept is typical in the literature, including the conventional RBC and New Keynesian models; (b) even when higher order terms are included in the numerical solutions of these models, the results barely change, as the aggregate macro variables are not volatile (see Figure 3) and therefore the omitted risk premia terms are small. While doing so, we nonetheless allow for risk-sharing wedges $\tilde{\zeta}_t$, which may proxy for risk premia shocks. In particular, note that $\psi_t = -\mathbb{E}_t \tilde{\zeta}_{t+1}$ represent a UIP shock. Indeed, assume that home households can additionally trade a home-currency bond with each other (without facing a wedge), then we can derive the UIP condition from combining the home Euler equations for home- and foreign-currency risk-free bonds (see footnote 10):

$$\mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left[R_t - e^{-\tilde{\zeta}_{t+1}} R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] \right\} = 0,$$

which upon log-linearization yeilds:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t. \quad (27)$$

In our earlier work (Itskhoki and Mukhin 2019), we have argued that such UIP shocks ψ_t are essential to explain the exchange rate disconnect behavior under the floating regime. Here, however, we show that they are not enough on their own to also explain the broad set of Mussa facts, which involves a change in the exchange rate regime.¹³

For concreteness, we focus on two types of shocks – a relative productivity shock $\tilde{a}_t = a_t - a_t^*$ and a risk-sharing wedge $\psi_t = -\mathbb{E}_t \tilde{\zeta}_{t+1}$ – which we assume both follow independent exogenous AR(1) processes with common persistence parameter $\rho \in (0, 1)$ and innovations $\sigma_a \varepsilon_t^a$ and $\sigma_\psi \varepsilon_t$ respectively. Our results generalize to the case with additional shocks, including monetary shocks and foreign-good demand shocks ξ_t , and we incorporate them in our quantitative analysis in Section 5.

The log-linear approximation to (26) is given by:

$$\mathbb{E}_t \{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \} = \psi_t. \quad (28)$$

Any solution to (28) can be written in the following form:

$$\sigma (c_t - c_t^*) - q_t = -\frac{\psi_t}{1 - \rho} + m_t, \quad (29)$$

where m_t is a martingale, that is $m_{t+1} = m_t + u_{t+1}$ with $\mathbb{E}_t u_{t+1} = 0$. Any fundamental solution to (28) in addition has u_{t+1} equal some linear combination of the innovations of the exogenous shock processes, $\sigma_a \varepsilon_{t+1}^a$ and $\sigma_\psi \varepsilon_{t+1}$. In what follows, we solve for u_{t+1} and prove that under various circumstances it is independent of the monetary policy regime, and thus so is $\sigma (c_t - c_t^*) - q_t$.

The fundamental solution for u_{t+1} must satisfy the intertemporal budget constraint. We log-linearize the flow budget constraint (22), using the definition of terms of trade (23), to obtain:

$$\beta b_{t+1} - b_t = \gamma \left[\frac{2(1 - \gamma)\theta - 1}{1 - 2\gamma} q_t + c_t^* - c_t \right], \quad (30)$$

where $b_t = \frac{d(\mathcal{R}_t \mathcal{B}_t)}{P_H Y / \beta}$ is the deviation of net foreign assets from zero (symmetric steady state NFA) in units of home nominal GDP. Solve forward, imposing the NPGC, to arrive at the intertemporal budget constraint:

$$b_t + \gamma \sum_{k=0}^{\infty} \beta^k \left[\frac{2(1 - \gamma)\theta - 1}{1 - 2\gamma} q_{t+k} - (c_{t+k} - c_{t+k}^*) \right] = 0. \quad (31)$$

Generalized Cole-Obstfeld case Consider the case with the following parameter restriction:

$$\sigma = \frac{1 - 2\gamma}{2(1 - \gamma)\theta - 1}. \quad (32)$$

¹³As we show in Section 4, explaining the Mussa puzzle requires a refinement of the theory of ψ_t shocks.

This nests the Cole-Obstfeld case above as $\sigma = \theta = 1$ is a special case of (32).

When (32) is satisfied, we can use (29) and (31) to solve for the equilibrium martingale innovation.¹⁴

$$\Delta m_t = u_t = \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)} \sigma_\psi \varepsilon_t,$$

which does not depend on the monetary policy rule. Substituting into the first difference of (29), we obtain:

$$\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} = \psi_t - \frac{\beta}{1 - \beta\rho} \sigma_\psi \varepsilon_{t+1}, \quad (33)$$

confirming that the statistical process for $\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}$ does not depend on the monetary regime. In particular, $\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}$ does not respond to productivity shocks under either regime and responds to the risk-premium/risk-sharing shock ψ_t in the same way independently of the monetary regime.

3.2.3 Equilibrium price dynamics

Outside the generalized Cole-Obstfeld case, the solution cannot be obtained independent of the equilibrium price dynamics, which is clearly sensitive to changes in monetary policy, and therefore constitutes our key focus of interest. To characterize the equilibrium dynamics in this case, we derive the final dynamic equation of the model, namely the Phillips curve for prices.

In Appendix A.3.2, we log-linearize the price setting equation (17) and combine it with labor and product market clearing conditions, to arrive at the following open-economy Phillips curve:

$$(1 - \beta L^{-1})[\pi_t - \pi_t^* - 2\gamma \Delta e_t] = \kappa[(c_t - c_t^*) + \gamma \kappa_q q_t - \kappa_a \tilde{a}_t], \quad (34)$$

where L is the lag operator and $L^{-1}z_t = \mathbb{E}_t z_{t+1}$, and $\kappa \equiv (1 - 2\gamma)[\sigma + (1 - 2\gamma)\varphi] \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$ is the slope coefficient of the Phillips curve, $\kappa_q \equiv \frac{2\theta(1-\gamma)\varphi+1}{\sigma+(1-2\gamma)\varphi} \frac{2}{1-2\gamma}$ and $\kappa_a \equiv \frac{1+\varphi}{\sigma+(1-2\gamma)\varphi}$. The term in the square brackets in (34) is the result of the goods market clearing condition under flexible prices, which may be violated when prices are sticky. In addition to the risk-sharing condition (28) and the budget constraint (30), the Phillips curve (34) provides the third equilibrium condition in the goods market, which ties together the equilibrium behavior of $(c_t - c_t^*)$ and q_t .

Our assumption on the conduct of monetary policy allows us to convert (34) into a sharp tool for characterizing the equilibrium behavior of the real exchange rate. We define:

$$k_R = \begin{cases} \kappa/\sigma, & R = \text{peg}, \\ \kappa/(2\gamma\sigma), & R = \text{float}, \end{cases}$$

¹⁴Using (29), the net present value of the innovation to the budget constraint (31) at time t , which must be zero, is given by

$$\frac{\gamma}{\sigma} \sum_{k=0}^{\infty} \beta^k \left[\frac{\rho^k \varepsilon_t}{1 - \rho} - u_t \right] = \frac{\gamma}{\sigma} \left[\frac{\varepsilon_t}{(1 - \rho)(1 - \beta\rho)} - \frac{u_t}{1 - \beta} \right] = 0.$$

This yields the solution in the text.

where under the ‘float’ the monetary authorities ensure $\pi_t = \pi_t^* \equiv 0$, while under the ‘peg’ $\Delta e_t = \pi_t^* = 0$. We then combine (34) with the definition of a real depreciation, $\Delta q_t = \Delta e_t + \pi_t^* - \pi_t$, to obtain:

Lemma 1 *The equilibrium dynamics of the real exchange rate satisfies:*

$$\Delta q_t = \beta \mathbb{E}_t \Delta q_{t+1} - \sigma k_R [(c_t - c_t^*) + \gamma \kappa_q q_t - \kappa_a \tilde{a}_t], \quad (35)$$

under both monetary regimes, $R \in \{\text{float}, \text{peg}\}$.

Equation (35) is a dynamic error correction model, where an overvalued real exchange rate from the point of view of flexible-price market clearing leads to a real depreciation, and vice versa. This general error correction structure is independent from the monetary regime, however, the speed of correction does depend on it – with a faster correction under the float, since $k_{\text{float}} > k_{\text{peg}}$ as $\gamma < 1/2$. Furthermore, a change from a peg to a float is equivalent to an increase in the slope of the Phillips curve κ , e.g. due to the increased flexibility of prices (lower λ) (cf. Engel 2018). Therefore, in general, the change in the monetary policy regime can affect the equilibrium comovement between consumption and the real exchange rate.¹⁵

To solve for the equilibrium cointegration between consumption and real exchange rate and alternative monetary regimes, we substitute (29) into (35) to obtain a second-order difference equation in q_t :

$$[1 + \beta + (1 + \gamma \sigma \kappa_q) k_R] q_t - q_{t-1} - \beta \mathbb{E}_t q_{t+1} = k_R z_t, \quad (36)$$

where $z_t \equiv \frac{\psi_t}{1-\rho} + \sigma \kappa_a \tilde{a}_t - m_t$. The unique non-explosive solution consistent with (36) is given by:

$$(1 - \delta_R L) q_t = k_R \delta_R \sum_{k=0}^{\infty} (\beta \delta_R)^k \mathbb{E}_t z_{t+k}, \quad (37)$$

where $\delta_R \in (0, 1)$ is the only such root of $\beta x^2 - [1 + \beta + (1 + \gamma \sigma \kappa_q) k_R] x + 1 = 0$, and is decreasing in k_R . Using the fact that ψ_t and \tilde{a}_t are AR(1) and m_t is random walk, we evaluate the right-hand side of (37), and then use the intertemporal budget constraint (31) to solve for the equilibrium innovation to m_t :¹⁶

$$\Delta m_t = u_t = \frac{1 - \beta}{1 - \beta \rho} \left[\frac{1 + \chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho^2} \sigma_\psi \varepsilon_t}{1 + \chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho}} + \frac{\chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho^2} \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t}{1 + \chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho}} \right], \quad (38)$$

where $\chi \equiv \sigma \frac{2(1-\gamma)\theta-1}{1-2\gamma} - 1$, which is equal to zero under the generalized Cole-Obstfeld parameter

¹⁵The two limiting cases of fully rigid or fully flexible prices are trivial exceptions, since in these cases $k_R \equiv \infty$ or $k_R \equiv 0$ respectively.

¹⁶Substituting (29) into (31) and imposing a zero innovation to budget constraint, implies:

$$\frac{\sigma_\psi \varepsilon_t}{(1-\rho)(1-\beta\rho)} - \frac{u_t}{1-\beta} + \chi \sum_{k=0}^{\infty} \beta^k IRF_k^q = 0,$$

where IRF_k^q is the impulse response function of the equilibrium RER process (37), which we use to solve for u_t in (38). The solution for q_t is given by (for further derivation see Appendix A.3.3):

$$q_t = \delta_R q_{t-1} + k_R \delta_R \left[\frac{\psi_t}{(1-\rho)(1-\beta\rho\delta_R)} + \frac{\sigma \kappa_a \tilde{a}_t}{1-\beta\rho\delta_R} - \frac{m_t}{1-\beta\delta_R} \right].$$

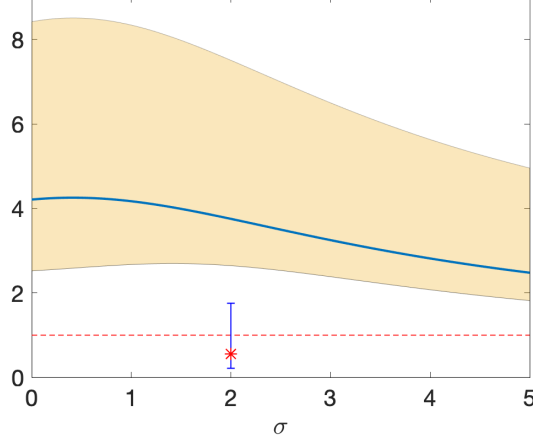


Figure 7: Ratio of $\text{std}(\tilde{\zeta}_t)$ after/during the Bretton Woods System, where $\tilde{\zeta}_t \equiv \sigma \Delta \tilde{c}_t - \Delta q_t$

Note: $\text{std}(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t)$ is computed for 1960–72 and 1973–89 for the RoW vs the U.S. for different value of σ . The dashed red line at 1 illustrates the prediction of Proposition 1 for the ratio of the standard deviation across the regimes. The red asterisk (and the simulated 90% blue confidence interval) correspond to the calibrated quantitative model, which is not nested as a special case of Proposition 1.

restriction (32). Note that, in general, u_t depends on the monetary regime via k_R and δ_R terms in (38), however, it is not the case under any of the following circumstances: (a) $\chi = 0$; (b) $k_R \equiv 0$ or $\delta_R \equiv 0$; (c) $\beta \rightarrow 1$; or (d) $\rho \rightarrow 1$, as we summarize in Proposition 1 below.

3.3 Empirical Falsification

The general characterization above suggests that the cointegration relationship (29) between consumption and the real exchange rate is in general endogenous to the monetary regime, as is evident from the presence of k_R and δ_R terms in (38). Nonetheless, the result below emphasizes a number of limiting cases when it does not depend on the monetary regime:

Proposition 1 *The cointegration relationship between relative consumption $c_t - c_t^*$ and the real exchange rate q_t does not depend on the exchange rate regime (peg vs float) under any of the following circumstances:*

1. *international financial autarky;*
2. *complete international asset markets (with or without international risk-sharing wedges);*
3. *generalized Cole-Obstfeld case, $\sigma = \frac{1-2\gamma}{2(1-\gamma)\theta-1}$, with or without risk-sharing wedges;*
4. *in the limit of both fully fixed and fully flexible prices;*
5. *in the limit of perfect patience, $\beta \rightarrow 1$, independently of the persistence of shocks $\rho < 1$;*
6. *in the limit of persistent shocks, $\rho \rightarrow 1$, independently of the discount factor $\beta < 1$.*

In all these case, the process for $\sigma(c_t - c_t^) - q_t$ is independent of the exchange rate regime, and in particular $\text{var}(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t)$ remains unchanged as the economy switches between a peg and a float.*

The first three cases are clear from the discussion above, while the remaining two cases require an additional explanation. The case of $\beta \rightarrow 1$ is intuitive, as prices become flexible in the medium run,

and hence the slope of the Phillips curve is becoming inessential from the point of view of a perfectly patient economy. As $\rho \rightarrow 1$, the ψ_t shock starts to dominate the volatility of the real exchange rate...

Proposition 1 is clearly falsified by the data, as we illustrate in Figure 7, as well as in Figure 5c and Table 1. While Proposition 1 suggests that the relationship between relative consumption and real exchange rate should not be affected by monetary policy, the data suggests a dramatic change in the volatility of $\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$, falsifying the models nested by the proposition. Such models are of course special limiting cases, however, quantitatively the prediction of Proposition 1 is accurate even outside these special cases — as we illustrate in Figure 5 and discuss further in the quantitative Section 5.

4 An Alternative Model of Non-neutrality

We now present an alternative explanation to the broad set of Mussa facts documented in Section 2. Specifically, we propose a model with monetary non-neutrality emerging due to financial market segmentation, rather than as a result of goods-market price stickiness. We build on the modeling environment of Section 3, but to emphasize the point assume away price stickiness (set $\lambda = 0$). The only other change to the environment is the modeling of the international financial market, as we describe next.

4.1 Segmented Financial Market

There are three types of agents participating in the financial market: households, noise traders and professional intermediaries. The home and foreign households trade local currency bonds only. In particular, the home households demand at time t a quantity B_{t+1} of the home-currency bonds. Similarly, foreign households demand a quantity B_{t+1}^* of the foreign-currency bonds. Both B_{t+1} and B_{t+1}^* can take positive or negative values, depending on whether the households save or borrow respectively. Therefore, in the notation of Section 3, $|J_t| = |J_t^*| = 1$ and $J_t \cap J_t^* = \emptyset$.

The trades of the households are intermediated by risk-averse intermediaries, or market makers. There are m symmetric intermediaries, who adopt a zero-capital *carry trade* strategy, that is take a long position in the foreign-currency bonds and a short position of equal value in the home-currency bonds, or vice versa. The return on this carry trade is therefore:

$$\tilde{R}_{t+1}^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \quad (39)$$

per one dollar invested in the foreign-currency bond and \mathcal{E}_t euros sold of home-currency bonds at time t . We denote the size of their position by d_{t+1}^* , which may take positive or negative values depending on whether they are long or short in the foreign-currency bond, and assume that intermediaries maximize the CARA utility of the real return on their investment in units of the foreign consumption good:

$$\max_{d_{t+1}^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* \right) \right\}, \quad (40)$$

where $\omega \geq 0$ is the risk aversion parameter.¹⁷ In aggregate, all m intermediaries invest $\frac{D_{t+1}^*}{R_t^*} = m d_{t+1}^*$ dollars in foreign-currency bonds, as one dollar at time t affords a quantity R_t^* of dollar bonds D_{t+1}^* , each paying out one dollar at $t+1$. The intermediaries also take an offsetting position of $\frac{D_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^*}{R_t^*}$ euros in home-currency bonds, resulting in a zero-value portfolio at time t .

Finally, there are n symmetric noise (or liquidity) traders, who have an exogenously-evolving demand for the foreign currency.¹⁸ Like intermediaries, noise traders take a zero-capital position long in the foreign currency and short equal value in the home currency, or vice versa if they have an excess demand for the home currency. The overall position of the noise traders is

$$\frac{N_{t+1}^*}{R_t^*} = n \left(e^{\psi_t} - 1 \right) \quad (41)$$

in foreign-currency bonds and respectively $\frac{N_{t+1}}{R_t} = -\mathcal{E}_t \frac{N_{t+1}^*}{R_t^*}$ in home-currency bonds. We refer to the noise trader shock ψ_t as the *financial shock*, and assume it follows an AR(1) process:

$$\psi_t = \rho \psi_{t-1} + \sigma_\psi \varepsilon_t, \quad \varepsilon_t \sim iid(0, 1), \quad (42)$$

where $\rho \in [0, 1]$ is the persistence of the financial shock and $\sigma_\psi \geq 0$ is its volatility. The incomes (and losses) of both intermediaries and noise traders are returned in the end of each trading period to the foreign households as lump sum payments together with the dividends of the foreign firms, Π_{t+1}^* .¹⁹

Both currency bonds are in zero net supply, and therefore financial market clearing requires that the positions of the households, intermediaries and the noise traders balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \quad \text{and} \quad B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0. \quad (43)$$

As both noise traders and intermediaries hold zero-capital positions, financial market clearing (43) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t \frac{B_{t+1}^*}{R_t^*} = 0$. In other words, the financial market merely intermediates the intertemporal borrowing between home and foreign households.

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would be happy to do so without risk premia, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected return on the carry trade, $\mathbb{E}_t \frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} = 0$. However, under risk aversion, $\omega > 0$, the intermediaries are not willing

¹⁷A property of the portfolio choice under CARA utility is that the solution does not depend on the level of wealth of the intermediaries, thus avoiding the need to carry it as an additional state variable, which would be the case under CRRA utility.

¹⁸The noise traders demand a certain position in home and foreign currency independently of the expected return on this portfolio, $\mathbb{E}_t \tilde{R}_{t+1}^*$, and of the other macroeconomic fundamentals reflected in the state variables of the home and foreign economies. Their demand for currency can be motivated as a liquidity demand, or alternatively as emerging from biased expectations about the exchange rate, $\mathbb{E}_t^n \mathcal{E}_{t+1} \neq \mathbb{E}_t \mathcal{E}_{t+1}$, as in [Jeanne and Rose \(2002\)](#).

¹⁹This generates an additional income of $\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*} \tilde{R}_{t+1}^*$ dollars for the foreign households. As a result of this transfer, the foreign country budget constraint becomes the same as the home country budget constraint (21), despite that foreign households face a generally different rate of return $\tilde{R}_t^* \neq R_t$. See Appendix ?? for details, where we also show that this transfer is second order and hence does not affect the first order dynamics of the equilibrium system.

to take a risky carry trade position without an appropriate compensation, resulting in equilibrium risk premia and deviations from the UIP. We characterize the equilibrium in the financial market in:

Lemma 2 *The equilibrium condition in the financial market, log-linearized around a symmetric steady state with $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$ and assuming that $\omega\sigma_e^2$ is asymptotically finite and non-zero, is:*

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \quad (44)$$

where $i_t \equiv \log R_t$, $i_t^* \equiv \log R_t^*$, $b_{t+1} \equiv \frac{1}{P_H Y} B_{t+1} = -\frac{\bar{\varepsilon}}{P_H Y} B_{t+1}^*$, the coefficients $\chi_1 \equiv \frac{n}{m} \omega \sigma_e^2$ and $\chi_2 \equiv \frac{\bar{P}_H \bar{Y}}{m} \beta \omega \sigma_e^2$, and $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ is the conditional volatility of the nominal exchange rate.

The equilibrium condition (3) is the *modified UIP* in our model with imperfect financial intermediation, where the right hand side corresponds to the departures from the UIP. Condition (3) arises from the combination of the financial market clearing (43) with the solution to the portfolio choice problem of the intermediaries (40), as we formally show in Appendix A.3.4. Intuitively, the optimal portfolio of the intermediaries d_{t+1}^* is proportional to the expected log return on the carry trade, $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, scaled by the risk absorption capacity of the intermediaries, $\omega\sigma_e^2$, i.e. the product of their risk aversion and the volatility of the carry trade return (namely, the exchange rate risk). As $\omega\sigma_e^2 \rightarrow 0$, the risk absorption capacity of the intermediaries increases, and the UIP deviations disappear in the limit as $\chi_1, \chi_2 \rightarrow 0$. With $\omega\sigma_e^2 > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. The noise-trader shocks ψ_t create exogenous UIP deviations, while all other shocks result in endogenous UIP deviations by means of their effect on the external imbalances b_{t+1} , which need to be intermediated by the financial sector. Note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond – by the noise traders and households, respectively – and hence result in a negative expected return on the foreign currency bond.²⁰

4.2 Mussa puzzle: limiting resolution

The general solution for the real exchange rate under the generalized UIP condition (44) is given by:

Lemma 3 (i) *Under floating regime, $\text{var}_t(\Delta e_{t+1}) > 0$ and hence $\chi_1, \chi_2 > 0$ in (44). The real exchange rate then follows ARMA(2,1):*

$$(1-\delta L)q_t = \frac{1}{1+\gamma\sigma\kappa_q} \frac{\beta\delta}{1-\beta\rho\delta} \left[(1-\beta^{-1}L)\chi_1\psi_t + \left(\frac{(1-\beta\delta)/(\beta\delta)}{1+\frac{\chi}{1+\gamma\sigma\kappa_q}}(1-\rho\delta L) + (1-\rho)(1-\beta^{-1}L) \right) \sigma\kappa_a \tilde{a}_t \right], \quad (45)$$

where $\delta \in (0, 1]$ and $\delta \rightarrow 1$ as χ_2 in (44) approaches 0. Furthermore, $e_t \equiv q_t$ and $\pi_t = \pi_t^* \equiv 0$.

(ii) *Under peg regime, $e_t \equiv 0$, and thus $\chi_1 = \chi_2 = 0$ in (44). Furthermore, $\pi_t^* \equiv 0$ and $\Delta q_t = -\pi_t$.*

²⁰Imperfect risk absorption capacity of the intermediaries results in the expected deviations from the UIP and thus expected profits in the financial market, which are returned lump sum to the foreign households. While the resulting UIP wedge is first order, the expected profits from the carry trade are second order (as the optimal portfolio is proportional to the expected UIP deviation), and hence it is negligible from the point of view of the linearized budget constraint of the foreign country.

The real exchange rate then follows an ARIMA(1,1,1):

$$\Delta q_t = \frac{1}{1 + \gamma\sigma\kappa_q} \frac{\beta}{1 - \beta\rho} \left[\frac{(1 - \beta)/\beta}{1 + \frac{\chi}{1 + \gamma\sigma\kappa_q}} (1 - \rho L) + (1 - \rho)(1 - \beta^{-1}L) \right] \sigma\kappa_a \tilde{a}_t. \quad (46)$$

The proof of Lemma 3 relies on the combination of the modified UIP condition (44) and the budget constraint (30) after expressing out consumption and interest rates using the goods market clearing condition and the household Euler equations for local bonds:

$$c_t - c_t^* = \kappa_a \tilde{a}_t - \gamma\kappa_q q_t, \quad (47)$$

$$r_t - r_t^* = \sigma\kappa_a \mathbb{E}_t \Delta \tilde{a}_{t+1} - \gamma\kappa_q \mathbb{E}_t \Delta q_{t+1}, \quad (48)$$

where $r_t = i_t - \mathbb{E}_t \pi_t = \sigma \mathbb{E}_t \Delta c_{t+1}$. Recall that κ_a and κ_q were defined in (34), and χ was defined in (38) such that $\chi = 0$ under the generalized Cole-Obstfeld parameter restriction (32). See the proof in Appendix A.3.5.

Proposition 2 *A change in the monetary policy rule from peg to float results in a sharp increase in volatility of both nominal and real exchange rates – arbitrary large when $\rho \approx 1$ – with the change in the behavior of the other macro variables vanishingly small when $\gamma \approx 0$.*

In addition to (47) and (48), we make use of the expression for the real wage:

$$w_t - p_t = a_t - \frac{\gamma}{1 - 2\gamma} q_t. \quad (49)$$

Under the peg, we can rewrite it in first difference as:

$$\Delta w_t = \Delta a_t - \left[1 + \frac{\gamma}{1 - 2\gamma} \right] \Delta q_t,$$

where we replaced $\pi_t = -\Delta q_t$. Under the float, we have instead:

$$\Delta w_t = \Delta a_t + \frac{\gamma}{1 - 2\gamma} \Delta q_t.$$

When γ is small, the real exchange rate volatility has little effect on the properties of the real quantities, such as $\Delta c_t - \Delta c_t^*$ and $r_t - r_t^*$, and thus the change in the exchange rate regime has little effect on their volatility, which is mostly due to the productivity shock anyways (see (47) and (48)).

The volatility of inflation changes from zero under the float to a positive number which is proportional to the volatility of the productivity shock. When $\rho \rightarrow 1$, the volatility of inflation (and the real exchange rate) under the peg is given by:

$$\text{std}(\pi_t) = \text{std}(\Delta q_t) = \frac{\sigma\kappa_a}{1 + \gamma\sigma\kappa_q + \chi} \text{std}(\Delta \tilde{a}_t).$$

Therefore, $\text{std}(\pi_t)$ is arbitrary smaller than $\text{std}(\Delta q_t)$ under the float, and it is equal to $\text{std}(\Delta q_t)$ under

the peg. Finally, for nominal wages, $\text{std}(\Delta w_t)$, additionally consider the limit with $\gamma \rightarrow 0$. Then:

$$\text{std}(\Delta w_t)|_{float} = \text{std}(\Delta a_t) \quad \text{and} \quad \text{std}(\Delta w_t)|_{peg} = \text{std}(\Delta a_t - \Delta q_t) = \frac{|1 + \chi - \sigma \kappa_a|}{1 + \chi} \text{std}(\Delta a_t).$$

5 Quantitative Exploration

Table 3: Quantitative models

Panel A: standard deviations															
	Δe_t			Δq_t			π_t			Δc_t			Δgdp_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Models w/o financial shock:															
IRBC	1.5	12.0	8.0	15.4	15.4	1.0	12.7	3.2	0.2	9.1	9.1	1.0	15.0	15.0	1.0
NKOE-1	1.5	12.0	8.0	4.2	12.8	3.0	3.1	1.8	0.6	7.1	6.8	1.0	17.7	11.7	0.7
NKOE-2	1.5	12.0	8.0	1.5	11.5	7.4	1.3	1.3	1.0	5.0	5.2	1.0	8.1	8.4	1.0
Models w/ exogenous financial shock:															
IRBC	1.5	12.0	8.0	11.0	11.0	1.0	10.2	0.9	0.1	1.8	1.8	1.0	2.5	2.5	1.0
NKOE-1	1.5	12.0	8.0	2.2	11.9	5.3	1.4	0.4	0.3	5.8	1.3	0.2	14.5	2.1	0.1
NKOE-2	1.5	12.0	8.0	2.1	11.8	5.7	1.3	0.3	0.2	5.8	1.1	0.2	8.6	1.8	0.2
Models w/ endogenous financial shock:															
IRBC	1.5	12.0	8.0	3.0	11.0	3.6	1.4	0.9	0.7	1.6	1.8	1.1	2.5	2.5	1.0
NKOE-1	1.5	12.0	8.0	1.7	11.9	6.9	0.4	0.4	1.0	1.1	1.3	1.1	1.9	2.1	1.1
NKOE-2	1.5	12.0	8.0	1.4	11.8	8.2	0.2	0.3	1.5	0.9	1.1	1.2	1.5	1.8	1.2

Panel B: correlations															
	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta n x_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$		β^{UIP}		
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	
Models w/o financial shock:															
IRBC	0.86	0.99	0.91	0.91	-0.10	-0.10	0.30	0.30	0.34	0.34	0.99	0.99	0.8	0.9	
NKOE-1	0.67	0.99	0.28	0.70	-0.10	-0.49	0.38	0.31	0.65	0.41	0.91	0.97	0.3	1.0	
NKOE-2	0.96	0.99	0.49	0.99	-0.10	0.05	0.95	0.30	0.97	0.33	1.00	1.00	1.0	1.0	
Models w/ exogenous financial shock:															
IRBC	0.86	0.99	-0.40	-0.40	0.93	0.93	0.30	0.30	0.15	0.15	0.88	0.88	0.0	-1.3	
NKOE-1	0.81	1.00	-0.88	-0.40	0.89	0.93	0.60	0.30	-0.06	0.32	0.99	0.84	-0.1	-1.6	
NKOE-2	0.82	1.00	-0.89	-0.40	0.92	0.97	0.51	0.30	-0.10	0.26	1.00	0.79	-0.1	-2.2	
Models w/ endogenous financial shock:															
IRBC	0.98	1.00	0.92	-0.40	-0.10	0.93	0.30	0.30	0.39	0.16	0.99	0.88	1.0	-1.4	
NKOE-1	0.98	1.00	0.84	-0.40	-0.10	0.93	0.44	0.30	0.54	0.32	0.96	0.84	1.0	-1.6	
NKOE-2	1.00	1.00	0.94	-0.40	-0.10	0.97	0.66	0.30	0.70	0.26	0.99	0.79	1.0	-2.3	

Note: in all calibrations, shocks are normalized to obtain $\text{std}(\Delta e_t) = 12\%$ under the float. Parameter ϕ_e in the Taylor rule is calibrated to generate 8-fold reduction in $\text{std}(\Delta e_t)$ between monetary regimes. When possible, relative volatilities of shocks are calibrated to match $\text{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ under the float and $\text{corr}(\Delta q_t, \Delta n x_t) = -0.1$ under the peg. The cross-country correlation of productivity/monetary shocks matches $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the float. Capital adjustment parameter ensures that $\frac{\text{std}(\Delta i_t)}{\text{std}(\Delta gdp_t)} = 2.5$ under the float. The moments are calculated by simulating the model for $T = 100,000$ quarters. NKOE models feature sticky wages and LCP sticky prices, while IRBC models feature no nominal rigidities; IRBC and NKOE-1 feature productivity shocks, while NKOE-2 features monetary (Taylor rule) shocks.

A Appendix

A.1 Additional Figures and Tables

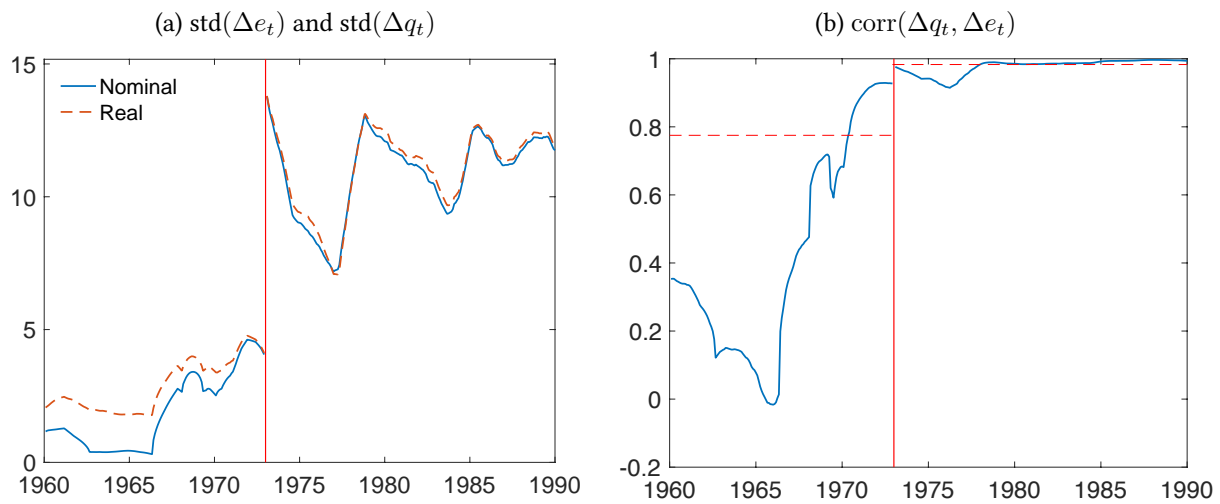


Figure A1: Volatility and correlation of nominal and real exchange rates over time

Note: The left panel plots standard deviations and the right panel plots correlation of Δe_t and Δq_t over time. Series from Figure 1; triangular moving averages with a window over 18 months before and after the date, treating 1973:01 as the end point for the two regimes (see Figure 3 and Appendix Figure A4 for other variables).

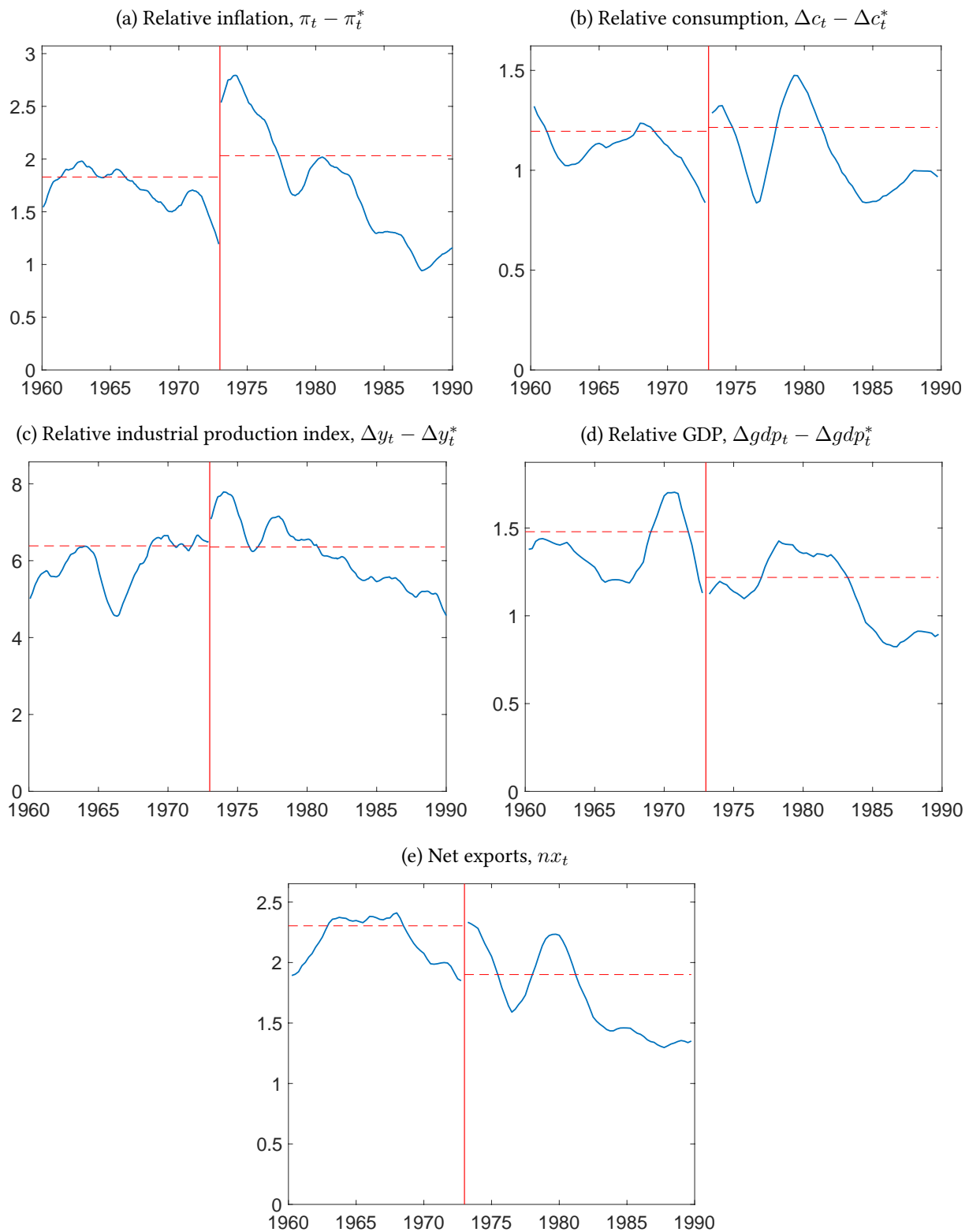


Figure A2: Volatility of macroeconomic variables over time

Note: The top four panels zoom in on the panels b, d, e and f in Figure 3. The bottom panel plots the standard deviations of nx_t defined as the ratio of export minus imports to the sum of exports and imports, for the US against the rest of the world.



Figure A3: Volatility over time: alternative break at 1971:08

Note: Like Appendix Figure A2, but with an alternative break date in 1971:08 (1971:Q3). **DIFFERENT DEFINITION**

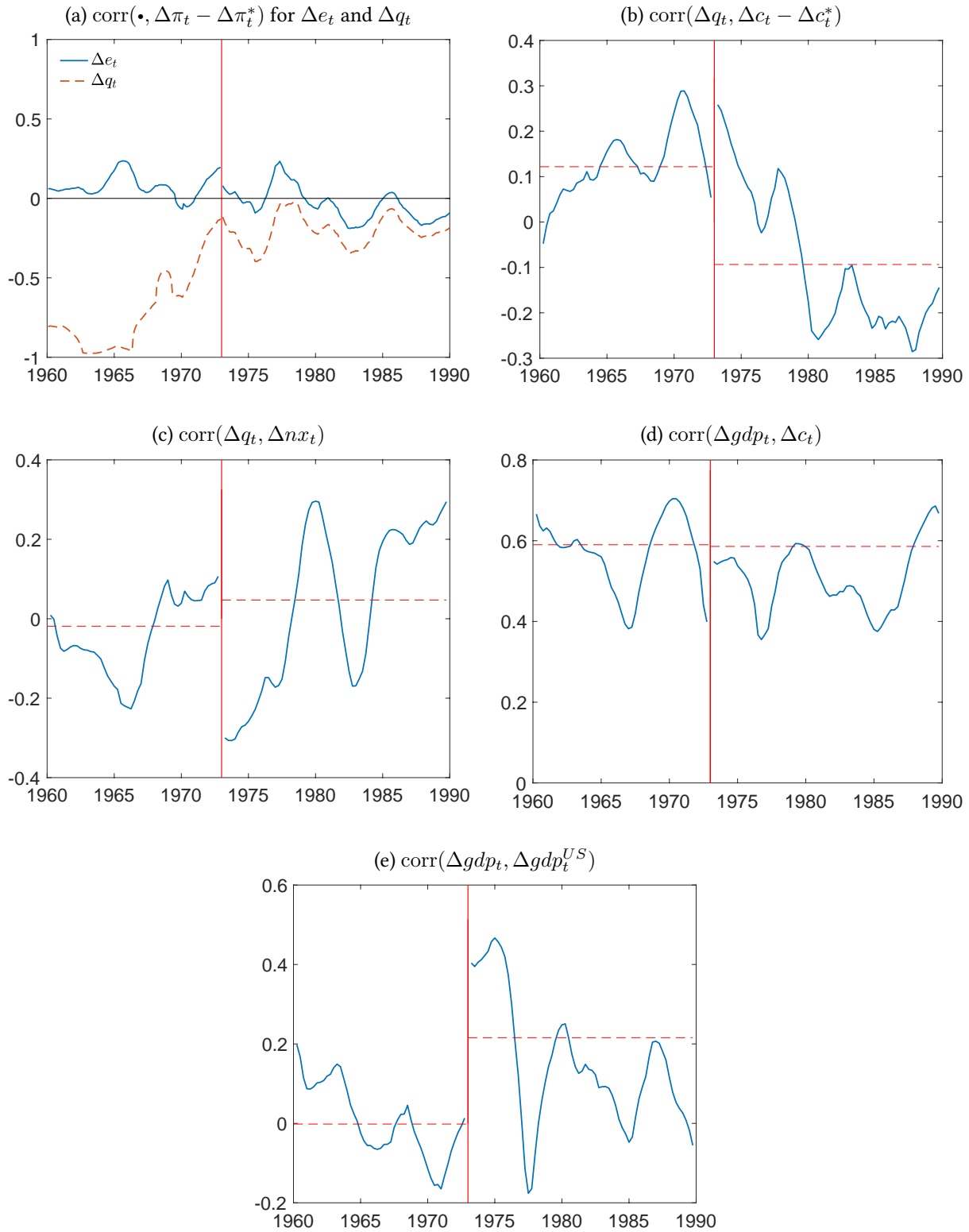
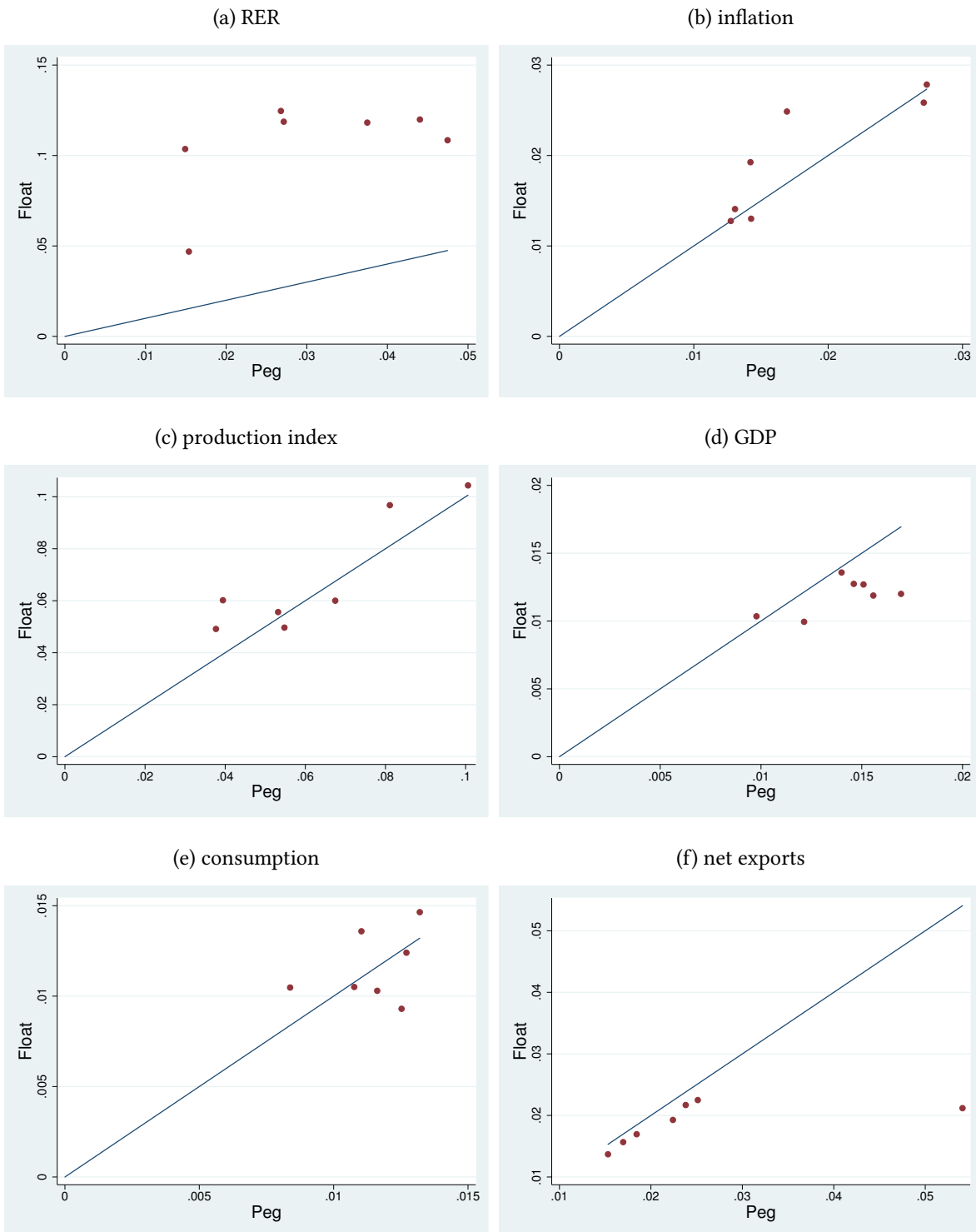


Figure A4: Correlations over time

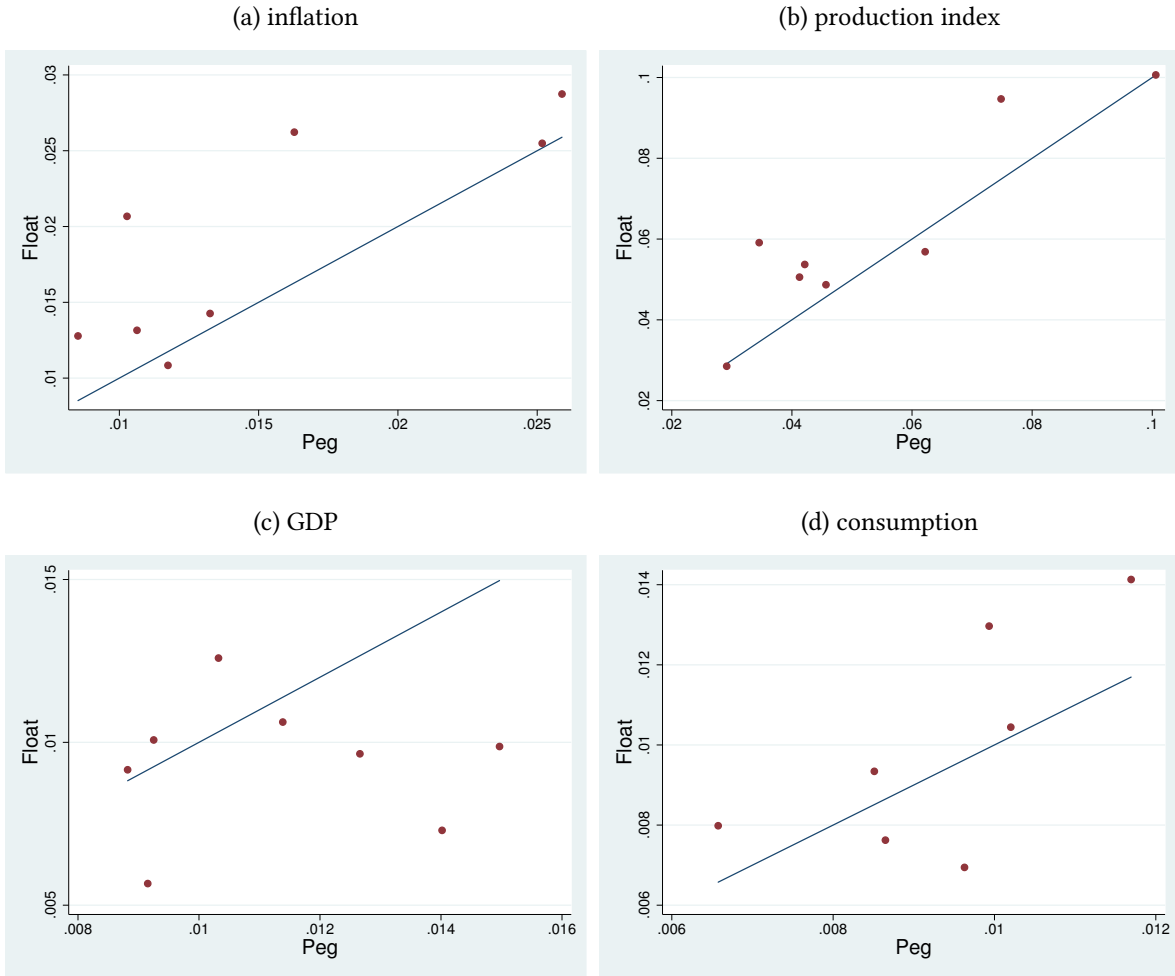
Note: the pictures shows the correlations for the RoW estimated separately before and after January 1973. The rolling window is up to 5 years for quarterly series – shorter at the corners – with linearly decreasing weights.

Figure A5: Scatter plots: volatility before and after the end of the Bretton-Woods System



Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

Figure A6: Scatter plots: country-level instead of cross-differences



Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

The scatter plots from Figures A5 show that the volatility of the RER is higher under the float than under the peg for every single country in our sample. Interestingly, the floating regime results in almost equal volatility of exchange rates across countries except for Canada, which retained partial peg to the dollar during 1970-80s. At the same time, the countries concentrate tightly along the 45-degree line for other macroeconomic variables indicating small changes in their volatilities across the regimes. (The only exception is Spain with an abnormally high volatility of net exports in 1960s.) Interestingly, there is more variation for country-level series instead of the cross-country differences (see Figure A6), but again we find no systematic differences for fundamentals between two regimes.

A.2 Data

Additional details for Section 2:

1. CPI data for Canada in 1960 is from OECD, but is downloaded from FRED and made consistent with the rest of the series.
2. **Outliers:**
 - (a) civil unrests in France in May–June 1968 led to a more than 20% fall in production; France also had abnormally volatile production index during the whole 1960s;
 - (b) earthquake and tsunami in Japan in March–April 2011 led to 17% fall in production. Since these observations are required for aggregation across non-U.S. countries, I replaced them with extrapolations using the values before and after the episodes.
 - (c) The same applies to Germany production index in 1984:06, which constitutes a clear measurement error, and Spain production index in 1960, which is missing.

A.3 Derivations

A.3.1 Equilibrium system, steady state, log-linearization

A.3.2 Phillips curve

The log-linearized price setting equation (17) is given by:

$$\bar{p}_{Ht} = (1 - \beta\lambda) \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \{w_{t+k} - a_{t+k}\},$$

that is a discounted weighted average of future marginal costs (14) conditional on the price staying in effect. We rewrite this condition as:

$$\bar{p}_{Ht} - p_{t-1} = (1 - \beta\lambda) \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \{w_{t+k} - a_{t+k} - p_{t+k}\} + \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \pi_{t+k}$$

or equivalently

$$(1 - \beta\lambda L^{-1})(\bar{p}_{Ht} - p_{H,t-1}) = (1 - \beta\lambda)(w_t - a_t - p_{Ht}) + \pi_{Ht}. \quad (\text{A1})$$

Next, due to the Calvo pricing structure, the evolution of the home-good inflation is then given by:

$$\pi_{Ht} = (1 - \lambda)(\bar{p}_{Ht} - p_{H,t-1}), \quad (\text{A2})$$

and thus we can combine the previous two equation to obtain the Phillips curve for π_{Ht} :

$$\pi_{Ht} = \beta \mathbb{E}_t \pi_{H,t+1} + \hat{\kappa}(w_t - a_t - p_{Ht}), \quad \hat{\kappa} \equiv \frac{(1 - \lambda)(1 - \beta\lambda)}{\lambda}. \quad (\text{A3})$$

Using the price index and the law of one price (15), we have:

$$p_t = (1 - \gamma)p_{Ht} + \gamma(p_{Ft}^* + e_t),$$

or equivalently:

$$p_t - p_{Ht} = \gamma s_t = \frac{\gamma}{1 - 2\gamma} q_t.$$

Substituting this into (A3):

$$\pi_t - \frac{\gamma}{1 - 2\gamma} \Delta q_t = \beta \mathbb{E}_t \left\{ \pi_{t+1} - \frac{\gamma}{1 - 2\gamma} \Delta q_{t+1} \right\} + \hat{\kappa} \left(\sigma c_t + \varphi y_t - (1 + \varphi) a_t + \frac{\gamma}{1 - 2\gamma} q_t \right),$$

where we have used (7) and (13) to establish that:

$$w_t - p_t - a_t = \sigma c_t + \varphi \ell_t - a_t = \sigma c_t + \varphi y_t - (1 + \varphi) a_t.$$

Next, taking the difference between π_t and the equivalent Phillips curve for π_t^* , we arrive at:

$$z_t = \beta \mathbb{E}_t z_{t+1} + \hat{\kappa} (1 - 2\gamma) x_t,$$

where

$$\begin{aligned} z_t &\equiv \frac{1}{1 - 2\gamma} \left[\pi_t - \pi_t^* - \frac{2\gamma}{1 - 2\gamma} \Delta q_t \right] = \pi_t - \pi_t^* - 2\gamma \Delta e_t, \\ x_t &\equiv \sigma (c_t - c_t^*) + \varphi (y_t - y_t^*) - (1 + \varphi) \tilde{a}_t + \frac{2\gamma}{1 - 2\gamma} q_t. \end{aligned}$$

The last step is to solve out $y_t - y_t^*$ using the market clearing condition:

$$\begin{aligned} y_t &= (1 - \gamma) [-\theta (p_{Ht} - p_t) + c_t] + \gamma [-\theta (p_{Ht} - e_t - p_t^*) + c_t^*] \\ &= (1 - \gamma) c_t + \gamma c_t^* + 2\theta \frac{\gamma(1 - \gamma)}{1 - 2\gamma} q_t, \end{aligned}$$

so that

$$y_t - y_t^* = (1 - 2\gamma)(c_t - c_t^*) + 4\theta \frac{\gamma(1 - \gamma)}{1 - 2\gamma} q_t,$$

and we can rewrite:

$$x_t = [\sigma + (1 - 2\gamma)\varphi] \left[(c_t - c_t^*) + \frac{2\gamma}{1 - 2\gamma} \frac{2\theta(1 - \gamma)\varphi + 1}{\sigma + (1 - 2\gamma)\varphi} q_t - \frac{1 + \varphi}{\sigma + (1 - 2\gamma)\varphi} \tilde{a}_t \right].$$

Note that $x_t = 0$ is goods market clearing under flexible prices (Backus-Smith resolution in disconnect).

A.3.3 Cointegration

See footnote 16. The impulse response of $(1 - \delta_R L)q_t$ is given by:

$$IRF_k^{(1 - \delta_R L)q_t} = \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \left[\frac{\rho^k}{1 - \beta \rho \delta_R} \left(\frac{\sigma_\psi \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) - \frac{u_t}{1 - \beta \delta_R} \right]$$

And therefore the impulse response of q_t is:

$$IRF_k^{q_t} = \sum_{j=0}^k \rho^{k-j} IRF_k^{(1 - \delta_R L)q_t}$$

We calculate:

$$\begin{aligned} \sum_{k=0}^{\infty} \beta^k IRF_k^{qt} &= \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \sum_{k=0}^{\infty} \sum_{j=0}^k \beta^k \rho^{k-j} \left[\frac{\rho^k}{1 - \beta \rho \delta_R} \left(\frac{\sigma_{\psi} \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) - \frac{u_t}{1 - \beta \delta_R} \right] \\ &= \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \left[\frac{1}{1 - \beta \rho \delta_R} \left(\frac{\sigma_{\psi} \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) \underbrace{\sum_{k=0}^{\infty} (\beta \rho)^k \frac{1 - \rho^{k+1}}{1 - \rho}}_{=\frac{1-\rho}{(1-\beta\rho)(1-\beta\rho^2)}} - \frac{u_t}{1 - \beta \delta_R} \underbrace{\sum_{k=0}^{\infty} \beta^k \frac{1 - \rho^{k+1}}{1 - \rho}}_{=\frac{1-\rho}{(1-\beta)(1-\beta\rho^2)}} \right] \end{aligned}$$

Substituting this back into the first expression in footnote 16 results in the solution (38) in the text.

Limiting case $\rho \rightarrow 1$

$$\begin{aligned} \Delta m_t &= u_t = \frac{\sigma_{\psi} \varepsilon_t}{1 - \rho}, \\ \Delta m_t - \frac{\Delta \psi_t}{1 - \rho} &= \frac{\sigma_{\psi} \varepsilon_t}{1 - \rho} - \frac{(\rho - 1) \psi_{t-1} + \sigma_{\psi} \varepsilon_t}{1 - \rho} = \psi_{t-1} \\ \sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t &= \psi_{t-1} \\ (1 - \delta_R L) \Delta q_t &= -\frac{k_R \delta_R}{1 - \beta \delta_R} \frac{1}{1 + \gamma \sigma \kappa_q} \psi_{t-1}. \end{aligned}$$

A.3.4 Segmented financial market

Lemma A4 *The solution to the portfolio choice problem (40) when the time periods are short is given by:*

$$d_{t+1}^* = -\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}, \quad (\text{A4})$$

where $i_t \equiv \log R_t$, $i_t^* \equiv \log R_t^*$, $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ and $\sigma_{e\pi^*} = \text{cov}_t(\Delta e_{t+1}, \Delta p_{t+1}^*)$.

Proof: See Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). ■

Note the extra terms in the numerator of (A4), which correspond to the Jensen inequality corrections to the expected log return on the carry trade.

Proof of Lemma 2 Consider the market clearing for the foreign-currency bond in (43) and substitute in (41) and (A4) to obtain after a few algebraical manipulations:

$$\frac{B_{t+1}^*}{R_t^*} + n(e^{\psi_t} - 1) = m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}.$$

The market clearing conditions in (43) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1} + N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^* + N_{t+1}^*}{R_t^*}$, results in the equilibrium balance

between home and foreign household asset positions:

$$\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B_{t+1}^*}{R_t^*}.$$

Therefore, we rewrite the equilibrium condition in the financial market as:

$$m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2} = n(e^{\psi_t} - 1) - \frac{B_{t+1}}{\mathcal{E}_t R_t}.$$

Next we log-linearize this condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$ and $\bar{B} = \bar{B}^* = 0$. Furthermore, we assume that shocks are small, resulting in σ_e^2 and $\sigma_{e\pi^*}$ being second order, however we adopt the asymptotics in which as σ_e^2 shrinks ω increases proportionally leaving $\omega \sigma_e^2$ constant, finite and nonzero. This results in the following log linearization:

$$\frac{m}{\omega \sigma_e^2} (i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}) = n\psi_t - \frac{\bar{P}_H \bar{Y}}{\bar{\mathcal{E}} \bar{R}} b_{t+1},$$

where $b_{t+1} = \frac{1}{\bar{P}_H \bar{Y}} B_{t+1}$, or equivalently

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \underbrace{\frac{n}{m} \omega \sigma_e^2}_{\equiv \chi_1} \cdot \psi_t - \underbrace{\frac{\beta \bar{P}_H \bar{Y}}{m} \omega \sigma_e^2}_{\equiv \chi_2} \cdot b_{t+1}.$$

This completes the proof of the lemma. \blacksquare

DISCUSS FINITE $\omega \sigma_e^2$ AND CONSTANT σ_e^2 ... Cite [Hansen and Sargent \(2011\)](#)

Profits and losses in the financial market Consider the profit and losses of the non-households participants in the financial market – the intermediaries and the noise traders:

$$\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*} \tilde{R}_{t+1}^* = \left(m d_{t+1}^* + n(e^{\psi_t} - 1) \right) \left[1 - e^{i_t - i_t^* - \Delta e_{t+1}} \right] R_t^*,$$

where we used the definition of \tilde{R}_{t+1}^* in (39). Using Lemma A4, the lower-order terms of these profits are:

$$\begin{aligned} - \left(-m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}}{\omega \sigma_e^2} + n\psi_t \right) (i_t - i_t^* - \Delta e_{t+1}) \bar{R}^* &= -\frac{n}{\beta} x_{t+1} \left(-\frac{1}{\chi_1} \mathbb{E}_t x_{t+1} + \psi_t \right) \\ &= \frac{n\chi_2}{\beta\chi_1} x_{t+1} b_{t+1} = \bar{P}_H \bar{Y} \cdot b_{t+1} x_{t+1}, \end{aligned}$$

where $x_{t+1} \equiv i_t - i_t^* - \Delta e_{t+1}$, and Lemma 2 implies $\mathbb{E}_t x_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}$, where $\chi_1 = \frac{n}{m} \omega \sigma_e^2$. Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the profits and losses in the financial markets are also first order, since $B_{t+1} = \bar{P}_H \bar{Y} \cdot b_{t+1}$ is first order around $\bar{B} = 0$. Intuitively, the profits and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position of the households; while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

Covered interest parity Consider the extension of the portfolio choice problem (40) of the intermediaries with the additional option to invest in the CIP deviations:

$$\max_{d_{t+1}^*, d_{t+1}^{F*}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \left[\frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* + \frac{R_t^{F*}}{P_{t+1}^*} d_{t+1}^{F*} + \frac{R_t^*}{P_{t+1}^*} \mathcal{W}_t^* \right] \right) \right\},$$

where the return on one dollar invested in the CIP deviation (long home-currency bond plus a forward and short foreign currency bond) is:

$$R_t^{F*} = \frac{\mathcal{E}_t}{\mathcal{F}_t} R_t - R_t^*,$$

since 1 dollar at t buys $\mathcal{E}_t R_t$ units of home-currency bonds, and $d_{t+1}^{F*} \geq 0$ is the period- t dollar size of this position. Note that we also allowed for non-zero wealth \mathcal{W}_t^* of the intermediaries, which is by default invested into the ‘riskless’ foreign-currency bond. Both CIP investment and wealth investment are subject to the foreign inflation risk only, but no risk of nominal return, unlike the carry trade d_{t+1}^* . Note that the CIP investment, just like the carry trade, requires no capital at time t .

The first order optimality condition with respect to the CIP investment is:

$$R_t^{F*} \cdot \mathbb{E}_t \left\{ \frac{1}{P_{t+1}^*} \exp \left(-\omega \left[\frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* + \frac{R_t^{F*}}{P_{t+1}^*} d_{t+1}^{F*} + \frac{R_t^*}{P_{t+1}^*} \mathcal{W}_t^* \right] \right) \right\} = 0.$$

However, since the expectation term is strictly positive for any $d_{t+1}^{F*} \in (-\infty, \infty)$, this condition can be satisfied only if $R_t^{F*} = 0$. If $R_t^{F*} > 0$, then the intermediaries will take an unbounded position in the CIP trade, $d_{t+1}^{F*} = \infty$, and vice versa.

A.3.5 Real exchange rate

Proof of Lemma 3

Equations (44) and (30) form a system of dynamic equations:

$$\begin{aligned} -(1 + \gamma \sigma \kappa_q) \mathbb{E}_t \Delta q_{t+1} &= \chi_1 \psi_t - \chi_2 b_{t+1}, \\ \beta b_{t+1} - b_t &= \gamma \left[\frac{2(1 - \gamma)\theta - 1}{1 - 2\gamma} q_t + \gamma \kappa_q q_t - \kappa_a \tilde{a}_t \right] \end{aligned}$$

where we used the fact that $\pi_t = \pi_t^* = 0$ under the floating regime and also that

$$\begin{aligned} c_t - c_t^* &= \kappa_a \tilde{a}_t - \gamma \kappa_q q_t, \\ i_t - i_t^* &= \mathbb{E}_t \{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) + (\pi_t - \pi_t^*) \} = \sigma \kappa_a \mathbb{E}_t \Delta \tilde{a}_{t+1} - \gamma \sigma \kappa_q \mathbb{E}_t \Delta q_{t+1}, \end{aligned}$$

where the first equation is the result of market clearing (see derivations in Appendix A.3.2) and the latter equation is the manipulation of the household Euler equations for local bonds (see footnote 10).

References

- ALVAREZ, F., A. ATKESON, AND P. J. KEHOE (2007): "If Exchange Rates are Random Walks, Then Almost Everything We Say About Monetary Policy is Wrong," *American Economic Review*, 97(2), 339–345.
- BACKUS, D. K., AND G. W. SMITH (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35(3–4), 297–316.
- BAXTER, M., AND A. C. STOCKMAN (1989): "Business cycles and the exchange-rate regime: Some international evidence," *Journal of Monetary Economics*, 23(3), 377–400.
- BERKA, M., M. B. DEVEREUX, AND C. ENGEL (2012): "Real Exchange Rate Adjustment in and out of the Eurozone," *American Economic Review*, 102(3), 179–85.
- CAMPBELL, J. Y., AND L. M. VICEIRA (2002): *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press.
- CHARI, V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.
- COLACITO, R., AND M. M. CROCE (2013): "International Asset Pricing with Recursive Preferences," *Journal of Finance*, 68(6), 2651–2686.
- COLE, H. L., AND M. OBSTFELD (1991): "Commodity trade and international risk sharing," *Journal of Monetary Economics*, 28(1), 3–24.
- DEVEREUX, M. B., AND V. V. HNATKOVSKA (2014): "Borders and Nominal Exchange Rates in Risk-Sharing," <http://faculty.arts.ubc.ca/vhnatkovska/research.htm>.
- ENGEL, C. (2018): "Real Exchange Rate Convergence: The Roles of Price Stickiness and Monetary Policy," *Journal of Monetary Economics*, forthcoming.
- FLOOD, R. P., AND A. K. ROSE (1995): "Fixing exchange rates A virtual quest for fundamentals," *Journal of Monetary Economics*, 36(1), 3–37.
- HANSEN, L. P., AND T. J. SARGENT (2011): "Robustness and ambiguity in continuous time," *Journal of Economic Theory*, 146(3), 1195–1223.
- ITSKHOKI, O., AND D. MUKHIN (2019): "Exchange Rate Disconnect in General Equilibrium," <https://scholar.princeton.edu/itskhoki/pages/papers>.
- JEANNE, O., AND A. K. ROSE (2002): "Noise Trading and Exchange Rate Regimes," *The Quarterly Journal of Economics*, 117(2), 537–569.
- KOLLMANN, R. (2005): "Macroeconomic effects of nominal exchange rate regimes: new insights into the role of price dynamics," *Journal of International Money and Finance*, 24(2), 275–292.
- MONACELLI, T. (2004): "Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy," *Journal of International Economics*, 62(1), 191–217.
- MUSSA, M. L. (1986): "Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications," *Carnegie-Rochester Conference Series on Public Policy*, 25(1), 117–214.
- NAKAMURA, E., AND J. STEINSSON (2018): "Identification in Macroeconomics," *Journal of Economic Perspectives*, 32(3), 59–86.

OBSTFELD, M., AND K. ROGOFF (2001): "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in *NBER Macroeconomics Annual 2000*, vol. 15, pp. 339–390.

PAVLOVA, A., AND R. RIGOBON (2007): "Asset prices and exchange rates," *Review of Financial Studies*, 20(4), 1139–1180.