Income, Price Dispersion and Risk Sharing

Eungsik Kim*
Department of Economics
University of Kansas

Stephen Spear†
Tepper School of Business
Carnegie Mellon University

Abstract

In this paper, we study how income-dependent prices under imperfect competition affect consumption allocations across states and ages in a life-cycle model with the incomplete market. We show that the income-dependent prices can reduce risk sharing by biasing consumption toward the rich more than the perfectly competitive economy. Thus, there exists additional consumption volatility when the goods markets are imperfectly competitive along with the financial market friction. In numerical analysis, we quantify the complementary welfare loss from the additional volatility between the two frictions in a parametrized version of the model and find out that the supermodular welfare loss takes about 50% of the welfare loss solely from the incomplete market. We also show that the income-dependent prices over the life-cycle can depress consumption smoothing behavior and, in fact, generate correlated consumption/income profiles without other frictions, as observed in data. We conduct a policy analysis which confirms that fiscal policy can improve welfare by reducing consumption variation across ages and states, whereas monetary policy might decrease welfare by generating more variable consumption profiles over the life-cycle via an intertemporal wedge, although it can mitigate consumption volatility between states.

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*Email address: eungsikk@ku.edu
†Email address: ss1f@andrew.cmu.edu

1 Introduction

One of the important topics in macroeconomics, financial economics, and public economics is to understand the consumption risk that households face in a world of incomplete financial markets. The household consumption risk is directly related to their welfare, and governments or insurance agencies need to understand this risk in order to design stabilization policies or insurance products.

There is abundant empirical evidence to show that prices are dispersed over the income of buyers for the same goods because of bulk discounts, bargaining power, and many other reasons. In developing countries, Rao (2000) shows using Indian villages data that the poor pay more for identical goods than the rich because of quantity premiums that they have to pay when buying goods in small quantities. He shows that the Gini coefficients of real incomes are indeed greater than that of nominal incomes in the economies of the Indian villages when considering the income-dependent price heterogeneity. We can also find similar observations of quantity premiums in developed economies. For example, consumers can get price discounts if they pay for a year of expensive services at once, such as childcare. The poor cannot afford this chunk of payment due to borrowing constraints. In addition, banks provide interest rate premiums to investors who save large amounts for longer periods. Again, such opportunities are not relevant to the poor's financial situation.

Consumption allocations are determined by not only household income but also goods prices that they face. Under income-dependent heterogeneous prices, more consumptions will be allocated to the group facing relatively lower prices. Thus, imperfectly competitive goods markets can affect consumption allocation, its inequality, and risk-sharing between households. However, most consumption risk-sharing papers assume that the commodity markets are competitive where all agents buy identical goods at the same prices.[1] The analysis based on the perfectly competitive economy might lead to the under- or overestimate of the consumption risk of households.

Hence, this paper studies the effects of imperfect competition on household consumption allocations in a general equilibrium model with financial market incompleteness. Then, we examine whether imperfect competition increases consumption volatility and reduces risk sharing when financial markets are incomplete, compared to a standard competitive economy. We also show that additional consumption volatility from imperfect competition generates a significant welfare loss. In addition, we study government policies that can improve long-run welfare when both frictions are present and quantify its welfare effects.

To address these issues, we embed the Shapley-Shubik market game into a three-period-lived monetary overlapping generations (OLG) model under pure exchange and endowment risk.[2] Here, agents live youth, middle-aged and retired periods. They save and partially insure against endowment risk via fiat money in an incomplete financial market. The trading mechanism dictated by the Shapley-Shubik market game is as follows. Players trade with other anonymous players by making offers of consumption goods and bids of money in a central-

[1] There are indeed no papers studying the implications the price heterogeneity in general equilibrium models other than a few empirical papers showing the existence of such heterogeneity (see Rao 2000 and Aguiar and Hurst 2007).

[2] This three-period OLG model remains tractable but still captures an inverted-U shaped endowment structure consistent with the life-cycle income profile in the data.

ized trading post. Each player is allocated a proportion of the aggregate commodity offer in the proportion that her bid bears to the aggregate bid. Similarly, she is assigned a share of the aggregate money bid in the proportion that her offer has to the aggregate offer of the good (see Shapley and Shubik 1977; Dubey and Shubik 1978 for more details about the Shapley-Shubik market game).[3]

We confirm that agents face different *effective* prices for identical goods in our model depending on market power characterized by their income.[4] Because agents have market power, the Shapley-Shubik market game mechanism generates economies of scale (in bid allocations) associated with the size of a player's offer. We refer to the bid benefit so generated as the (money) return the agent enjoys. Players with large offers thus experience high return rates which imply that they need to give up less saving to increase their current bid for an additional unit of the good. Therefore, those with large offers can purchase identical goods at lower effective prices, and thus price heterogeneity is observed.

In this paper, we first analyze the type of equilibria that we will examine to address our questions because different types of equilibria generate different welfare implications and impose different degrees of computational difficulty in these kinds of dynamic games. We first check that simple short memory rational expectations equilibria do not exist in the OLG model with strategic interactions, as in Henriksen and Spear (2012). The non-existence of short memory monetary Nash equilibria implies that any rational expectations equilibrium must consider lagged endogenous state variables. Thus, we focus on the (forward-looking) pure strategy Markov Nash equilibrium as an equilibrium concept for our analysis. Our focus on this equilibrium concept is motivated in part by the fact that computing this type of recursive equilibrium is tractable because agents do not monitor all the past actions of others, and condition their decisions only on current states. Moreover, Kubler and Polemarchakis (2004) examine the existence of (stationary) Markov ϵ-equilibria in OLG economies which are good enough for numerical work due to its inherent rounding and truncation errors.

To understand the effects of imperfect competition on intertemporal allocations, we first study the equilibrium outcome in the model without endowment risk. Here, we analytically show that perfect consumption smoothing fails because the inverted-U shape endowment structure yields a price heterogeneity over one's life-cycle under the market game trading mechanism. Agents face the lowest effective prices in the middle-aged period when they offer the largest amount of the good and have the highest bid return rate. To intertemporally optimize, they transfer their lifetime wealth to the second period of life in order to consume more cheaply. Thus, an agent's consumption flow will follow her endowment stream in our model, consistent with the empirical stylized facts of the correlated consumption/income profiles.[5] This result indicates that a model with imperfect competition generates a distinct consumption stream compared to the perfectly competitive economy which smoothes out consumption.

Going back to the economy with both frictions, we find the primary result of this paper that consumption allocations are more biased to the advantaged agents by the endowment

[3] Fiat money in our model works not only as a store of value and an instrument to hedge risk but also as a medium of exchange in the market game.

[4] We introduce here the distinction between average prices – those given by dividing aggregate bids by aggregate offers – and effective prices – which equal agents' marginal rates of substitution at any best response because it is important for our analysis of the effects of imperfect competition.

[5] We will frequently refer to this correlation by saying that the income and consumption profiles are parallel.

shock relative to the competitive economy. Thus, imperfect competition increases consumption inequality and reduces risk sharing under financial market incompleteness. Intuitively, the trading mechanism results in lower effective prices to the advantaged group via the bid-returning process where those offering more endowments face higher return rates. Thus, the doubly lucky agents can purchase more goods in the economy with imperfect competition simply because of both higher income and lower prices. At the same time, we observe larger consumption variation between agents, which indicates weaker consumption risk-sharing.

In our numerical analysis, we check that within-age consumption volatility in the long-run equilibrium increases 4.0% for the young, 2.9% for the middle-aged and 2.6% for the old when adding imperfect competition to the incomplete market. This additional consumption volatility generates a complementary welfare loss. In other words, the welfare loss under imperfect competition and market incompleteness is bigger than the sum of welfare losses from each friction separately. As the size of shocks increases, the additional consumption volatility and its supermodular welfare loss grow as well. Our numerical analysis implies that the complementary welfare loss constitutes about 50% of the welfare loss solely from the incomplete market friction. This numerical result implies that there exists a significant supermodular welfare loss when one considers the price heterogeneity on the top of the financial market incompleteness.

To check the robustness of the results, we work with other parameter values for the time discount factor and risk aversion. Changes in these parameters still induce the same qualitative result of the complementary welfare loss from both frictions, although they can attenuate the loss quantitatively. We find that the structure of an endowment shock matters for the size of the additional consumption volatility. Unlike an idiosyncratic endowment shock, an aggregate endowment shock generates small additional consumption volatility because endowments are positively correlated across agents, and they experience similar movements in effective prices. Thus, there are no specific groups which can purchase identical goods at significantly lower prices. The within-age consumption volatility increases only 1.52% for the young, 0.47% for the middle-aged and 0.8% for the old under an aggregate shock in a parametrized version of the model.

We analyze two types of government policies to improve long-run social welfare: monetary and fiscal policies. We measure social welfare by the *ex-ante* expected utility of an individual being born into an economy. The government increases its monetary base to transfer new money to the low-income group facing a bad endowment shock. This expansionary monetary policy reduces consumption risk between states. In the economy with only incomplete markets, the government can increase long-run social welfare with the help of this monetary policy. However, the monetary policy can decrease social welfare in the presence of imperfect competition. When both frictions are present, an intertemporal wedge generated by the inflation tax strengthens the consumption variation over one's life-cycle.[6] Thus, even as the within-age consumption volatility decreases, overall welfare is not improved. As an alternative, the government can employ the combination of a linear endowment tax and lump-sum transfer to the disadvantaged group. We show that this fiscal policy pools the consumption risk and weakens the welfare loss from imperfect competition by reducing the share of consumption traded under strategic interactions. As income tax rates rise, the policy generates more consumption

[6] Imperfect competition already generates non-smooth consumption over the life-cycle. Further consumption variation over different ages will reduce welfare significantly under a concave utility function.

smoothing over the life-cycle and less volatility of consumption between states. Thus, the fiscal policy can improve social welfare even if both frictions are present. From this analysis, we note that the introduction of imperfect competition generates an asymmetry between the effects of fiscal and monetary policy actions, unlike in the competitive economy.

We also consider a possible extension of the model by incorporating the search activity of agents for price discount opportunities to reflect empirical findings in Aguiar and Hurst (2007). They find that households with different hourly wages spend different amounts of time searching for discount changes, and thus can experience distinct prices for identical goods through this channel. In this extended model, we assume that search effort is also exogenously determined, and both offering goods and exercising search effort reduces effective prices. Following Aguiar and Hurst (2007), those receiving good endowment shocks face bad search effort shocks. This assumption reflects less search effort by high-income groups due to their high opportunity cost of search from their high wage. Therefore, whether imperfect competition increases or decreases consumption risk-exposure depends on who faces lower effective prices between high and low-income groups given the inverse relationship of search effort with income.

This paper contributes to the literature on several dimensions. To the best of our knowledge, this is the first paper to study a general equilibrium model with both incomplete markets and price heterogeneity from imperfect competition. We show the importance of the price effect channel in risk-sharing, due to the fact that the consumption risk of households may be underestimated if we ignore price dispersion across households for identical goods. Our result implies that the government stabilization policy should be designed by recognizing the sources of inefficient outcomes and the existence of the complementary welfare losses between the incomplete market and imperfect competition frictions. Monetary policy decreases overall social welfare since it further distorts lifecycle consumption variation generated by the lifetime price heterogeneity when both frictions exist. Unlike competitive economies, the monetary and fiscal policies generate asymmetric welfare results because the fiscal policy resolves the inefficiency due to each friction. Lastly, our model with imperfect competition provides a distinct explanation for the correlated consumption/income profiles observed in data, independently of the mechanisms in the existing models with capital market constraints or impatient consumers.[7]

In the literature, there are a few applied papers which examine price dispersion across households for identical goods (see Rao 2000; Aguiar and Hurst 2007). However, they restrict their attention to looking for evidence of price heterogeneity. Thus, they do not ask its effect on consumption volatility and/or welfare effects. They do not also analyze welfare-enhancing government policies under both market incompleteness and imperfect competition. From the strategic market game literature, we find a few papers that extend the Shapley-Shubik market game to an intertemporal economy with intrinsic uncertainty. Giraud and Weyers (2004) show that imperfect competition can generate subgame-perfect equilibrium allocations that Pareto-dominate the competitive equilibria of the corresponding economy with only market incompleteness if players can condition their present actions on the history of prices. Although their paper studies constrained efficiency with trigger strategies, it does not explicitly mention

[7] In the Permanent Income Hypothesis literature, it is well known that consumption and income profiles are closely correlated (see Carroll and Summers 1991; Carroll 1997 and many others). There are two types of models that are typically used to explain this stylized fact. The first model is the life-cycle model with liquidity constraints and precautionary saving motives. The other one is the Keynesian model inhabited by impatient or hand-to-mouth consumers who exhaust their disposable income in every period.

the bid returning process, heterogeneous prices and their effects on consumption risk. Giraud and Stahn (2008) demonstrate the existence of a generalized Nash equilibrium in a two-period economy with the Shapley-Shubik market game on both assets and commodity. This paper also lacks the welfare implication of imperfect competition and Pareto-improving policies.

We organize this paper as follows. Section 2 briefly describes the three-period OLG model incorporating the Shapley-Shubik market game with financial market incompleteness. In Section 3, we examine how the imperfect competition generates non-smooth consumption allocations over the life-cycle in the model without income risk. Section 4 studies how imperfect competition generates additional consumption volatility and calculates its complementary welfare loss when both frictions exist. In Section 5, we check the robustness of our results. We examine two types of government policies to improve social welfare in Section 6. In Section 7, we extend the model by integrating the search behavior of agents for price discounts. Finally, Section 8 concludes this paper. Appendices provide proofs and numerical algorithms.

2 Model

In this section, we develop a pure exchange overlapping generation model incorporating the Shapley-Shubik market game. Time is discrete and indexed by t from 1 to infinity. In each period, $n > 0$ finite agents are born and live three periods labeled as young, middle-aged and old. There is no population growth. They consume a single good and can save via accumulations of fiat money. We will explain the trading mechanism under the Shapley-Shubik market game below. We assume that n agents within the same cohort are identical in both preferences and endowments, and thus we focus on symmetric Nash equilibria in the entire analysis below.

There is an exogenous shock with two states of nature, $s \in \{\alpha, \beta\}$, which affects endowments. The shock process is assumed to be independent and identically distributed (IID) across time with the state probability given by $0 < \pi^s < 1$ for $s \in \{\alpha, \beta\}$, where $\pi^\alpha + \pi^\beta = 1$. Agents' endowment profiles are given by a stochastic nonnegative vector $\boldsymbol{\omega}^s = \left(\omega_1^s, \omega_2^{s'}, \omega_3^{s''}\right)$ where ω_1^s is endowment when young in state s, $\omega_2^{s'}$ is endowment when middle-aged in state s', and $\omega_3^{s''}$ is endowment when old in state s''. We assume $\omega_i^s \gg 0$ for $\forall i$ and $\forall s$. Note that endowments in each age depend only on the current realization of the exogenous shock.

A consumption vector for the symmetric agents born in state s at time t is given by $\boldsymbol{c}_t^s = \left(c_{1,t}^s, c_{2,t+1}^{s'}, c_{3,t+2}^{s''}\right)$. Here, $c_{1,t}^s$ is the first-period consumption given state s in time t, $c_{2,t+1}^{s'}$ is the second-period consumption given state s' in time $t+1$, and $c_{3,t+2}^{s''}$ is the last period consumption given state s'' in time $t+2$. Hereafter, we denote by $x_{i,j}^s$ the value of x in the i-th age of an agent's life given state s at time j.

Consumer preferences are given by an additively time-separable utility function $U : \mathbb{R}_+^7 \to \mathbb{R} \cup \{-\infty\}$ with U specified by:

$$U\left(\boldsymbol{c}_t^s\right) = u\left(c_{1,t}^s\right) + \delta \sum_{s' \in \{\alpha,\beta\}} \pi^{s'} \left\{ u\left(c_{2,t+1}^{s'}\right) + \delta \sum_{s'' \in \{\alpha,\beta\}} \pi^{s''} u\left(c_{3,t+2}^{s''}\right) \right\} \tag{1}$$

where the one-period utility function $u : \mathbb{R}_+ \to \mathbb{R} \cup \{-\infty\}$ is C^3, strictly increasing, strictly concave, $u'''(c) > 0$ for $\forall c > 0$ and satisfies the Inada condition, and $0 < \delta \leq 1$ is the time discount factor.

Agents trade outside money for the single commodity in a central trading post under strategic interactions. The symmetric agents born in state s at time t make the same non-negative lifetime offers of goods $\boldsymbol{q}_t^s = \left(q_{1,t}^s, q_{2,t+1}^{s'}, q_{3,t+2}^{s''}\right) \gg \mathbf{0}$ to receive money, and the same non-negative lifetime bids of money $\boldsymbol{b}_t^s = \left(b_{1,t}^s, b_{2,t+1}^{s'}, b_{3,t+2}^{s''}\right) \gg \mathbf{0}$ to buy goods. They also save the same amount of money for tomorrow and the day after with money holdings given by $\boldsymbol{m}_t^s = \left(m_{1,t}^s, m_{2,t+1}^{s'}\right)$ to optimize consumption intertemporally and self-insure against endowment risk. Therefore, $\left\{(\boldsymbol{q}_t^s, \boldsymbol{b}_t^s, \boldsymbol{m}_t^s) \in \mathbb{R}^{17} \mid \boldsymbol{\omega}^s \gg \mathbf{0}\right\}$ denotes the strategy set of the symmetric agents born in state s at time t. We let $\left(Q_{1,t}^s, Q_{2,t+1}^{s'}, Q_{3,t+2}^{s''}\right) = \left(nq_{1,t}^s, nq_{2,t+1}^{s'}, nq_{3,t+2}^{s''}\right)$ and $\left(B_{1,t}^s, B_{2,t+1}^{s'}, B_{3,t+2}^{s''}\right) = \left(nb_{1,t}^s, nb_{2,t+1}^{s'}, nb_{3,t+2}^{s''}\right)$ be the sums of lifetime offers and bids of n symmetric agents born in state s in period t. The aggregate offer of the good in time t is the sum of offers made in period t by all consumers born in periods $t-2$, $t-1$ and t: $Q_t^s = Q_{3,t}^s + Q_{2,t}^s + Q_{1,t}^s$. Likewise, the aggregate bid of money in time t is the sum of the bids made in period t by all consumers born in periods $t-2$, $t-1$ and t: $B_t^s = B_{3,t}^s + B_{2,t}^s + B_{1,t}^s$.

Given offers and bids, the trading mechanism under the Shapley-Shubik market game allocates goods and money as follows. Each consumer is assigned a share of the aggregate bid of money in the proportion that her offer has to the aggregate offer. Similarly, each consumer is allocated a proportion of the aggregate offer of the commodity in the proportion that her bid bears to the aggregate bid. Under these notations and the trading mechanism, we write the budget constraints faced by agents born in time t and state s:

$$
\begin{aligned}
b_{1,t}^s + m_{1,t}^s &= \frac{q_{1,t}^s}{Q_t^s} B_t^s \\
b_{2,t+1}^{s'} + m_{2,t+1}^{s'} - m_{1,t}^s &= \frac{q_{2,t+1}^{s'}}{Q_{t+1}^{s'}} B_{t+1}^{s'} \qquad (2) \\
b_{3,t+2}^{s''} - m_{2,t+1}^{s'} &= \frac{q_{3,t+2}^{s''}}{Q_{t+2}^{s''}} B_{t+2}^{s''}
\end{aligned}
$$

where $\frac{B_t^s}{Q_t^s}$ can be interpreted as the average price of the single good in terms of the money in period t and state s. These budget constraints show that agents receive money by offering goods, bid money to buy goods, and save the rest.

Agents born in time t and state s consume goods as follows under the trading mechanism of the market game

$$
\begin{aligned}
c_{1,t}^s &= \omega_1^s - q_{1,t}^s + \frac{b_{1,t}^s}{B_t^s} Q_t^s \\
c_{2,t+1}^{s'} &= \omega_2^{s'} - q_{2,t+1}^{s'} + \frac{b_{2,t+1}^{s'}}{B_{t+1}^{s'}} Q_{t+1}^{s'} \qquad (3) \\
c_{3,t+2}^{s''} &= \omega_3^{s''} - q_{3,t+2}^{s''} + \frac{b_{3,t+2}^{s''}}{B_{t+2}^{s''}} Q_{t+2}^{s''}
\end{aligned}
$$

where agents consume the sum of their endowments and goods purchased by bidding money, net of goods offers to get money.

We are interested in the pure strategy symmetric Nash equilibria, and thus we express prices or the inverse of prices in (2) and (3) in terms of the offers and bids of other agents. For this, we introduce new variables: $B^s_{t,-i} = B^s_t - b^s_{i,t}$ and $Q^s_{t,-i} = Q^s_t - q^s_{i,t}$ for $\forall i \in \{1,2,3\}$. With these notations, we can rewrite (2):

$$
\begin{aligned}
b^s_{1,t} + m^s_{1,t} &= \left(\frac{B^s_{t,-1} - m^s_{1,t}}{Q^s_{t,-1}} \right) q^s_{1,t} \\
b^{s'}_{2,t+1} + m^{s'}_{2,t+1} - m^s_{1,t} &= \left(\frac{B^{s'}_{t+1,-2} - m^{s'}_{2,t+1} + m^s_{1,t}}{Q^{s'}_{t+1,-2}} \right) q^{s'}_{2,t+1} \\
b^{s''}_{3,t+2} - m^{s'}_{2,t+1} &= \left(\frac{B^{s''}_{t+2,-3} + m^{s'}_{2,t+1}}{Q^{s''}_{t+2,-3}} \right) q^{s''}_{3,t+2}
\end{aligned} \tag{4}
$$

By equating the right-hand sides of (2) and (4), we obtain:

$$
\frac{Q^s_t}{B^s_t} = \frac{Q^s_{t,-1}}{B^s_{t,-1} - m^s_{1,t}}, \; \frac{Q^{s'}_{t+1}}{B^{s'}_{t+1}} = \frac{Q^{s'}_{t+1,-2}}{B^{s'}_{t+1,-2} - m^{s'}_{2,t+1} + m^s_{1,t}} \text{ and } \frac{Q^{s''}_{t+2}}{B^{s''}_{t+2}} = \frac{Q^{s''}_{t+2,-3}}{B^{s''}_{t+2,-3} + m^{s'}_{2,t+1}} \tag{5}
$$

Substituting (4) and (5) into (3) then yields:

$$
\begin{aligned}
c^s_{1,t} &= \omega^s_1 - \left(\frac{Q^s_{t,-1}}{B^s_{t,-1} - m^s_{1,t}} \right) m^s_{1,t} \\
c^{s'}_{2,t+1} &= \omega^{s'}_2 - \left(\frac{Q^{s'}_{t+1,-2}}{B^{s'}_{t+1,-2} - m^{s'}_{2,t+1} + m^s_{1,t}} \right) \left(m^{s'}_{2,t+1} - m^s_{1,t} \right) \\
c^{s''}_{3,t+2} &= \omega^{s''}_3 + \left(\frac{Q^{s''}_{t+2,-3}}{B^{s''}_{t+2,-3} + m^{s'}_{2,t+1}} \right) m^{s'}_{2,t+1}
\end{aligned} \tag{6}
$$

The optimization problem of a typical young agent is then defined, given the offers and bids of all other agents, by:

$$
\begin{aligned}
\max_{(\boldsymbol{q}^s_t, \boldsymbol{b}^s_t, \boldsymbol{m}^s_t)} U\left(\boldsymbol{c}^s_t\right) = {} & u\left(\omega^s_1 - \left(\frac{Q^s_{t,-1}}{B^s_{t,-1} - m^s_{1,t}} \right) m^s_{1,t} \right) \\
& + \delta \sum_{s' \in \{\alpha,\beta\}} \pi^{s'} u \left(\omega^{s'}_2 - \left(\frac{Q^{s'}_{t+1,-2}}{B^{s'}_{t+1,-2} - m^{s'}_{2,t+1} + m^s_{1,t}} \right) \left(m^{s'}_{2,t+1} - m^s_{1,t} \right) \right) \\
& + \delta^2 \sum_{(s',s'') \in \{\alpha,\beta\}^2} \pi^{s'} \pi^{s''} u \left(\omega^{s''}_3 + \left(\frac{Q^{s''}_{t+2,-3}}{B^{s''}_{t+2,-3} + m^{s'}_{2,t+1}} \right) m^{s'}_{2,t+1} \right)
\end{aligned} \tag{7}
$$

subject to (4)

The first order conditions with respect to $m_{1,t}^{s}$ and $m_{2,t+1}^{s'}$ are respectively:

(8)

$$u'\left(c_{1,t}^{s}\right)\left(\frac{Q_{t,-1}^{s}}{B_{t,-1}^{s}-m_{1,t}^{s}}+\frac{Q_{t,-1}^{s}}{\left(B_{t,-1}^{s}-m_{1,t}^{s}\right)^{2}}m_{1,t}^{s}\right)$$
$$=\delta\sum_{s'\in\{\alpha,\beta\}}\pi^{s'}u'\left(c_{2,t+1}^{s'}\right)\left(\frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'}-m_{2,t+1}^{s'}+m_{1,t}^{s}}+\frac{Q_{t+1,-2}^{s'}}{\left(B_{t+1,-2}^{s'}-m_{2,t+1}^{s'}+m_{1,t}^{s}\right)^{2}}\left(m_{2,t+1}^{s'}-m_{1,t}^{s}\right)\right)$$

and

$$u'\left(c_{2,t+1}^{s'}\right)\left(\frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'}-m_{2,t+1}^{s'}+m_{1,t}^{s}}+\frac{Q_{t+1,-2}^{s'}}{\left(B_{t+1,-2}^{s'}-m_{2,t+1}^{s'}+m_{1,t}^{s}\right)^{2}}\left(m_{2,t+1}^{s'}-m_{1,t}^{s}\right)\right)$$
$$=\delta\sum_{s''\in\{\alpha,\beta\}}\pi^{s''}u'\left(c_{3,t+2}^{s''}\right)\left(\frac{Q_{t+2,-3}^{s''}}{B_{t+2,-3}^{s''}+m_{2,t+1}^{s'}}-\frac{Q_{t+2,-3}^{s''}}{\left(B_{t+2,-3}^{s''}+m_{2,t+1}^{s'}\right)^{2}}m_{2,t+1}^{s'}\right) \tag{9}$$

To derive income-dependent heterogenous effective prices generated by imperfect competition, we simplify the expressions in parentheses in the first-order conditions in (8) and (9), using (5):

$$u'\left(c_{1,t}^{s}\right)\frac{B_{t,-1}^{s}}{Q_{t,-1}^{s}}\left(\frac{Q_{t}^{s}}{B_{t}^{s}}\right)^{2}$$
$$=\delta\sum_{s'\in\{\alpha,\beta\}}\pi^{s'}u'\left(c_{2,t+1}^{s'}\right)\frac{B_{t+1,-2}^{s'}}{Q_{t+1,-2}^{s'}}\left(\frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}}\right)^{2} \tag{10}$$

and

$$u'\left(c_{2,t+1}^{s'}\right)\frac{B_{t+1,-2}^{s'}}{Q_{t+1,-2}^{s'}}\left(\frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}}\right)^{2}$$
$$=\delta\sum_{s''\in\{\alpha,\beta\}}\pi^{s''}u'\left(c_{3,t+2}^{s''}\right)\frac{B_{t+2,-3}^{s''}}{Q_{t+2,-3}^{s''}}\left(\frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}}\right)^{2} \tag{11}$$

where $\frac{Q_t^s}{B_t^s}$, $\frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}}$ and $\frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}}$ are the inverses of average good prices in time t, $t+1$ and $t+2$ respectively.

As the number of symmetric agents converges to infinity, $\frac{B_{t,-1}^{s}}{Q_{t,-1}^{s}}$ goes to $\frac{B_t^s}{Q_t^s}$ and it will be canceled out with $\frac{Q_t^s}{B_t^s}$ in (10). Likewise, $\frac{B_{t+1,-2}^{s'}}{Q_{t+1,-2}^{s'}}$ and $\frac{B_{t+2,-3}^{s''}}{Q_{t+2,-3}^{s''}}$ will be canceled out with $\frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}}$ and $\frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}}$ in (10) and (11). Thus, the first-order conditions in an imperfectly competitive economy will be the usual optimality conditions in the perfect competition model in the limit.

We call the inverse of $\frac{B^s_{t,-i}}{Q^s_{t,-i}}\left(\frac{Q^s_t}{B^s_t}\right)^2$ the *effective price* of the commodity that age-i agents pay to purchase an additional unit of the consumption good in time t and state s. The derivation of the *effective price* is straightforward from the allocation rule and individual budget constraints. In time t and state s, age-i agents should bid $\frac{\triangle c^s_{i,t}(B^s_t)^2}{Q^s_t B^s_{t,-i}-\triangle c^s_{i,t}B^s_t}$ to get additional consumption of $\triangle c^s_{i,t}$ from the allocation rule. The budget constraint implies that the age-i agents get back some of their own bids via their offers as a share of the aggregate offer, $\frac{q^s_{i,t}}{Q^s_t}$. We call this the bid-returning process. Thus, the age-i agents need to give up an amount $\frac{Q^s_{t,-i}}{Q^s_t}$ of money from their saving to increase their current bid by 1. From these results, we know that the age-i agents have to reduce their saving by $\frac{(B^s_t)^2}{Q^s_t B^s_{t,-i}-\triangle c^s_{i,t}B^s_t}\frac{Q^s_{t,-i}}{Q^s_t}$ to raise their current consumption by 1. Writing $\frac{(B^s_t)^2}{Q^s_t B^s_{t,-i}-\triangle c^s_{i,t}B^s_t}\frac{Q^s_{t,-i}}{Q^s_t} = \frac{Q^s_{t,-i}}{B^s_{t,-i}}\left(\frac{B^s_t}{Q^s_t}\right)^2 \frac{Q^s_t B^s_{t,-i}}{Q^s_t B^s_{t,-i}-\triangle c^s_{i,t}B^s_t}$, as $\triangle c^s_{i,t} \longrightarrow 0$, this expression reduces to $\frac{Q^s_{t,-i}}{B^s_{t,-i}}\left(\frac{B^s_t}{Q^s_t}\right)^2$ which we call the *effective price* of the good for an agent of age i in time t and state s. We stress here that the offer-dependent bid return rates are the main cause of the heterogeneous *effective prices* across agents.

We now discuss the well-known indeterminacy issue in the market game models. Note that once the money demands are determined by (8) and (9), either the offers or bids of agents are indeterminate in (4). In other words, an agent's net trade can be achieved by an infinite combination of offers and bids given other agents' offers and bids (see Peck et al. 1992). For example, when increasing offers more than one's endowments, raising bids buys back the additional offers so that (4) is satisfied. Thus, either offers or bids should be exogenously determined to avoid the indeterminacy issue. Following a standard assumption in the market game literature, we focus on the offer constrained game in which agents must offer all their endowments, so-called sell-all strategy, for the rest of the paper. As shown in Peck et al. (1992), the equilibria of the offer constrained game would be those of the unconstrained game, as long as exogeneous offers yield interior bids.

In period 1, there are n middle-aged consumers who live in period 1 and 2, and n old consumers who live only in period 1. The initial middle-aged and old carry money holdings of $m_{1,0}$ and $m_{2,0}$ respectively where $n\left(m_{1,0}+m_{2,0}\right) = nM$. We assume that the aggregate supply of money is fixed at nM from period 1 onward in the base model. Hence, the money market clearing condition is given by $n\left(m^{s_t}_{1,t}+m^{s_t}_{2,t}\right) = nM$ for $\forall t$.[8] We can define the problems of the initial middle-aged and old generations in time 1 similar to the problems given above for agents born in time 1 and onward by considering the fact that the lifetime left is one and two periods for the initial old and middle-aged respectively. Thus, their symmetric strategy sets are given by $\left\{\left(q^{s_1}_{3,1}, b^{s_1}_{3,1}\right) \in \mathbb{R}^2 \mid \omega^{s_1}_3 \gg \mathbf{0}\right\}$ and $\left\{\left(q^{s_1}_{2,1}, q^{s_2}_{3,2}, b^{s_1}_{2,1}, b^{s_2}_{3,2}, m^{s_1}_{2,1}\right) \in \mathbb{R}^7 \mid \left(\omega^{s_1}_2, \omega^{s_2}_3\right) \gg \mathbf{0}\right\}$ respectively.

We define the offer constrained market game under the sell-all strategy in OLG models as below.

Definition 1. The following elements describe the offer-constrained OLG market game with sell-all strategy.

[8] By Walras's law, we can ignore the market clearing for the consumption good.

1. Symmetric $3n$ players in each period
2. A finite set $\Phi = \{\alpha, \beta\}$ of states of shocks. Shocks follow an IID process with the state probability, $0 < \pi^s < 1$ for $s \in \{\alpha, \beta\}$ where $\pi^\alpha + \pi^\beta = 1$
3. Stochastic endowments $\boldsymbol{\omega}^s = \left(\omega_1^s, \omega_2^{s'}, \omega_3^{s''}\right)$ where $(s, s', s'') \in \{\alpha, \beta\}^3$ for all periods
4. The time-separable von Neumann-Morgenstern utility function U
5. The strategy set $\left\{(\boldsymbol{q}_t^s, \boldsymbol{b}_t^s, \boldsymbol{m}_t^s) \in \mathbb{R}^{17} \mid \boldsymbol{\omega}^s \gg \mathbf{0}\right\}$
6. Offers constrained at endowments, $\boldsymbol{q}_t^s = \boldsymbol{\omega}^s$ for $\forall t$

In this model, we are interested in symmetric Nash equilibria with pure strategies because we assume that symmetric agents within the same cohort bid and save equal amounts. We further concentrate on monetary Nash equilibria where the price of money is positive. Thus, we assume appropriate endowment profiles such that the resulting aggregate money demands are positive.

We now define a symmetric monetary Nash equilibrium with pure strategies in an offer constrained market game under sell-all strategy in OLG models.

Definition 2. For the offer constrained market game under sell-all strategy in the three-period OLG models, a symmetric monetary Nash equilibrium in pure strategies is a sequence of bids and money demands $\left(b_{3,1}^{s_1}, b_{2,1}^{s_1}, b_{3,2}^{s_2}, m_{2,1}^{s_1}, \boldsymbol{b}_t^{s_t}, \boldsymbol{m}_t^{s_t}\right)_{t=1,2,\ldots}$ such that:

1. Offers are given at endowments $\left(q_{3,1}^{s_1}, q_{2,1}^{s_1}, q_{3,2}^{s_2}, \boldsymbol{q}_t^{s_t}\right)_{t=1,2,\ldots} = \left(\omega_3^{s_1}, \omega_2^{s_1}, \omega_3^{s_2}, \boldsymbol{\omega}^{s_t}\right)_{t=1,2,\ldots}$ where s_t is the state realization in period t.
2. Every agent's strategies $\left(b_{3,1}^{s_1}, b_{2,1}^{s_1}, b_{3,2}^{s_2}, m_{2,1}^{s_1}, \boldsymbol{b}_t^{s_t}, \boldsymbol{m}_t^{s_t}\right)_{t=1,2,\ldots}$ are the best response to the actions of other agents taken as given.
3. For $\forall t$, $n\left(m_{1,t}^{s_t} + m_{2,t}^{s_t}\right) = nM$, where nM is the stock of fiat money from period 1 onward.

For the numerical analysis below, we need the symmetric monetary Nash equilibria to be computationally tractable. Thus, we focus on a subset of those equilibria which we are able to compute via dynamic programming. As one of the simplest equilibrium concept, we first examine short memory symmetric monetary Nash equilibria where equilibrium allocations depend only on a history of finite past shocks. Such equilibrium requires knowing the values of equilibrium allocations over finite combinations of past shock realizations.

In the following proposition, we show that the short memory symmetric monetary Nash equilibria do not indeed exist generically. Here, the genericity means that the set of economies for which such equilibria do not exist is dense in the offer constrained market game defined as above.

Proposition 1. There are no short memory (or T-memory) symmetric monetary Nash equilibria generically in the offer constrained market game under sell-all strategy in the three-period OLG models.

Proof. See Appendix A.1. □

The exclusion of the memoryless equilibria implies that any rational expectations equilibrium must include the entire history of past shocks or lagged endogenous state variables as sufficient statistics for history. Here, we consider recursive Markov Nash equilibria, which take the distribution of asset holdings across agents as the endogenous state variables, as an alternative for several reasons. First, the Markov equilibrium does not allow any trigger strategies monitoring the past actions of others so that agents condition their decisions only on current states, which makes computing such equilibrium tractable. In addition, we can find previous studies which show the existence of Markov ϵ-equilibria in OLG economies that we use for our numerical analysis as argued below.

Let $\sigma_t = \left[m_{1,t-1}^{s_{t-1}}, s_t\right] \in \Sigma \subset R \times \Phi$ represent the state variables – the lagged money holdings carried by the current middle-aged and the realization of the current shock, where $\hat{\Sigma}$ is the state space of both endogenous and exogenous state variables. With this notation, we now state the definition of the symmetric recursive Markov monetary Nash equilibria in our model.

Definition 3. For the offer constrained market game under sell-all strategy in the three-period OLG models, symmetric recursive Markov monetary Nash equilibria in pure strategies consist of policy functions for bids and money demands, $\{b_1(\sigma_t), b_2(\sigma_t), b_3(\sigma_t), m_1(\sigma_t), m_2(\sigma_t)\}$, which are the best responses to the actions of other agents taken as given and clear the money market.

Note that the lagged money holdings of the current old can be ignored in the space of the endogenous state variables by the money market clearing condition.

[HG theorem here with checking the genericity of its hyperbolicity + larger shocks with justifications from Kubler and Polemarchakis (2004).]

Kubler and Polemarchakis (2004) examine the existence of (stationary) Markov ϵ-equilibria in OLG economies which clear the market and are within ϵ-bound of true utility maximizing choices. They show that such Markov ϵ-equilibria exist for all $\epsilon > 0$ and converge to true equilibria as $\epsilon \longrightarrow 0$. They stress that only Markov ϵ-equilibria can be computed in numerical work because of rounding and truncation errors. With the justification of Kubler and Polemarchakis (2004), we focus on computing Markov ϵ-equilibria in the welfare analysis. We numerically check that such Markov ϵ-equilibria can be found in our model with strategic interactions under any sizes of shocks for any ϵ-error bounds.

3 Correlated consumption/income profiles

This section studies the effects of lifecycle price heterogeneity on intertemporal allocations in the model with strategic interactions. Thus, we first examine the equilibrium outcomes in the economy only with imperfect competition, but no endowment risk. To make this analysis more simple, we focus on the case with $\delta = 1$.

Under these restrictions, we observe equal lifetime consumption allocations at the steady states under competitive economies because agents face the same effective prices for identical goods over the lifecycle. However, in the economy with imperfect competition, agents experience a price heterogeneity across the life-cycle according to their income as seen in (10) and (11). Thus, agents consume more in periods with lower effective prices, which leads to unequal lifetime consumption allocations. This is the main content of the following proposition.

Proposition 2. Perfectly smoothed consumptions cannot be the stationary allocations in the offer constrained market game under sell-all strategy in the three-period OLG models with $\delta = 1$ and no endowment risk. The steady-state allocations are characterized by more consumption in period with higher endowments.

Proof. See Appendix A.2. □

The results imply that perfect consumption smoothing fails over the life-cycle and a typical agent's consumptions rather follow her endowment stream. We give intuition behind these results in detail as follows. The Shapley-Shubik market game mechanism generates a feedback effect from the fact that agents return a portion of their own bids via their simultaneous offers to sell the consumption good. The rate of return equals one's share of the aggregate offer. Thus, there are higher return rates for the ages with larger endowments under the sell-all strategy. A high return rate allows agents to give up less saving to increase the current bid for an additional unit of the good. This result implies that agents can purchase identical goods at lower effective prices in the periods with larger endowments. The intertemporally optimizing consumers transfer their wealth to periods with lower effective prices to purchase consumption more cheaply. Hence, we observe that consumption growth closely parallels income growth at the stationary equilibria. The correlated consumption/income profiles lead to the failure of perfect consumption smoothing in the model with strategic interactions. Even under equal lifetime incomes, the perfectly smoothed consumptions cannot be the outcome of the steady state equilibrium because such allocations violate the money market clearing condition as noted in the proof of Proposition 2.

To be consistent with data, we now restrict our attention to inverted-U shape lifetime endowment profiles. We assume that agents retire in the old period and receive zero endowments. The result in Proposition 2 indicates that the lifetime consumptions will also show a hump-shaped distribution. In the following corollary, we summarize this finding and other interesting features for the stationary allocations under the hump-shaped endowment structures with $\omega_3 = 0$.

Corollary 1. The stationary consumption allocations are hump-shaped if the endowment structures are hump-shaped in the offer constrained market game under sell-all strategy in the three-period OLG models with $\delta = 1$ and no endowment risk. Assuming $\omega_3 = 0$, the steady-state allocations in this economy satisfy the relationship that $c_1 \gtreqless \frac{1}{3}(\omega_1 + \omega_2)$, $c_2 > \frac{1}{3}(\omega_1 + \omega_2)$, and $c_3 < \frac{1}{3}(\omega_1 + \omega_2)$. $c_1 > \frac{1}{3}(\omega_1 + \omega_2)$ if ω_1 is smaller than but close enough to $\frac{\Omega}{2n}$. Furthermore, $c_1 < \frac{1}{3}(\omega_1 + \omega_2)$ if ω_1 is larger than but sufficiently close to $\frac{\Omega}{3n}$.

Proof. See Appendix A.3. □

This result implies that a model with imperfect competition provides a distinctive explanation for the stylized fact of the hump-shaped consumption profiles (given the hump-shaped income profiles) even with patient consumers and without capital market imperfections such as liquidity constraints and precautionary saving motives. It is also worth noting why income-dependent prices produce a welfare loss in the OLG market game economy. In the competitive economy, prices adjust to balance the purchasing power of coexisting generations so that they face the same effective prices for identical goods. The income-neutral prices allow perfect consumption smoothing. On the other hand, price heterogeneity generates variations in the life-cycle consumptions which leads to a welfare loss in the imperfectly competitive economy. Indeed, this is a general feature of the way imperfect competition in the Shapley-Shubik market game generates inefficient allocations over the lifecycle and across states, as we will see below. Because agents face different effective prices (via the non-linear budget constraints), aligning the utility gradients at an equilibrium allocation is impossible.

4 Consumption Volatility and Risk Sharing

In this section, we return to the economy with both imperfect competition and incomplete market frictions. We examine how strategic interactions affect consumption allocations across not only lifetimes but also across states through a numerical analysis. At the same time, we analyze how imperfect competition affects consumption volatility between states and the exposure of consumption to endowment risk on top of the financial market incompleteness. Lastly, we quantify the welfare loss from the effects of interactions between the two frictions on consumption allocations compared to the benchmark, deterministic competitive economy.

In our numerical analysis, we first consider an idiosyncratic shock across generations where the young and middle-aged face endowment risk in the opposite direction. We maintain the assumption that $\delta = 1$. These assumptions make the welfare analysis clearer because the benchmark allocations in the frictionless economy will be the perfectly smoothed consumptions over ages and states, and then one can identify the welfare loss by comparing the certainty equivalent consumptions in the frictional economies with the benchmark ones.

In addition, we assume constant relative risk aversion utility functions, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ where $\sigma = 2$. It is assumed that there is one representative agent born in each generation – $n = 1$ – to highlight the effect of strategic interactions on risk-sharing. The total money quantity, M, is normalized to be one. The total endowment is also normalized to be one and the shares of the endowments are assumed to be $\frac{3}{8}$, $\frac{5}{8}$ and 0 for the young, the middle-aged and the retired in the economy without the endowment risk: $\{\omega_1, \omega_2, \omega_3\} = \left\{\frac{3}{8}, \frac{5}{8}, 0\right\}$.

The idiosyncratic shock follows a simple symmetric IID Bernoulli process with two states. In our base case, we consider the idiosyncratic endowment streams given by $\left\{\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha\right\} = \left\{\frac{5}{16}, \frac{11}{16}, 0\right\}$ and $\left\{\omega_1^\beta, \omega_2^\beta, \omega_3^\beta\right\} = \left\{\frac{7}{16}, \frac{9}{16}, 0\right\}$ where ω_i^s is the endowment of age i in state s. Thus, the baseline idiosyncratic shock generates the standard deviation of $\frac{1}{16}$ for the idiosyncratic endowments. Here, the young cohort is advantaged in state β and disadvantaged in state α because they receive more endowments in state β and fewer endowments in state α than in the economy without the endowment risk. We also work with other parameter values for the time discount factor and risk aversion commonly used in the macroeconomics literature and different structures for the endowment risk in the robustness section below.

Given the parameters' values above, we compute the symmetric recursive Markov Nash equilibria for a welfare analysis in the three types of economies: the first with only incomplete markets; the second with only imperfect competition; and the third with both frictions.[9] We simulate the three economies for 21,000 periods and ignore the first 1000 periods to avoid the effect of initial conditions on the results. Time averages and cross-sectional averages will be the same because of the ergodicity in the recursive Markov equilibria.[10] This property allows us to calculate the ex-ante expected utility at the steady-states with the simulation data. Then, we find the certainty equivalent (CE) consumption that achieves the same ex-ante expected utility in the frictional economies. The welfare loss from each friction is then measured by the difference between its CE consumption and the benchmark perfect smoothing consumption as the percentage of the benchmark allocation. One can view the welfare loss measure or the CE consumption loss rate as a relative risk-premium. By comparing the welfare losses in the three economies, we can identify the complementary welfare loss from the additional consumption volatility generated via interactions between the incomplete market and imperfect competition. Specifically, we compute such supermodular welfare loss as the welfare loss in the economy with both frictions net of the sum of the welfare losses in the other two economies with each friction.

Table 1 summarizes the welfare analysis results for the three types of economies above relative to the benchmark economy under the baseline parameter values. In the second and third columns, we display the mean of consumption simulation data for each age conditional on the state of the current shock. The fourth column represents the average consumption level of each cohort without conditioning on the current shock state. To express the consumption volatility faced by each cohort, we use the percentage coefficient of variation (CV). The percentage CV is calculated by dividing the standard deviation of unconditional consumption in each age with its mean and then multiplying by 100%. We write the CV values of each cohort in the last column. In the last panel, we address CE consumption and their percentages out of the benchmark CE consumption for all the three economies to evaluate the welfare loss from each frictional economy.

In the economy with only imperfect competition, the lifetime consumption profile exhibits a hump-shaped structure following the endowment profile, which verifies the implication in Corollary 1. Intuitively, the middle-aged face the lowest effective prices via the bid-returning process because their offers are the highest. Hence, the young agents save more, and the middle-aged save less than what they do in the competitive economy. This disparate saving behavior leads to a consumption variation across ages observed in the inverted-U shaped consumption profile. Such a deviation from the perfectly smoothed consumption allocations generates a welfare loss of 2.52% in the baseline economy with strategic interactions.

There is almost no variation in the unconditional mean lifetime consumption allocations in the economy with only incomplete markets since agents face almost the same goods prices across ages. However, there is consumption volatility between states within each age because fiat money alone cannot perfectly share the birth-state and successive endowment risk. The consumptions of the young are higher in state β than in state α whereas the middle-aged consume more in state α than in state β because the young are advantaged in state β and the middle-aged are advantaged in state α given the shock structure. Our numerical analysis indi-

[9] We describe the algorithm computing the recursive Markov Nash equilibria in Appendix B.

[10] The proof of this property can be obtained by a straightforward generalization of the technique introduced by Duffie et al. (1994) for stationary Markov equilibria.

Table 1: Welfare analysis under the base economy

Benchmark consumption without frictions				
Age	State α	State β	Mean	CV (%)
Young	0.3333	0.3333	0.3333	0.00%
Middle-aged	0.3333	0.3333	0.3333	0.00%
Old	0.3333	0.3333	0.3333	0.00%
Equilibrium consumption only with imperfect competition				
Age	State α	State β	Mean	CV (%)
Young	0.3286	0.3286	0.3286	0.00%
Middle-aged	0.4007	0.4007	0.4007	0.00%
Old	0.2707	0.2707	0.2707	0.00%
Equilibrium consumption only with incomplete market				
Age	State α	State β	Mean	CV (%)
Young	0.3146	0.3455	0.3301	4.68%
Middle-aged	0.3639	0.3064	0.3352	8.58%
Old	0.3214	0.3480	0.3347	3.98%
Equilibrium consumption with both frictions				
Age	State α	State β	Mean	CV (%)
Young	0.2975	0.3544	0.3260	8.72%
Middle-aged	0.4493	0.3567	0.4030	11.48%
Old	0.2532	0.2889	0.2711	6.58%
Welfare analysis				
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3249	0.3310	0.3214
CE consumption as (%) of benchmark	100%	97.48%	99.29%	96.42%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

cates a welfare loss of 0.71% from the consumption volatility arising under the financial market incompleteness in the baseline economy.

In the economy with both frictions, the equilibrium consumptions are volatile across both ages and states as expected. One of the interesting results in this economy is that the consumption volatility between states is much larger for all cohorts than that of the economy with only the incomplete market friction. Specifically, the within-age consumption volatility increases by 4.04% for the young, 2.9% for the middle-aged, and 2.6% for the old. The additional consumption volatility implies that imperfect competition reduces the risk sharing between states among generations, and generates a supermodular welfare loss of 0.35% since the total welfare loss is 3.58% in the economy with both frictions, whereas the sum of welfare losses from each friction alone is 3.23%.

Intuitively, the income-dependent effective prices allocate more consumption to the advantaged generations – who receive a good shock – under imperfect competition because the market game mechanism assigns lower effective prices to those offering more, and thus the advantaged cohort can purchase more goods relative to the competitive economy from both higher incomes and lower prices. This biased allocation conflicts with the requirements for perfect risk-sharing, which necessitates a transfer from the advantaged cohort to the disadvantaged one. Thus, imperfect competition increases consumption volatility between states, which implies the larger risk-exposure of consumption to the endowment shock.

Next, we consider two different sizes of shocks with standard deviations (SD) of $\frac{2}{16}$ and $\frac{3}{16}$ to study how the size of shocks affects the complementary welfare loss. In the former shock, the idiosyncratic endowment profile is described by $\left\{\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha\right\} = \left\{\frac{4}{16}, \frac{12}{16}, 0\right\}$ and $\left\{\omega_1^\beta, \omega_2^\beta, \omega_3^\beta\right\} = \left\{\frac{8}{16}, \frac{8}{16}, 0\right\}$. For the latter shock, it is given by $\left\{\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha\right\} = \left\{\frac{3}{16}, \frac{13}{16}, 0\right\}$ and $\left\{\omega_1^\beta, \omega_2^\beta, \omega_3^\beta\right\} = \left\{\frac{9}{16}, \frac{7}{16}, 0\right\}$.

Comparing Tables 1, 2, and 3, one sees that the welfare loss from only the incomplete market increases as the size of the shock increases: 2.88% for the shock with SD $\frac{2}{16}$ and 6.73% for the shock with SD $\frac{3}{16}$. This result is straightforward because larger shocks raise the consumption volatility within ages unless perfect consumption smoothing is obtained. Here, the interesting point of the result is that the welfare loss from the financial market incompleteness grows roughly by four or ten times although we amplify the size of the shock by two or three times from the base case due to the concavity of the utility function.

We also stress that the larger the size of the shock, the higher the supermodular welfare loss from interactions between imperfect competition and market incompleteness: 1.47% for the shock with SD $\frac{2}{16}$ and 3.19% for the shock with SD $\frac{3}{16}$. As the size of the shock rises, the gap between effective prices across generations expands. Thus, the favored cohort can buy goods at much cheaper prices under a larger shock than a smaller shock, which increases the additional consumption volatility. In the base case, the additional consumption volatilities are about 4.04%, 2.9%, and 2.6% for the young, middle-aged and old, respectively. When the size of the shock doubles, those volatilities are 7.9%, 6.07%, and 6.07%. If the size of the shock rises up to three times, the corresponding volatilities are 11.87%, 9.03%, and 9.62%. Therefore, the supermodular welfare loss rises as the size of the shock increases. Lastly, we emphasize that the complementary welfare loss takes about 50% of the welfare loss solely from the incomplete market for any sizes of shocks, as seen in the results above.

Table 2: Welfare analysis under an idiosyncratic shock with SD $\frac{2}{16}$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.2884	0.3537	0.3211	10.17%
Middle-aged	0.3970	0.2834	0.3402	16.70%
Old	0.3147	0.3629	0.3388	7.11%
Equilibrium consumption with both friction				
Young	0.2612	0.3764	0.3188	18.07%
Middle-aged	0.5034	0.3167	0.4101	22.77%
Old	0.2354	0.3069	0.2712	13.18%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3249	0.3237	0.3104
CE consumption as (%) of benchmark	100%	97.48%	97.12%	93.13%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

Table 3: Welfare analysis under an idiosyncratic shock with SD $\frac{3}{16}$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.2557	0.3606	0.3081	17.03%
Middle-aged	0.4326	0.2644	0.3485	24.14%
Old	0.3117	0.3750	0.3434	9.22%
Equilibrium consumption with both friction				
Young	0.2192	0.3973	0.3082	28.90%
Middle-aged	0.5618	0.2819	0.4219	33.17%
Old	0.2191	0.3208	0.2699	18.84%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3249	0.3109	0.2918
CE consumption as (%) of benchmark	100%	97.48%	93.27%	87.54%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

5 Robustness Check

In this section, we change the parameters' values in the numerical analysis to check the robustness of our results in this paper. First, we set the time discount factor commonly used in the literature. Then, we consider high risk-aversion value. Lastly, we examine a different endowment shock structure.

5.1 Changing Time Discount Factor

We set a new time discount factor at 0.54 because a widely used annual subjective discount factor is 0.97 from the literature and one period in our model takes 20 years in the real economy. We keep other parameters as in the base case.

Table 4: Welfare analysis under the economy with $\delta = 0.54$

Age	State α	State β	Mean	CV (%)
Benchmark consumption without frictions				
Young	0.4396	0.4396	0.4396	0.00%
Middle-aged	0.3230	0.3230	0.3230	0.00%
Old	0.2374	0.2374	0.2374	0.00%
Equilibrium consumption only with imperfect competition				
Young	0.4071	0.4071	0.4071	0.00%
Middle-aged	0.3913	0.3913	0.3913	0.00%
Old	0.2016	0.2016	0.2016	0.00%
Equilibrium consumption only with incomplete market				
Young	0.4086	0.4572	0.4329	5.62%
Middle-aged	0.3648	0.2890	0.3269	11.60%
Old	0.2266	0.2538	0.2402	5.66%
Equilibrium consumption with both friction				
Young	0.3670	0.4375	0.4023	8.76%
Middle-aged	0.4468	0.3423	0.3946	13.25%
Old	0.1862	0.2202	0.2032	8.39%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3539	0.3467	0.3500	0.3417
CE consumption as (%) of benchmark	100%	97.96%	98.88%	96.53%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %

- CE consumption represents certainty equivalent consumption

From the first panel in Table 4, the stationary consumption allocations decrease in ages in the frictionless economy with a low time discount factor because agents now prefer present consumption to future consumption. However, the middle-aged consume as much as the young in the economy with imperfect competition due to the lowest effective prices for the middle-aged. Thus, the correlation between life-cycle consumption and endowment profiles is 0.88 at the steady-state of this economy, which is much larger than the one in the frictionless economy, 0.52. This result supports our finding that strategic interactions make consumption correlated with income over the life-cycle, even with a realistic time discounting factor.

Looking at the welfare loss from each friction, we find, interestingly, that a lower time discount factor attenuates the welfare loss due to imperfect competition from 2.52% in the base case to 2.04%, but intensifies the welfare loss due to the incomplete market from 0.71% to 1.12%. One can get an intuition for these results by considering the extreme case of $\delta = 0$. In this case, agents care about consumption in youth only. Thus, one's saving behavior and consumption allocations are not affected by the heterogenous effective prices across ages under the imperfectly competitive market. This result implies that there is less room for strategic interactions to generate welfare losses. Hence, by continuity, a lower time discount factor yields a relatively smaller welfare loss from imperfect competition.

Under a low δ, agents regard future consumption as being less valuable than current consumption, and thus there is a limited amount of borrowing and saving among adjacent generations. For example, the middle-aged generation facing a bad shock can borrow only a limited amount (relative to the higher δ case) from the young generation who save less due to discounting future consumption. Thus, a low time discount factor reduces risk-sharing between cohorts and makes consumption more volatile. The consumption inequality within each age indeed increases by 1-3% compared to the base case. Therefore, we observe a large welfare loss from the incomplete market friction under a low δ.

The additional consumption volatilities for all ages are smaller under a low time discount factor compared to the base case because consumption allocations are less distorted by strategic interactions as δ decreases as described above. Those volatilities are 3.14% for the young, 1.65% for the middle-aged, and 2.73% for the old, which are about 1% lower than the corresponding ones in the base case. Therefore, the complementary welfare loss is 0.31% in the discounting economy, which is smaller than 0.35% in the base case. This constitutes about 27% of the welfare loss due solely to incomplete markets. This result implies that although its quantitative welfare effect can be changed, imperfect competition still increases the exposure of consumption to endowment risk even under a realistic time discount factor.

5.2 Changing Risk Aversion

Now, we examine consumption variation across ages and states in frictional economies under a high-risk aversion, $\sigma = 6$. We also keep other parameters as in the base case.

After increasing the risk aversion, we still observe the correlated (hump-shaped) consumption/income profiles over the life-cycle under strategic interactions as implied in Corollary 1, although the lifetime consumption profile becomes closer to the perfectly smoothed one. In the second panel of Table 5, the consumption volatility between states generated by the incomplete market friction is also lower compared to the base case. This smaller consumption variation across both ages or states arises from the strong demand of more risk-averse agents for risk sharing and consumption smoothing.

Table 5: Welfare analysis under the economy with $\sigma = 6$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with imperfect competition				
Young	0.3321	0.3321	0.3321	0.00%
Middle-aged	0.3591	0.3591	0.3591	0.00%
Old	0.3088	0.3088	0.3088	0.00%
Equilibrium consumption only with incomplete market				
Young	0.3224	0.3373	0.3298	2.27%
Middle-aged	0.3546	0.3150	0.3348	5.91%
Old	0.3231	0.3477	0.3354	3.67%
Equilibrium consumption with both friction				
Young	0.3154	0.3410	0.3282	3.91%
Middle-aged	0.3894	0.3322	0.3608	7.93%
Old	0.2952	0.3268	0.3110	5.08%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3296	0.3303	0.3254
CE consumption as (%) of benchmark	100%	98.89%	99.09%	97.64%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

The changes in welfare losses under each friction show different patterns. Under a high risk-aversion, the welfare loss due to imperfect competition decreases from 2.52% to 1.01%, whereas it increases from 0.71% to 0.91% under financial market incompleteness. To explain the discrepancy in the welfare results, we should note that a high degree of risk-aversion reinforces the disutility from consumption volatility even as it decreases consumption variation across both ages or states. Therefore, whether the welfare loss increases or decreases depends on which effect dominates the other between the former volatility effect and the latter disutility effect. In the economy with only strategic interactions, the disutility effect dominates the volatility one and thus there is a smaller welfare loss. However, it is the opposite with only the incomplete market friction, which results in a larger welfare loss.

Under a high-risk aversion, the imperfect competition friction adds consumption volatility up to 1.64% for the young, 2.02% for the middle-aged, and 1.41% for the old. These volatilities are much smaller than in the base case because of the strong demand for enhanced risk sharing by more risk-averse agents. However, since agents with a higher risk-aversion experience more significant disutility from consumption variation, the complementary welfare loss is 0.34%, which is comparable to 0.35% in the base case. Such welfare loss takes roughly 37% of the welfare loss solely from the incomplete market. The results so far support our view that strategic interactions raise the risk-exposure of consumption to the shock, even in a high-risk aversion world.

5.3 Changing Shock Structure

In this subsection, we examine how the structure of the endowment shocks matters for the additional consumption volatility and its supermodular welfare effect under the imperfect competition. For this, we consider an aggregate shock where the stochastic endowments streams are positively correlated between generations: $\left\{\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha, \Omega^\alpha\right\} = \left\{\frac{5}{16}, \frac{9}{16}, 0, \frac{14}{16}\right\}$ and $\left\{\omega_1^\beta, \omega_2^\beta, \omega_3^\beta, \Omega^\beta\right\} = \left\{\frac{7}{16}, \frac{11}{16}, 0, \frac{18}{16}\right\}$ in which Ω^s is the total endowment in state s. We set the other parameters as in the base case. Note that this aggregate shock has the same SD as the idiosyncractic shock in the base case from the viewpoints of each age.

From the second panel of Table 6, we still observe correlated consumption/endowment profiles over the life-cycle and additional consumption variation due to strategic interactions even under the aggregate shock. However, imperfect competition does not increase the consumption inequality in each age as much as in the idiosyncratic shock case. Indeed, the within-age consumption volatility rises by 1.52% for the young, 0.47% for the middle-aged, and 0.8% for the old when adding imperfect competition to market incompleteness. The intuition behind these results is that the aggregate shock does not differentiate advantaged and disadvantaged cohorts and thus contemporary cohorts experience similar changes in the effective prices of goods across states. Hence, there are no specific groups which can purchase identical goods at significantly lower prices. This restricts the extent of the additional consumption volatility and reduces the risk-sharing in a very limited manner.

Looking at Table 1 and 6, the consumption inequality defined by CV is larger for the young and the old and smaller for the middle-aged under the aggregate shock than the idiosyncratic shock, given the same SD. These different patterns in CV come mainly from the distinct behaviors of prices between the two models. In the model with an aggregate shock, the value of

Table 6: Welfare analysis under the economy with an aggregate shock

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.2939	0.3672	0.3306	11.09%
Middle-aged	0.3096	0.3499	0.3297	6.11%
Old	0.2715	0.4079	0.3397	20.07%
Equilibrium consumption with both friction				
Young	0.2850	0.3673	0.3262	12.61%
Middle-aged	0.3711	0.4234	0.3972	6.58%
Old	0.2188	0.3342	0.2765	20.87%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %

money is much more volatile between states because the aggregate saving is low in state α but high in state β. However, it is quite stable in the model with an idiosyncratic shock since the aggregate saving is similar across states due to the same total endowments regardless of states. The young agents save less in the good state when facing volatile prices than when facing stable prices because of downward risk on the price of money in the next period. On the other hand, they save more in the bad state under volatile prices than stable ones due to the upward price risk of money. Thus, saving variation within the young cohort is smaller under the aggregate shock than the idiosyncratic shock, which results in a larger consumption variation for this age. The consumption variation of the old age group is solely determined by the volatility in the value of money holdings because they receive zero endowments. The large price fluctuations under the aggregate shock make the value of the old's asset holdings volatile although the saving of the middle-aged is stable. Therefore, we observe a larger consumption variation for this age under the aggregate shock than under the idiosyncratic shock. Since the young and old age groups absorb the risk-exposure of consumption to the endowment shocks, the middle-aged hedge against the consumption risk significantly and face a smaller consumption variation under the aggregate shock. It is the opposite in the model with the idiosyncratic shock. The young and old agents, in this case, share the consumption risk extensively, and thus the middle-aged absorb the largest portion of the risk and show a larger consumption variation.

5.4 Further Discussion

In the robustness checks above, we do not explicitly consider an idiosyncratic shock within cohorts because the welfare implications will be straightforward based on the results we have derived so far. *Ex-ante* identical agents become *ex-post* heterogenous under this idiosyncratic shock. Agents receiving a good shock can purchase goods cheaply whereas those receiving a bad shock will pay more to buy the same goods in the imperfectly competitive market. Therefore, the strategic interaction will also increase consumption volatility within ages under this type of shock.

For the entire numerical analysis, we have concentrated on the symmetric shocks with equal probabilities. As long as the endowment risk differentiates gaining and losing cohorts given a state, there exists effective price dispersion for identical goods over generations. Thus, the strategic interaction will add consumption volatility even under an asymmetric shock. However, the asymmetry of the shock can affect the size of the welfare loss. For example, if a probability distribution is skewed to one state, it will produce a small variance in consumption allocations between states. Thus, imperfect competition generates a smaller complementary welfare loss, as seen above.

6 Policy Analysis

In this section, we study monetary and fiscal policies to improve long-run social welfare measured by the *ex-ante* expected utility of an individual at the stationary equilibria in the base economy in Section 4. We quantify the welfare improvement by government interventions with how much the CE consumption increases from the base economy.

6.1 Monetary Policy

We first consider an expansionary monetary policy under which the government issues new money and transfers it to the age group receiving a bad endowment shock among the young and middle-aged. We assume that the total money supply grows at a constant rate in every period. This policy can improve social welfare by rebalancing the amount of money holdings and promoting risk-sharing between gaining and losing cohorts. One can interpret this monetary policy as redistributing wealth implicitly from those who get a good shock to their counterpart by inducing an inflation tax.

Under active monetary policy, the new money market clearing condition is given by:

$$M_t = M_{t-1} + \Delta m_t = (1+g)\, M_{t-1} \tag{12}$$

and the household budget constraints become:

$$\begin{aligned}
\hat{b}^{s}_{1,t} + \hat{m}^{s}_{1,t} &= \frac{q^{s}_{1,t}}{Q^{s}_{t}} \hat{B}^{s}_{t} + \Delta\hat{m}_t I\left(\omega^{s}_{1} = \omega^{B}_{1}\right) \\
\hat{b}^{s'}_{2,t+1} + \hat{m}^{s'}_{2,t+1} - \frac{\hat{m}^{s}_{1,t}}{(1+g)} &= \frac{q^{s'}_{2,t+1}}{Q^{s'}_{t+1}} \hat{B}^{s'}_{t+1} + \Delta\hat{m}_{t+1} I\left(\omega^{s'}_{2} = \omega^{B}_{2}\right) \\
\hat{b}^{s''}_{3,t+2} - \frac{\hat{m}^{s'}_{2,t+1}}{(1+g)} &= \frac{q^{s''}_{3,t+2}}{Q^{s''}_{t+2}} \hat{B}^{s''}_{t+2}
\end{aligned} \tag{13}$$

where M_t is the total money supply at time t, Δm_t denotes new money supply transferred to the age facing a bad shock, and g is the growth rate of the total money supply. Here, $I(\cdot)$ is an indicator function which equals one if the age i cohort is in the bad endowment state, ω^{B}_{i}, and zero otherwise. For computation, we normalize variables by dividing them by the total money supply in their corresponding periods and denote normalized variables with the circumflex (aka hat) symbol, $\hat{}$. For example, $\hat{b}^{s}_{1,t} = \frac{b^{s}_{1,t}}{M_t}$. From equation (12), $\Delta\hat{m}_t = \frac{g}{1+g}$.

From the market game trading mechanism, a agent born in time t obtains the following consumption allocations:

$$
\begin{aligned}
c_{1,t}^{s} &= \omega_1^{s} - q_{1,t}^{s} + \frac{\hat{b}_{1,t}^{s}}{\hat{B}_t^{s}} Q_t^{s} \\
c_{2,t+1}^{s'} &= \omega_2^{s'} - q_{2,t+1}^{s'} + \frac{\hat{b}_{2,t+1}^{s'}}{\hat{B}_{t+1}^{s'}} Q_{t+1}^{s'} \qquad (14)\\
c_{3,t+2}^{s''} &= \omega_3^{s''} - q_{3,t+2}^{s''} + \frac{\hat{b}_{3,t+2}^{s''}}{\hat{B}_{t+2}^{s''}} Q_{t+2}^{s''}
\end{aligned}
$$

As in (4), we can rewrite (13) as:

$$
\begin{aligned}
\hat{b}_{1,t}^{s} + \hat{m}_{1,t}^{s} &= \left(\frac{\hat{B}_{t,-1}^{s} - \hat{m}_{1,t}^{s} + \Delta\hat{m}_t I_1}{Q_{t,-1}^{s}} \right) q_{1,t}^{s} + \Delta\hat{m}_t I_1 \\
\hat{b}_{2,t+1}^{s'} + \hat{m}_{2,t+1}^{s'} - \frac{\hat{m}_{1,t}^{s}}{(1+g)} &= \left(\frac{\hat{B}_{t+1,-2}^{s'} - \hat{m}_{2,t+1}^{s'} + \frac{\hat{m}_{1,t}^{s}}{(1+g)} + \Delta\hat{m}_{t+1} I_2}{Q_{t+1,-2}^{s'}} \right) q_{2,t+1}^{s'} + \Delta\hat{m}_{t+1} I_2 \qquad (15)\\
\hat{b}_{3,t+2}^{s''} - \frac{\hat{m}_{2,t+1}^{s'}}{(1+g)} &= \left(\frac{\hat{B}_{t+2,-3}^{s''} + \frac{\hat{m}_{2,t+1}^{s'}}{(1+g)}}{Q_{t+2,-3}^{s''}} \right) q_{3,t+2}^{s''}
\end{aligned}
$$

where $I_1 = I\left(\omega_1^{s} = \omega_1^{B}\right)$ and $I_2 = I\left(\omega_2^{s'} = \omega_2^{B}\right)$ with a convenient abuse of notation.

By equating the right-hand sides of (13) and (15), we derive an equation similar to (5):

(16)

$$
\frac{Q_t^{s}}{\hat{B}_t^{s}} = \frac{Q_{t,-1}^{s}}{\hat{B}_{t,-1}^{s} - \hat{m}_{1,t}^{s} + \Delta\hat{m}_t I_1},\ \frac{Q_{t+1}^{s'}}{\hat{B}_{t+1}^{s'}} = \frac{Q_{t+1,-2}^{s'}}{\hat{B}_{t+1,-2}^{s'} - \hat{m}_{2,t+1}^{s'} + \frac{\hat{m}_{1,t}^{s}}{(1+g)} + \Delta\hat{m}_{t+1} I_2},\ \frac{Q_{t+2}^{s''}}{\hat{B}_{t+2}^{s''}} = \frac{Q_{t+2,-3}^{s''}}{\hat{B}_{t+2,-3}^{s''} + \frac{\hat{m}_{2,t+1}^{s'}}{(1+g)}}
$$

Substituting these equations with (15) into (14), we generate the following consumption allocations in the economy with the active monetary policy:

(17)

$$
\begin{aligned}
c_{1,t}^{s} &= \omega_1^{s} - \left(\frac{Q_{t,-1}^{s}}{\hat{B}_{t,-1}^{s} - \hat{m}_{1,t}^{s} + \Delta\hat{m}_t I_1} \right) \left(\hat{m}_{1,t}^{s} - \Delta\hat{m}_t I_1 \right) \\
c_{2,t+1}^{s'} &= \omega_2^{s'} - \left(\frac{Q_{t+1,-2}^{s'}}{(1+g)\left(\hat{B}_{t+1,-2}^{s'} - \hat{m}_{2,t+1}^{s'} + \Delta\hat{m}_{t+1} I_2 \right) + \hat{m}_{1,t}^{s}} \right) \left((1+g)\left(\hat{m}_{2,t+1}^{s'} - \Delta\hat{m}_{t+1} I_2 \right) - \hat{m}_{1,t}^{s} \right) \\
c_{3,t+2}^{s''} &= \omega_3^{s''} + \left(\frac{Q_{t+2,-3}^{s''}}{(1+g)\,\hat{B}_{t+2,-3}^{s''} + \hat{m}_{2,t+1}^{s'}} \right) \hat{m}_{2,t+1}^{s'}
\end{aligned}
$$

Under the expansionary monetary policy, the first-order conditions with respect to $\hat{m}_{1,t}^{s}$ and

$\hat{m}^{s'}_{2,t+1}$ are given by:

$$
\begin{aligned}
&u'\left(c^s_{1,t}\right)\frac{\hat{B}^s_{t,-1}Q^s_{t,-1}}{\left(\hat{B}^s_{t,-1}-\hat{m}^s_{1,t}+\Delta\hat{m}_t I_1\right)^2} \\
&=\delta\sum_{s'\in\{\alpha,\beta\}}\pi^{s'}u'\left(c^{s'}_{2,t+1}\right)\frac{\hat{B}^{s'}_{t+1,-2}Q^{s'}_{t+1,-2}}{(1+g)\left(\left(\hat{B}^{s'}_{t+1,-2}-\hat{m}^{s'}_{2,t+1}+\Delta\hat{m}_{t+1}I_2\right)+\frac{\hat{m}^s_{1,t}}{(1+g)}\right)^2}
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
&u'\left(c^{s'}_{2,t+1}\right)\frac{\hat{B}^{s'}_{t+1,-2}Q^{s'}_{t+1,-2}}{\left(\left(\hat{B}^{s'}_{t+1,-2}-\hat{m}^{s'}_{2,t+1}+\Delta\hat{m}_{t+1}I_2\right)+\frac{\hat{m}^s_{1,t}}{(1+g)}\right)^2} \\
&=\delta\sum_{s''\in\{\alpha,\beta\}}\pi^{s''}u'\left(c^{s''}_{3,t+2}\right)\frac{\hat{B}^{s''}_{t+2,-3}Q^{s''}_{t+2,-3}}{(1+g)\left(\hat{B}^{s''}_{t+2,-3}+\frac{\hat{m}^{s'}_{2,t+1}}{(1+g)}\right)^2}
\end{aligned}
\tag{19}
$$

Using the equation (16), one can simplify (17), (18) and (19) as follows:

$$
\begin{aligned}
c^s_{1,t}&=\omega^s_1-\frac{Q^s_t}{\hat{B}^s_t}\left(\hat{m}^s_{1,t}-\Delta\hat{m}_t I_1\right)\\
c^{s'}_{2,t+1}&=\omega^{s'}_2-\frac{Q^{s'}_{t+1}}{\hat{B}^{s'}_{t+1}}\left(\left(\hat{m}^{s'}_{2,t+1}-\Delta\hat{m}_{t+1}I_2\right)-\frac{\hat{m}^s_{1,t}}{(1+g)}\right)\\
c^{s''}_{3,t+2}&=\omega^{s''}_3+\frac{Q^{s''}_{t+2}}{\hat{B}^{s''}_{t+2}}\frac{\hat{m}^{s'}_{2,t+1}}{(1+g)}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&u'\left(c^s_{1,t}\right)\frac{\hat{B}^s_{t,-1}}{Q^s_{t,-1}}\left(\frac{Q^s_t}{\hat{B}^s_t}\right)^2\\
&=\delta\sum_{s'\in\{\alpha,\beta\}}\pi^{s'}u'\left(c^{s'}_{2,t+1}\right)\frac{1}{(1+g)}\frac{\hat{B}^{s'}_{t+1,-2}}{Q^{s'}_{t+1,-2}}\left(\frac{Q^{s'}_{t+1}}{\hat{B}^{s'}_{t+1}}\right)^2
\end{aligned}
\tag{21}
$$

and

$$
\begin{aligned}
&u'\left(c^{s'}_{2,t+1}\right)\frac{\hat{B}^{s'}_{t+1,-2}}{Q^{s'}_{t+1,-2}}\left(\frac{Q^{s'}_{t+1}}{\hat{B}^{s'}_{t+1}}\right)^2\\
&=\delta\sum_{s''\in\{\alpha,\beta\}}\pi^{s''}u'\left(c^{s''}_{3,t+2}\right)\frac{1}{(1+g)}\frac{\hat{B}^{s''}_{t+2,-3}}{Q^{s''}_{t+2,-3}}\left(\frac{Q^{s''}_{t+2}}{\hat{B}^{s''}_{t+2}}\right)^2
\end{aligned}
\tag{22}
$$

With these new equilibrium conditions, we run a numerical welfare analysis under the active monetary policy following the simulation steps conducted above. In the numerical exercise, we keep all other parameters as in the base case without policies, except the growth rates

Table 7: Welfare analysis in the economy with $g = 5\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.3260	0.3522	0.3391	3.87%
Middle-aged	0.3585	0.3105	0.3345	7.17%
Old	0.3156	0.3373	0.3264	3.33%
Equilibrium consumption with both friction				
Young	0.3066	0.3591	0.3329	7.88%
Middle-aged	0.4443	0.3604	0.4024	10.43%
Old	0.2490	0.2805	0.2648	5.94%
	Benchmark	**Inc. Mk.**	**Both**	
CE consumption	0.3333	0.3316	0.3212	
CE consumption as (%) of benchmark	100%	99.48%	96.35%	

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

Table 8: Welfare analysis in the economy with $g = 10\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.3370	0.3584	0.3477	3.08%
Middle-aged	0.3540	0.3139	0.3340	6.01%
Old	0.3090	0.3277	0.3183	2.94%
Equilibrium consumption with both friction				
Young	0.3150	0.3636	0.3393	7.17%
Middle-aged	0.4394	0.3638	0.4016	9.40%
Old	0.2457	0.2726	0.2591	5.19%
	Benchmark	**Inc. Mk.**	**Both**	
CE consumption	0.3333	0.3318	0.3207	
CE consumption as (%) of benchmark	100%	99.53%	96.21%	

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

Table 9: Welfare analysis in the economy with $g = 20\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with incomplete market				
Young	0.3568	0.3702	0.3635	1.84%
Middle-aged	0.3453	0.3203	0.3328	3.75%
Old	0.3095	0.2979	0.3037	1.91%
Equilibrium consumption with both friction				
Young	0.3302	0.3720	0.3511	5.94%
Middle-aged	0.4305	0.3702	0.4004	7.53%
Old	0.2393	0.2578	0.2485	3.73%
	Benchmark	**Inc. Mk.**	**Both**	
CE consumption	0.3333	0.3311	0.3191	
CE consumption as (%) of benchmark	100%	99.33%	95.72%	

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

of the total money supply. We test three different money growth rates: 5%, 10%, and 20%. Three tables below summarize the welfare results for each growth rate.

In the economy with only market incompleteness, the monetary policy reduces consumption volatility between states by implicitly redistributing wealth from the endowment rich cohort to the poor one via an inflation tax. However, it increases consumption variation over lifetimes by generating a declining consumption-age profile. Intuitively, the expansionary monetary policy generates an intertemporal wedge on saving because the new money supply in every period increases the prices of goods tomorrow compared to today and thus decreases gross interest rates. This wedge discourages individual saving, which brings about the declining life-cycle consumption stream.

Compared to the base case without government interventions, the active monetary policy improves long-run social welfare for all money growth rates by promoting risk-sharing despite generating lifetime consumption variation, when there is only the incomplete market friction. The 10% growth rate advances welfare most, and then 5% and 20 % in order. The larger the growth rate, the more risk-sharing but larger consumption variation over the life-cycle. Thus, the welfare improvement can be worst when the money growth rate is 20% because such a high rate will generate the largest intertemporal wedge among the three rates and the welfare loss from it can dominate the benefit of mitigating the consumption volatility between states.

If both frictions are present, all expansionary monetary policies worsen long-run social welfare compared to the base economy although the within-age consumption volatilities decrease. Imperfect competition already induces consumption variation over the life-cycle as implied by the correlated consumption/endowment profiles. Further consumption variation generated by the intertemporal wedge via the active monetary policies will drop welfare significantly under a concave utility function. Thus, the monetary policy can decrease social welfare under both frictions. For this reason, the 5% money growth rate decreases social welfare the least from the base case among the three rates, by distorting the life-cycle consumption allocations least.

We do not show the numerical analysis results about the role of the monetary policy in the economy only with imperfect competition because there is no consumption volatility between states for the policy to reduce and the monetary policy cannot adjust the effective prices, which are solely determined by endowment offers. Thus, we conjecture that introducing monetary policy will aggravate welfare in this economy by producing intertemporal wedges.

6.2 Fiscal Policy

In the previous subsection, we showed that monetary policy enhances inter-generational risk-sharing against the idiosyncratic endowment risk. However, it indeed decreases social welfare when both frictions are present because the active monetary policy produces an intertemporal wedge and further distorts consumption variation over the life-cycle.

It is well-known in the market game literature that allowing wash-sales decreases the welfare loss from strategic interactions by equating the effective prices among agents with different levels of endowments (see Peck and Shell (1990)).[11] However, if wash-sales are prohibited because of commitment issues or other exogenous constraints, then a linear endowment tax and transfer can be an alternative. This fiscal policy reduces the amount of goods traded in the imperfect market. Thus, it limits the extent of distortion across the life-cycle by the strategic interaction. In addition, the fiscal policy can improve endowment risk-sharing via intergenerational redistribution and reduction of the consumption volatility between states.

Therefore, in this subsection, we propose a fiscal policy to improve welfare even if both frictions are present. We focus on a time-invariant linear endowment tax and lump-sum transfer to all living generations. The government balances its budget. We assume that agents offer only the after-tax endowments, not the transfer. Thus, the timeline is as follows: the government collects tax revenue first, agents offer all endowments left under the sell-all strategy, and then they receive a transfer.

Under the fiscal policy, there are no changes in the individual budget constraints. However, the consumption allocation rules take the form:

$$\begin{aligned} c_{1,t}^{s} &= (1-\tau)\,\omega_1^{s} - q_{1,t}^{s} + \frac{b_{1,t}^{s}}{B_t^{s}} Q_t^{s} + T^{s} \\ c_{2,t+1}^{s'} &= (1-\tau)\,\omega_2^{s'} - q_{2,t+1}^{s'} + \frac{b_{2,t+1}^{s'}}{B_{t+1}^{s'}} Q_{t+1}^{s'} + T^{s'} \\ c_{3,t+2}^{s''} &= (1-\tau)\,\omega_3^{s''} - q_{3,t+2}^{s''} + \frac{b_{3,t+2}^{s''}}{B_{t+2}^{s''}} Q_{t+2}^{s''} + T^{s''} \end{aligned} \tag{23}$$

where τ is a time-invariant proportional endowment tax and $T^s = \frac{\tau\Omega^s}{3n}$ denotes a lump-sum

[11] When agents trade under the heterogeneous effective prices, they optimize decisions at different effective rates of substitution. Thus, Pareto-improving allocations can exist for the equilibrium under the market game. If wash-sales are allowed, then agents can offer more than their endowments. If they increase offers, the return rates of bidding will get closer between agents. In the limit, the effective prices will be identical, and thus the market game allocations will converge to Pareto-efficient ones.

transfer in state s. The after-tax offers are given by $q_{i,t}^s = (1-\tau)\,\omega_i^s$ and $Q_t^s = (1-\tau)\,\Omega^s$ for $\forall\,(i,s)$.

We can transform the consumption allocations above into:

$$\begin{aligned}
c_{1,t}^{s} &= (1-\tau)\,\omega_1^s - \left(\frac{Q_{t,-1}^s}{B_{t,-1}^s - m_{1,t}^s}\right) m_{1,t}^s + T^s \\
c_{2,t+1}^{s'} &= (1-\tau)\,\omega_2^{s'} - \left(\frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'} - m_{2,t+1}^{s'} + m_{1,t}^s}\right)\left(m_{2,t+1}^{s'} - m_{1,t}^s\right) + T^{s'} \\
c_{3,t+2}^{s''} &= (1-\tau)\,\omega_3^{s''} + \left(\frac{Q_{t+2,-3}^{s''}}{B_{t+2,-3}^{s''} + m_{2,t+1}^{s'}}\right) m_{2,t+1}^{s'} + T^{s''}
\end{aligned} \tag{24}$$

With the equilibrium conditions under the fiscal policy above, we compute the symmetric recursive monetary Nash equilibria. In the numerical analysis, we keep parameter values as in the base economy with no policies, other than the rates of the endowment tax. We examine three different tax rates: 10%, 20%, and 30%. In the following three tables, we summarize the numerical welfare results from different tax rates.

Table 10: Welfare analysis in the economy with $\tau = 10\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with imperfect competition				
Young	0.3289	0.3289	0.3289	0.00%
Middle-aged	0.3991	0.3991	0.3991	0.00%
Old	0.2720	0.2720	0.2720	0.00%
Equilibrium consumption only with incomplete market				
Young	0.3160	0.3452	0.3306	4.42%
Middle-aged	0.3613	0.3086	0.3350	7.86%
Old	0.3227	0.3462	0.3344	3.51%
Equilibrium consumption with both friction				
Young	0.3000	0.3538	0.3269	8.24%
Middle-aged	0.4437	0.3587	0.4012	10.59%
Old	0.2563	0.2874	0.2719	5.72%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3253	0.3313	0.3223
CE consumption as (%) of benchmark	100%	97.58%	99.39%	96.69%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

The numerical analysis shows that consumption variations fall across ages in the economy with only the imperfect competition friction as tax rates rise, because the fiscal policy shrinks

Table 11: Welfare analysis in the economy with $\tau = 20\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with imperfect competition				
Young	0.3293	0.3293	0.3293	0.00%
Middle-aged	0.3973	0.3973	0.3973	0.00%
Old	0.2735	0.2735	0.2735	0.00%
Equilibrium consumption only with incomplete market				
Young	0.3173	0.3448	0.3310	4.16%
Middle-aged	0.3582	0.3110	0.3346	7.06%
Old	0.3245	0.3442	0.3344	2.95%
Equilibrium consumption with both friction				
Young	0.3023	0.3532	0.3278	7.75%
Middle-aged	0.4372	0.3608	0.3990	9.57%
Old	0.2605	0.2860	0.2732	4.67%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3257	0.3317	0.3233
CE consumption as (%) of benchmark	100%	97.71%	99.50%	96.98%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %
- CE consumption represents certainty equivalent consumption

Table 12: Welfare analysis in the economy with $\tau = 30\%$

Age	State α	State β	Mean	CV (%)
Equilibrium consumption only with imperfect competition				
Young	0.3297	0.3297	0.3297	0.00%
Middle-aged	0.3950	0.3950	0.3950	0.00%
Old	0.2753	0.2753	0.2753	0.00%
Equilibrium consumption only with incomplete market				
Young	0.3189	0.3442	0.3315	3.82%
Middle-aged	0.3555	0.3134	0.3344	6.30%
Old	0.3256	0.3424	0.3340	2.52%
Equilibrium consumption with both friction				
Young	0.3049	0.3523	0.3286	7.20%
Middle-aged	0.4303	0.3625	0.3964	8.55%
Old	0.2852	0.2647	0.2750	3.72%
	Benchmark	**Imp. Comp.**	**Inc. Mk.**	**Both**
CE consumption	0.3333	0.3262	0.3320	0.3243
CE consumption as (%) of benchmark	100%	97.86%	99.60%	97.28%

- CV stands for the coefficient of variation which is the standard deviation of consumption divided by its mean measured in %

- CE consumption represents certainty equivalent consumption

the volume of goods transacted in the trading post and thus limit the distortion by strategic interactions. In addition, the fiscal policy reduces the consumption volatility between states in the economy with only the incomplete market friction by redistributing wealth from the rich cohort to the poor one. Thus, this government intervention decreases the welfare loss from each friction. Moreover, it also improves social welfare even when both frictions exist.

However, we should stress that there is no distortion from the linear tax in the endowment economy. Thus, taxing all endowments and distributing equally to living generations can achieve stationary allocations in the frictionless economy. It is natural to derive the result that the higher the tax rates, the closer to the benchmark allocation and the higher the welfare improvement. Thus, if labor supply is elastic and taxed, then there will be an upper bound for the tax rate to advance the long-run ex-ante expected utility because of a welfare loss in the labor supply distortion. However, the labor income tax will not generate an intertemporal wedge like the monetary policy and still smoothes consumption over the life-cycle as in the endowment tax model. Thus, a labor distorting tax/transfer policy also has an asymmetric welfare effect relative to the monetary policy case under strategic interactions.[12]

We have assumed that agents offer only the endowments net of taxes under the sell-all strategy. Even if we allow them to offer the transfer as well, the results will remain the same. The fiscal policy redistributes wealth from the working ages to the retired and from the rich to the poor. Thus, it smoothes offers across ages and states even if the offers include the transfer. Hence, there will be a smaller gap between the effective prices of different generations, and fiscal policy, in this different scheme, will also reduce the consumption variations over both ages and states.

7 Extension

In developing economies, the effective prices largely depend on the income of buyers, due to the quantity premium as seen in Rao (2000). However, Aguiar and Hurst (2007) show that the search activity of agents for price discount opportunities also generates heterogeneous prices for identical goods across households in developed countries such as the U.S. Thus, the low-income group can rather pay less for identical goods than its counterpart because of their intensive search to find better prices under the low opportunity cost of search from their low wage. Hence, the welfare implication of this paper should be carefully applied to the developed economies, where both the quantity premium and the search activity differentiated by income levels advantage high and low-income groups in the opposite direction.

In this section, we extend the current model to incorporate the effects of search activity for price discounts on the effective prices to obtain the welfare implications of imperfect competition for developed economies as well. For simplicity, we assume that the search effort is exogenously given as the goods endowment, and its distribution is inversely related to the income levels of agents or the opportunity cost of search activity across types and ages following the results in Aguiar and Hurst (2007). For example, low-wage/income workers receive a large

[12] In a perfectly competitive economy, there is no distortion in the lifetime consumption profile to reduce by the fiscal policy. However, it works in reducing the consumption variation across age under imperfect competition, which the monetary policy increases. We regard this discrepancy as to the asymmetric welfare effects of fiscal and monetary policy actions.

search endowment, whereas high-wage/income earners receive a small one.

As a reduced form model, we assume that both offering consumption goods and conducting search behavior reduce effective prices via the bid-returning process. In other words, agents either offering more goods or exercising more intensive search have higher bid return rates and face lower effective prices. With this assumption, we write new individual budget constraints:

$$\begin{aligned} b^s_{1,t} + m^s_{1,t} &= \left(\lambda \frac{q^s_{1,t}}{Q^s_t} + (1-\lambda)\frac{s^s_{1,t}}{S^s_t}\right) B^s_t \\ b^{s'}_{2,t+1} + m^{s'}_{2,t+1} - m^s_{1,t} &= \left(\lambda \frac{q^{s'}_{2,t+1}}{Q^{s'}_{t+1}} + (1-\lambda)\frac{s^{s'}_{2,t+1}}{S^{s'}_{t+1}}\right) B^{s'}_{t+1} \\ b^{s''}_{3,t+2} - m^{s'}_{2,t+1} &= \left(\lambda \frac{q^{s''}_{3,t+2}}{Q^{s''}_{t+2}} + (1-\lambda)\frac{s^{s''}_{3,t+2}}{S^{s''}_{t+2}}\right) B^{s''}_{t+2} \end{aligned} \tag{25}$$

where we assume that agents offer all their goods and search endowments under the sell-all strategy. The s variables (not in the superscripts) denote search endowment, and the weight parameter λ determines the relative importance of the quantity premium compared to search effort in increasing bid returns rates and thus reducing effective prices.

As done in the base model in Section 2, we can derive the following effective prices that generation i faces in time t in the extended model:

$$\frac{\left(1 - \lambda \frac{q^s_{i,t}}{Q^s_t} - (1-\lambda)\frac{s^s_{i,t}}{S^s_t}\right) Q^s_t B^s_{t,-i}}{\left(c^s_{i,t} - Q^s_t\right)^2} \tag{26}$$

This equation re-confirms that the more ones offer goods and search endowments, the lower their effective prices are. We stress again that we assume a negative correlation between the goods and search endowment distributions in this model. Thus, which income group will face lower effective prices depends on the value of λ. If λ is high enough, then offering goods is a more significant channel to reduce the effective prices than exercising search. Thus, those receiving a high endowment shock are advantaged as in the base model above. On the other hand, if λ is low enough, the search activity is a more significant factor than the income effect. In this case, the price heterogeneity will be more favorable toward low wage earners because they can put more search efforts.

The results of this analysis imply that whether the search effect dominates the income effect or the opposite is an empirical question. One can answer this by applying data to estimating λ. Note that we have assumed the shares out of total bids assigned to each agent in the parentheses of (25) linearly depend on both goods offers and search efforts. Instead of the linear form, one can use a more general non-linear share formula given by $f\left(q^s_{i,t}, s^s_{i,t}; Q^s_t, S^s_t\right)$ in the estimation, where f increases in both goods and search endowments offers. We do not pursue this empirical study in our paper since it is beyond the scope of the paper.

8 Conclusion

In this paper, we study whether imperfect competition can increase consumption variation across states and ages by interacting with financial market incompleteness. We show that income-dependent prices for identical goods under imperfect competition bias consumption toward agents who receive high-income shocks relative to the competitive economy and thus reduce risk-sharing by generating additional consumption volatility. We quantify the additional consumption volatility and its welfare loss in a parameterized version of the model. Our numerical analysis shows that the complementary welfare loss adds about 50% of the welfare loss due solely to market incompleteness. We also find that price heterogeneity across the lifecycle in our model can break down the perfect consumption smoothing result and generate the stylized fact of correlated consumption/income profiles without other frictions such as capital market imperfections or impatient consumers. To check the robustness of our results, we use other values for the time discount factor and risk-aversion parameters and different shock structures. We observe that the implication of imperfect competition on welfare remains the same qualitatively, but the quantitative welfare implications can vary according to the parameters' values. From a policy analysis perspective, we find that both monetary and fiscal policy reduces consumption volatility between states. However, monetary policy decreases welfare by further increasing consumption variation over the lifetime via an intertemporal wedge, while fiscal policy increases welfare by reducing the share of goods traded under the strategic interactions and mitigating lifecycle consumption variation. Thus, the introduction of the imperfect competition results in an asymmetric welfare effect between fiscal and monetary policy, unlike in competitive models.

A Proofs

A.1 Proposition 1

We prove the generic non-existence of strongly stationary symmetric monetary Nash equilibria (no-recall equilibria) by contradiction. It is straightforward to extend this result to short memory symmetric monetary Nash equilibria following arguments in Citanna and Siconolfi (2007) and Henriksen and Spear (2012) once the strongly stationary one does not exist.

We assume that there is a strongly stationary symmetric monetary Nash equilibrium where bids and money holdings only depend on the state of the current shock. Then, we can write the consumption allocations, the first-order conditions, and the market clearing condition under this type of equilibria as:

$$\begin{aligned} c_1^s &= \omega_1^s - \left(\frac{Q_{-1}^s}{B_{-1}^s - m_1^s}\right) m_1^s \\ c_2^{ss'} &= \omega_2^{s'} - \left(\frac{Q_{-2}^{s'}}{B_{-2}^{s'} - m_2^{s'} + m_1^s}\right)\left(m_2^{s'} - m_1^s\right) \\ c_3^{s's''} &= \omega_3^{s''} + \left(\frac{Q_{-3}^{s''}}{B_{-3}^{s''} + m_2^{s'}}\right) m_2^{s'} \end{aligned} \tag{A.1}$$

where $\{s, s', s''\} \in \{\alpha, \beta\}^3$.

$$\begin{aligned} & u'\left(c_1^{\alpha}\right) \frac{B_{-1}^{\alpha}}{Q_{-1}^{\alpha}} \left(\frac{Q^{\alpha}}{B^{\alpha}}\right)^2 \\ = \delta \sum_{s' \in \{\alpha,\beta\}} & \pi^{s'} u'\left(c_2^{\alpha s'}\right) \frac{B_{-2}^{s'}}{Q_{-2}^{s'}} \left(\frac{Q^{s'}}{B^{s'}}\right)^2 \end{aligned} \tag{A.2}$$

$$\begin{aligned} & u'\left(c_1^{\beta}\right) \frac{B_{-1}^{\beta}}{Q_{-1}^{\beta}} \left(\frac{Q^{\beta}}{B^{\beta}}\right)^2 \\ = \delta \sum_{s' \in \{\alpha,\beta\}} & \pi^{s'} u'\left(c_2^{\beta s'}\right) \frac{B_{-2}^{s'}}{Q_{-2}^{s'}} \left(\frac{Q^{s'}}{B^{s'}}\right)^2 \end{aligned} \tag{A.3}$$

$$\begin{aligned} & u'\left(c_2^{s\alpha}\right) \frac{B_{-2}^{\alpha}}{Q_{-2}^{\alpha}} \left(\frac{Q^{\alpha}}{B^{\alpha}}\right)^2 \\ = \delta \sum_{s'' \in \{\alpha,\beta\}} & \pi^{s''} u'\left(c_3^{\alpha s''}\right) \frac{B_{-3}^{s''}}{Q_{-3}^{s''}} \left(\frac{Q^{s''}}{B^{s''}}\right)^2 \end{aligned} \tag{A.4}$$

and

$$
\begin{aligned}
(A.5)\qquad & u'\left(c_2^{s\beta}\right)\frac{B_{-2}^{\beta}}{Q_{-2}^{\beta}}\left(\frac{Q^{\beta}}{B^{\beta}}\right)^2 \\
&= \delta \sum_{s''\in\{\alpha,\beta\}} \pi^{s''} u'\left(c_3^{\beta s''}\right)\frac{B_{-3}^{s''}}{Q_{-3}^{s''}}\left(\frac{Q^{s''}}{B^{s''}}\right)^2
\end{aligned}
$$

where $s \in \{\alpha, \beta\}$.

$$
(A.6)\qquad (m_1^s + m_2^s) = M \text{ for } s \in \{\alpha, \beta\}
$$

where $s \in \{\alpha, \beta\}$.

The equations above have the following implications. The young age consumptions depend on only the current shock state whereas the middle-aged and old period consumptions depend (potentially) on both the current and lagged shock realizations because agents' money holdings are affected by the state in which the asset was purchased. However, the right-hand-sides of the first-order conditions of the middle-aged in (A.4) and (A.5) are independent of the lagged state, s. Thus, $c_2^{\alpha\alpha} = c_2^{\beta\alpha} = c_2^{\alpha}$ and $c_2^{\alpha\beta} = c_2^{\beta\beta} = c_2^{\beta}$ which implies that the middle-aged period consumptions also depend only on the current shock realizations. The good market clearing condition requires that $c_1^s + c_2^s + c_3^{s's} = \omega_1^s + \omega_2^s + \omega_3^s$ for $\{s', s\} \in \{\alpha, \beta\}^2$. Therefore, we also obtain that $c_3^{\alpha\alpha} = c_3^{\beta\alpha} = c_3^{\alpha}$ and $c_3^{\alpha\beta} = c_3^{\beta\beta} = c_3^{\beta}$.

We now demonstrate that the money holdings are state-independent. For this, we derive the following equations by imposing the results that $c_2^{\alpha s} = c_2^{\beta s}$ and $c_3^{\alpha s} = c_3^{\beta s}$ for $s \in \{\alpha, \beta\}$ on (A.1):

$$
(A.7)\qquad \omega_2^s - \left(\frac{Q_{-2}^s}{B_{-2}^s - m_2^s + m_1^{\alpha}}\right)(m_2^s - m_1^{\alpha}) = \omega_2^s - \left(\frac{Q_{-2}^s}{B_{-2}^s - m_2^s + m_1^{\beta}}\right)\left(m_2^s - m_1^{\beta}\right)
$$

and

$$
(A.8)\qquad \omega_3^s + \left(\frac{Q_{-3}^s}{B_{-3}^s + m_2^{\alpha}}\right)m_2^{\alpha} = \omega_3^s + \left(\frac{Q_{-3}^s}{B_{-3}^s + m_2^{\beta}}\right)m_2^{\beta}
$$

To satisfy these equations, $m_1^{\alpha} = m_1^{\beta} = m_1$ and $m_2^{\alpha} = m_2^{\beta} = m_2$. The state-independent money holdings simplify the budget constraints faced by the overlapped generations:

$$
(A.9)\qquad
\begin{aligned}
b_1^s + m_1 &= \frac{q_1^s}{Q^s}B^s \\
b_2^s + m_2 - m_1 &= \frac{q_2^s}{Q^s}B^s \\
b_3^s - m_2 &= \frac{q_3^s}{Q^s}B^s
\end{aligned}
$$

where $s \in \{\alpha, \beta\}$.

The three equations in (A.9) are linearly dependent on each other since we obtain an identical equation by summing these equations and multiplying both sides by n:

$$nb_1^s + nb_2^s + nb_3^s = \frac{nq_1^s}{Q^s}B^s + \frac{nq_2^s}{Q^s}B^s + \frac{nq_3^s}{Q^s}B^s = B^s \tag{A.10}$$

where $s \in \{\alpha, \beta\}$.

Thus, we end up with a system of 15 equations: 6 equations from the consumption allocation rule, 4 equations from the first-order conditions, 4 equations from the budget constraints, and 1 equation from the money market clearing condition. However, there are 14 variables: $c_1^\alpha, c_1^\beta, c_2^\alpha, c_2^\beta, c_3^\alpha, c_3^\beta, b_1^\alpha, b_1^\beta, b_2^\alpha, b_2^\beta, b_3^\alpha, b_3^\beta, m_1$ and m_2. Because there are more equations than variables, the strongly stationary monetary Nash equilibria do not exist generically following the arguments in Spear (1985) and Citanna and Siconolfi (2007).

A.2 Proposition 2

We show that perfect consumption smoothing cannot be the outcome of the steady states in the offer constrained OLG market game under pure strategy with $\delta = 1$ and no endowment risk by contradiction.

Thus, we start by assuming that the perfectly smoothed consumption allocations are stationary outcomes, i.e. $c_1 = c_2 = c_3$. Then, $b_1 = b_2 = b_3$ at the steady states by the consumption allocation rule under the sell-all strategy. The first-order conditions require $q_1 = q_2 = q_3$ for the equal lifetime consumptions to be individual optimal choices. However, this equivalence of offers only holds when endowments are identical across ages. Even in the case with the equal endowment profile, the relationships among consumptions, bids and offers indicate $m_1 = m_2 = 0$ from the budget constraints which is contradictory to the money market clearing condition where $m_1 + m_2 = M > 0$. Therefore, perfect consumption smoothing does not hold at the steady states for any endowment streams in the OLG economy with $\delta = 1$ and no endowment risk if there are strategic interactions.

The steady-state consumption allocations rather vary over ages since agents consume more in a certain age with lower effective prices depending on their income as follows. We first write the Euler equations at the stationary allocations under the sell-all strategy:

$$u'\left(\frac{b_1}{B}\Omega\right)\frac{B_{-1}}{\Omega_{-1}} = u'\left(\frac{b_2}{B}\Omega\right)\frac{B_{-2}}{\Omega_{-2}} = u'\left(\frac{b_3}{B}\Omega\right)\frac{B_{-3}}{\Omega_{-3}} \tag{A.11}$$

where $\Omega = n(\omega_1 + \omega_2 + \omega_3)$ and $\Omega_{-i} = \Omega - \omega_i$ for $i \in \{1, 2, 3\}$.

Without loss of generality, we normalize the total bid to be one. Then, (A.11) becomes:

$$\frac{u'(b_1\Omega)(1-b_1)}{\Omega_{-1}} = \frac{u'(b_2\Omega)(1-b_2)}{\Omega_{-2}} = \frac{u'(b_3\Omega)(1-b_3)}{\Omega_{-3}} \tag{A.12}$$

Here, $u'(x\Omega)(1-x)$ is strictly decreasing in x for $\forall x \in [0,1]$ under the standard assumptions on the utility function. We focus on the cases where agents' bids are positive. Then, $\frac{1}{\Omega_{-i}}$ is proportional to the endowment of age i. Thus, b_i should be larger for ages with higher endowments to satisfy (A.12). This relationthip results in larger consumptions in ages with higher endowments.

Intuitively speaking, agents face lower effective prices when offering more and thus, they transfer their wealth to ages with large endowments to purchase goods more cheaply. Therefore, the graph of the consumption stream is parallel to that of the income stream at the stationary equilibria under strategic interactions.

A.3 Corollary 1

According to Proposition 2, consumptions are proportional to endowments at the stationary equilibria in the economy with strategic interactions. Thus, when $\omega_2 > \omega_1 > \omega_3 = 0$, we obtain that $c_2 > c_1 > c_3$. We first consider the case that $c_2 \leq \frac{1}{3}(\omega_1 + \omega_2)$, then $c_1 < \frac{1}{3}(\omega_1 + \omega_2)$ and $c_3 < \frac{1}{3}(\omega_1 + \omega_2)$ because of the order of the lifetime consumptions. This case violates the good market clearing condition because $c_1 + c_2 + c_3 < \omega_1 + \omega_2$. Therefore, $c_2 > \frac{1}{3}(\omega_1 + \omega_2)$. Likewise, $c_3 < \frac{1}{3}(\omega_1 + \omega_2)$. If not, the good market clearing condition will be violated. Unlike c_2 and c_3, c_1 can be larger than, equal to or smaller than $\frac{1}{3}(\omega_1 + \omega_2)$ which depends on endowment ratios between the young and the middle-aged and utility functions.

To characterize the condition for c_1 to be lager than $\frac{1}{3}(\omega_1 + \omega_2)$, we consider the endowment structure with $\omega_1 = \omega_2 = \frac{\Omega}{2n}$. In this case, the first equality in (A.12) becomes $u'(b_1\Omega)(1 - b_1) = u'(b_2\Omega)(1 - b_2)$. To satisfy this equation, $b_1 = b_2$ since $u'(x\Omega)(1 - x)$ is strictly decreasing in x. This result implies that $c_1 = c_2 > \frac{1}{3}(\omega_1 + \omega_2)$. By continuity, $c_1 > \frac{1}{3}(\omega_1 + \omega_2)$ if ω_1 is smaller than but close enough to $\frac{\Omega}{2n}$.

The endowment structure with $\omega_1 = \frac{\Omega}{3n}$ and $\omega_2 = \frac{2\Omega}{3n}$ transforms (A.12) to:

$$\frac{u'(b_1\Omega)(1 - b_1)}{3n - 1} = \frac{u'(b_2\Omega)(1 - b_2)}{3n - 2} = \frac{u'(b_3\Omega)(1 - b_3)}{3n} \tag{A.13}$$

This equation implies that $u'(b_1\Omega)(1 - b_1) = \frac{u'(b_2\Omega)(1-b_2)+u'(b_3\Omega)(1-b_3)}{2}$. Here, $u'(x\Omega)(1 - x)$ is strictly convex in x from the assumption that $u'''(\cdot) > 0$. Then, by Jensen's inequality, $b_1 < \frac{b_2+b_3}{2}$. From this result, we know that $b_1 < \frac{B}{3n}$ and thus, $c_1 < \frac{1}{3}(\omega_1 + \omega_2)$. Hence, if ω_1 is larger than but sufficiently close to $\frac{\Omega}{3n}$, $c_1 < \frac{1}{3}(\omega_1 + \omega_2)$ by continuity.

B Algorithm

In this appendix, we explain how to compute the symmetric recursive Markov Nash equilibria in the three-period OLG models with the offer constrained Shapley-Shubik market game under sell-all strategy. We adopt the projection method to approximate the equilibrium policy functions with high-degree Chebyshev polynomials. It is enough to interpolate the young's money demand function and the bidding functions for all ages because the money demands of the middle-aged are redundant with the young's ones from the money market clearing condition. We use the young's money holdings as a unique endogenous state variable. We apply a Newton-type nonlinear equation solver to find the coefficients of the policy functions satisfying the equilibrium conditions.

We summarize the algorithm in step by step as follows.

1. Define the type of polynomials to approximate the equilibrium policy functions.

- In this paper, we use the Chebyshev polynomials of which domain is $[-1, 1]$.

2. Set the degree of polynomials, N, for the unique endogenous state variable in the policy functions.

   - We use the same degree of polynomials for all approximated functions, $\{T_k(\cdot)\}_{k=0}^{N}$.
   - We let $\{\theta_k^{m_1,s}\}_{k=0}^{N}$ and $\left\{\theta_k^{b_i,s}\right\}_{k=0}^{N}$ be the coefficients of the policy functions for the young's money holding and the bidding in age i in state s.
   - The total number of coefficients is (# of policy functions) $\times$ (# of states) $\times$ $(N+1) = 4 \times 2 \times (N+1)$.

3. Generate $(N+1)$ nodes to apply the projection method.

   - We produce $(N+1)$ Chebyshev grids in $[-1, 1]$.

4. Approximate the policy functions as follows.

(B.1)
$$\widehat{m_1^s} = \sum_{k=0}^{N} \theta_k^{m_1,s} T_k\left(\tilde{m}_{1,-1}\right)$$

(B.2)
$$\widehat{b_i^s} = \sum_{k=0}^{N} \theta_k^{b_i,s} T_k\left(\tilde{m}_{1,-1}\right) \text{ for } i \in \{1, 2, 3\}$$

   where $\tilde{m}_{1,-1}$ is the lagged money holding which is converted into $[-1, 1]$ using an appropriate interval for the unique endogenous state variable, $[m_{1,min}, m_{1,max}]$.

5. Solve for the state-dependent coefficients satisfying the equilibrium conditions on the grids from Step 3.

   - In this problem, the equilibrium conditions consist of two first-order conditions for the young and the middle-aged combined with the consumption allocation rule and two independent individual budget constraints. Note that the last budget constraint is linearly dependent on the other two.
   - We construct a system of non-linear equations in the set of coefficients by evaluating the policy functions on each grid and inserting them into the equilibrium conditions. Then, we use a Newton-type solver to find the state-dependent coefficients at once.

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