Commitment and Conflict in Multilateral Bargaining

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Abstract

We extend the Baron and Ferejohn (1989) model of multilateral bargaining by allowing the players to attempt tying their hands prior to negotiating. A commitment binds a player not to accept any share of the pie below one's commitment. We compare the unanimity rule with majority rules. We find that precommitments lead to inefficiencies and delay under unanimity only. There are potentially many symmetric stationary subgame perfect equilibria that are inefficient under the unanimity rule. These can be ordered from the least to most inefficient/aggressive according to how how many uncommitted players are needed for an agreement to arise, or large a share of the pie each player commits to. With more aggressive commitments, the delay is longer and a greater number of uncommitted players are required. The most aggressive commitment profile exists independently of the number of players in the game, and the delay in this equilibrium is increasing in the number of players in the game. Yet, less aggressive commitment profiles cannot be sustained in equilibrium if the number of players in the game is sufficiently large. The delay and inefficiency at the least and at the most efficient equilibrium increase as the number of players increases. There is also an inefficient asymmetric equilibrium where a unique party holds others as hostage requiring lions share of the surplus, reflecting the stylized pattern in many multilateral negotiations in the field. Under any supermajority rule, however, there is no equilibrium with delay or inefficiency as competition to be included in the winning coalition ensures that no aggressive attempts to force concessions will be made. The predicted patterns are by and large consistent with observed inefficiencies in many international arenas including the European Union, WTO, and UNFCCC. The results suggest that the unanimity rule is particularly damaging if the number of legislators is large.

1 Introduction

"The unanimity rule has meant that some key proposals for growth, competitiveness and tax fairness in the Single Market have been blocked for years." (European Commission press release, Jan 15th 2019)

Efficient and effective governance of global market failures requires a great deal of international agreement. Currently, there are a number of fora where key issues for sustainable development of the planet are negotiated. However, many of these are characterized by a prolonged negotiation process or even a negotiation impasse. First, The United Nations Framework Convention on Climate Change (UNFCCC) has not been able to reach a comprehensive and binding agreement on how to limit carbondioxide emissions (Pizer 2006).[1] Second, the World Trade Organization's (WTO) Doha Development Round[2] has missed a number of deadlines, and by the present day still has not resulted in a comprehensive agreement (Ehlermann and Ehring 2005; Bagwell, Bown, and Staiger 2016).[3] Alternative plurilateral agreements, such as the WTO

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[1] See also Nordhaus (2006) and Stern (2008).

[2] The round was launched in 2001 and focused among other things on market access for agricultural goods – especially from the developing to the markets of the developed countries – and the reduction of subsidies in this sector by the developed countries (Meltzer 2011).

[3] The WTO was not able to meet the December 2002 deadline imposed in paragraph 6 of the Doha declaration on the TRIPS agreement and Public Health since a single member prevented consensus (Ehlermann and Ehring 2005). Instead a considerably weaker Trade Facilitation Agreement was agreed upon in 2013.

TRIPS agreement, have been adopted instead between a considerably smaller number of countries. Finally, the European Commission strives to reach the member countries' support for such issues as common environmental policy, measures to curb tax competition between member countries, and a refugee policy which would lift the pressure from the Mediterranian countries and share the responsibility more evenly between all member states. No considerable progress has been reached in these fronts. As a consequence, the European Commission, including Jean-Claude Juncker, the Commission president, and Pierre Moscovici (the Commissioner for Economic, Financial Affairs, Taxation and Customs) has recently proposed that the unanimity rule should be abandonned in favor of the so called *qualified majority rule*[4] in EU taxation policy. In popular media and among negotiation delegates, the unanimity requirement according to which all parties must reach a consensus for an agreement to arise, has been blamed for its ineffectiveness. Within the WTO and the EU, the criticism seems to have gained momentum as the number of member states in each organization has grown.

Existing models of rational multilateral bargaining, building upon the seminal model by Baron and Ferejohn (1989), do not provide much reason to challenge the unanimity rule. Rather, the basic model under stationary equilibria predicts immediate agreement without delay, independetly of whether unanimity or any type of majority is required (Banks and Duggan 2000). Moreover, unanimity is claimed to result in more equalitarian terms of agreement than the majority rules in stationary environments according to the existing models (Eraslan and Merlo 2017).[5]Models of asymmetric information and/or reputation for obstinacy (Myerson 1991; Abreu and Gul 2000; Compte and Jehiel 2002) may predict delay and inefficiency but they are typically fairly complicated or even intractable in multilateral settings, and have thus mostly focused on the bilateral case.[6]

How can the role of unanimity in contributing to delay, impasse and ineffciency then be understood from the perspective of rational interactive decision making, i.e. game theory? In this paper we propose a tractable complete information model of commitment and conflict of multilateral bargaining in an infinite horizon setting which builds on the seminal Baron-Ferejohn model and adds the capacity to pre-commit into the model[7] (Crawford 1982; Levenotoğlu and Tarar 2005; Ellingsen and Miettinen 2008; Ellingsen and Miettinen 2014; Miettinen and Perea 2015). At the commitment stage of each round, each player can attempt to tie her hands and commit to reject any proposal below a threshold. The commitment positions are common knowledge at the bargaing stage of each round and a successful commitment automatically rejects offers where the committed player receives less than the share indicated in her committed. Any such attempted commitment to reject low offers fails and decays with an exogenously given probability. We adopt the simplest model such model where each player's probability of failing is independent of the failures of the commitment attempts of other players and, moreover, each individual commitment attempt has an equal chance of failing. Again for the sake of simplicity, we assume that commitments have to be re-established at the commitment stage of each round. As in Ellingsen and Miettinen (2014), commitments are costly to an extent that two strategies, which lead to otherwise identical payoffs for a player, are ranked ranked lexicographically with preference given to the strategy that does not require commitment. Once commitment attempts have been made and their success determined, one of the players is randomly drawn to make a proposal to which all others respond to by either accepting or rejecting the offer. Agreement arises if, according to an exogenously given consent rule, sufficiently many players give their consent to the proposal.

The key focus of the paper is to compare the unanimity rule with various majorty rules under these circumstances. We find that under the unanimity rule delay and inefficiency are commonplace and often unavoidable. Under a wide range of relevant parameter values, every symmetric stationary equilibrium is associated with delay. Moreover, as the number of players grows larger, the delay and ineffciencies become more severe: both the maximal and the minimal expected conflict duration over all equilibria increase. To the contrary, there is never delay or inefficiency in any equilibrium under the majority or any supermajority rule. Ruling out unanimity and requiring all but one party to agree is enough to restore efficiency in our non-cooperative model since players compete for being included into the winning coalition. Comparing the agreements between unanimity and (super)majority rules, negotiation outcomes are not necessarily more equal under unanimity, and inequality increases with the number of players in the unanimity case. Thus our model provides a clear rationale for favoring the majority rule over the unanimity rule. This is precisely what is suggested by many practitioners and policy makers involved in international negotiations (see the first paragraph of the introduction).

The paper contributes to understanding the role of bargaining frictions in multilateral bargaining, and especially the mediating role of the number of negotiating parties in aggravating such frictions. There is a growing need for better

[4]See the European Commission press release, Jan 15th 2019. The *qualified majority rule* applied in the EU decision making requires 55% of member states to vote in favour and these countries must represent at least 65% of the total EU population.

[5]Comparison between majority and unanimity. Maggi and Morelli (2006) consider repeated voting without enforcement of outcomes and find that majority rule is more efficient than unanimity when preferences are positively correlated and players are patient.

[6]Ma (2018) provides an analysis of the three-player case. Kiefer (1988) and Kessler (1996) suggest that hazard rates in labor disputes are inconsistent with the incomplete information explanation.

[7]The seminal ideas were presented by Schelling (1956).

understaning of such frictions from an applied perspective, as pointed out by Bagwell, Bown, and Staiger (2016, pp. 97) in their survey article on the economics of WTO, for instance. Naturally, our model abstracts from the complexities generated by cross-externalities, asymmetric and incomplete information, and the shadow of alternative pluri- and bilateral treaties which characterize the WTO and UNFCCC negotiations (See Harstad 2012, for instance). Nevertheless, the present paper provides a tractable and simple framework to understand the role of unanimity in multilateral bargaining where precommitent is feasible.

The paper builds on the complete information strategic pre-commitment literature on bargaining starting with te seminal contributions of Schelling (1956). Crawford (1982) formalized some of Schelling's arguments in a bilateral bargaing framework with both strategic ex ante pre-commitment and ex-post revoking of commitments. He showed that with sufficiently low success probability of individual precommitments, both players make aggressive pre-commitments in the unique equilibrium which is inefficient since the commitments are mutually incompatible. Ellingsen and Miettinen (2008) show that impasse may be considerably more likely and inefficiency more severe if there is a small cost of commitment. Ellingsen and Miettinen (2014) generalize the results to a dynamic infinite horizon setting. We generalize the Ellingsen and Miettinen (2014) model to a multilateral case. Yet, we consider commitments which can be renewed at each round.[8]

In our model, the success of the commitment is not based on an attempt to mimic a behavioral type and the time that it takes for an opponent to credibly screen the committing players true type as in Myerson (1991), Abreu and Gul (2000), and Compte and Jehiel (2002). Rather the commitment technology is exogenously given. Our modelling approach is thus considerably simpler. In a multilateral bargaining model with obstinate types, each player would have to track the beliefs about the obstinacy of each of the other other players and these would have to be updated both based on the proposals and the rejections made. The optimal actions would then depend on these beliefs. Thus the dimensionality of the model grows exponentially with the number of players. The simplicity and tractability of the present complete information commitment model makes it scalable from a bilateral to a multilateral bargaining setting.

The key lesson in the multilateral bargaining literature is that when players' valuations are heterogeneous, those who need to be least compensated for, will be included in the winning coalition. Moreover under unanimity players with lowest valuations, will be monetarily compensated to buy them into the agreement. This theoretical idea receives empirical support in the experiment by Miller, Montero, and Vanberg (2018). This also illustrates why signalling or mispresenting a private information about valuation (Tsai and Yang 2010; Eraslan and Chen 2014), or sending delegates with induced valuations higher or lower than those of the principal (Harstad 2010) may pay off individually in these settings. Such mispresentation would result in inefficient delay.

The paper is structured as follows. Section 2 presents the model. 3 analyzes the simplest multilateral case of three players. Section 4 has the general analysis and the main results. Section 5 concludes.

2 The model

2.1 The negotiation game

The bargaining game, denoted by G, has infinite horizon. There are n players in the game. In the main analysis we assume that players have identical preferences and technologies. A player's utility is assumed to be linear in the player's share of the surplus. Players are impatient, with per period discount factor δ. The size of the surplus is normalized to one. The rules of the game, the parameters of the model and the rationality of the players are common knowledge.

In each period, indexed by t, actions are taken in two stages – the commitment stage and the bargaining stage. At the commitment stage, players can attempt making short-lived commitments which last at most the current period, i.e. , each player i chooses a commitment attempt $x_i \in [0, 1]$ or stays flexible in which case $x_i = w$. In between the two stages, commitments may decay, each with an independent probability $1 - q$. The probability that a commitment sticks is q. The realization of the attempt, the commitment status, is denoted by s_i and, given the technology, it equals x_i with probabiliy q and 0 with probability $1 - q$. The commitment status s_i equals 0 also if $x_i = w$ was chosen at the commitment stage. At the bargaining stage a proposer is drawn and she makes a proposal and the uncommitted responders choose whether to accept or reject the proposal.

At the bargaining stage, each player becomes the proposer with probability $1/n$ (each player has an equal chance of becoming the proposer) in which case the commitment loses its strength. With probability $(n - 1)/n$, a player becomes a responder. Each responder's commitment has an i.i.d. probability of $1 - q$ of losing its strength between the commitment and the bargaining stages. In this case the commitment status falls to zero. The probability that none of the responders

[8] See also Muthoo (1996) and Levenotoğlu and Tarar (2005).

has a loophole equals q^{n-1}, for instance. Each responder decides whether to accept or reject once the proposer has offered a share d_i to i, . Yet, a player i with commitment status $d_i < s_i$ will automatically reject the offer.

If a deal with player i share d_i is implemented player i's payoff equals $\delta^{t-1}d_i$ but i prefers d_i to be implemented at any time t without having attempted to commit to d_i to be implemented at time t so that i was committed. This way commitment is negligibly or lexicographically costly. Even more drastic commitment costs could be modelled but at the expense of more complicated exposition of the model.

The assumption that each period is divided into a commitment stage and a bargaining stage closely matches Crawford (1982) and Ellingsen and Miettinen (2008; 2014). Indeed, the basic assumption of this literature is that negotiators always have the opportunity to make unilateral commitments before they sit down to engage in bilateral talks.

In addition to the number of number of parties in the negotiations, the key difference with respect to the durable commitment model of Ellingsen and Miettinen (2014) is that in the present model commitments endure at most for the present period only. In that sense, the model is reminiscient to Miettinen and Perea (2015). This assumption is made to simplify analysis: there are no commitment states to be kept track of and thus we use simply the stationary subgame perfect equilibrium rather than Markov perfect equilibrium as our solution concept.

2.2 Histories, strategies and equilibrium

A history h_t consists of a sequence of commitment attempts, stochastic commitment status randomizations (determining whether each attempt succeeds or fails) and recognizations of the proposer, proposals and responses at consecutive periods, and the two stages of each period. A behavioral strategy $\psi_i$of player i is a collection of randomizations of actions of player i, one for each history of the game. A behavioral strategy profile ψ is an n-tuple of strategy profiles, one for each player $i = 1, ..., n$. A subgame conditional on history h_t is denoted by $G(h_t)$ and $\psi|h_t$ is a strategy profile of the game, consistent with history h_t, and thus a strategy in the subgame $G(h_t)$. A subgame perfect equilibrium is an n-tuple of strategy profiles which is a Nash equilibrium in every subgame, at each history h_t. A stationary subgame perfect equilibrium is a subgame perfect equilibrium where $\psi|h_t$ coincides across h_t at any commitment stage. For the proposers at the bargaining stage, the proposals for each player i coincide across histories where the current commitment status profiles coincide. For the responders at the bargaing stage, the responses depend on the current proposal s, the proposer, and the commitment status profile. In a symmetric stationary subgame perfect equilibrium, the commitments and proposals coincide for players with identical commitment statuses. We consider stationary subgame perfect equilibria (SSPE) of the game and when necessary focus on the symmetric SSPE. The restriction to symmetric equilibria is clearly indicated whenever we do so.

Let v_i and denote the equilibrium value for player i at SSPE and $(x_1^*, ..., x_n^*)$ the profile of stationary commitment attempts along the equilibrium path in that equilibrium. Then i accepts an offer at a history where she is a responder if $d_i \geq= \hat{x}_i = max\{\delta v_i, s_i\}$ where s_i is the player's current commitment status. Let index l order the commitment statuses of the responders in an (weakly) ascending order. Let player j be the proposer in the current round. Then if $1 - \sum_{l=1}^{n-1} \hat{x}_l \geq \delta v_j$, the proposer proposes $\hat{x}_l$ to each responder $l < \underline{n}$, 0 to the other responders, and $\sum_{l=1}^{n-1} 1 - \hat{x}_l$ to herself. This proposal will be accepted by responders $l < \underline{n}$ resulting in responder continuation payoff, $\pi_{i,R}$, equal to the proposed shares, and the proposer payoff equals $\pi_{j,P} = 1 - \sum_{l=1}^{n-1} \hat{x}_l$ in that case. Otherwise responder continuation payoff equals $\pi_{i,R} = \delta v_i$ and proposer continuation payoff equals $\pi_{j,P} = \delta v_j$. At the commitment stage each player i chooses an optimal commitment to maximize the expected payoff

$$argmax_{x_i \in [0,1] \cup \{w\}} E\{\pi_i(x_i, x_{-i}^*)\} = argmax_{x_i \in [0,1] \cup \{w\}} \frac{1}{n} E\{\pi_{i,P}|x_i, x_{-i}^*\} + \frac{n-1}{n} E\{\pi_{i,R}|x_i, x_{-i}^*\}.$$

3 The case of three players

In this section, we illustrate the main difference in the viability of commitment between the unanimity rule and any majority rule by means of a simple three-player model. We extend the model to allow for n players in Section 4.

There are three negotiators or players, $i = 1, 2, 3$. At the commitment stage of period 1, each player chooses a commitment as described in Section 2Each player is recognized as the proposer with probability 1/3 (each player has an equal chance of becoming the proposer) in which case the player's commitment loses its strength. With probability 2/3, a player becomes a responder. Each responder's commitment has an i.i.d. probability of $1 - q$ of losing its strength. Thus the probability that neither of the responders has a loophole equals q^2, that one of one of the responders has a loophole equals $2q(1 - q)$, and the probability that both have a loopholde equals $(1 - q)^2$.

In the bargaining stage of period 1, players' commitment statuses and negotiation roles are common knowledge. The proposer makes a proposal assigning a share d_j, $j = 1, 2, 3$, to each of the three players. Each responder decides whether to accept or reject the offer made by the proposer. Yet, any player i with commitment status $s_i > d_i$ will automatically reject the offer.

We look for a symmetric equilibrium in which all players make a commitment attempt $x^*$and players who succeed with their attempt thus have a commitment status $s^* = x^*$. Denote the (common) expected discounted equilibrium payoff by v^*. Due to stationarity, this is also the continuation value of each player. The following characterizes the equilibrium.

Proposition 1. *In the three-player game under unanimity rule, there exists an equilibrium with aggressive precommitments and delay. Moreover, this is the only symmetric and stationary equilibrium when $q < 1/2$.*

At the commitment stage of each period t, each uncommitted player chooses a commitment attempt

$$x^* = 1 - 2\delta v^*, \tag{1}$$

where

$$v^* = \frac{1 - q^2}{3(1 - \delta q^2)}$$

is the expected equilibrium payoff for each player. The c^* commitments are targeted to leave the proposer and the responder with a loophole exactly as much of the pie to make them indifferent betweeen agreeing and rejecting the agreement. The proposer and the responder with a loophole receive the continuation value, δv^*,and the player who is the only one to succeed with her commitment receives the residual, i.e. her commitment $1 - 2\delta v^*$. These shares sum up to one. Yet, the equilibrium is inefficient. This is due to the fact that the commitments are incompatible and whenever both players who are drawn to respond succeed in their commitment, there will be delay. The inefficiency is exhibited in the individual equilibrium payoff which is strictly below 1/3. The expression closely resembles the unique and inefficient Markov-perfect equilibrum payoff of Ellingsen and Miettinen (2014): in the present formula a factor δ is absent in the numerator since there is no discounting between the commitment stage and the bargaining stage and thus there is no exogenously imposed delay in the present model. Moreover, in the present model the proposer's commitment automatically fails and thus the endogenous delay is only driven by the commitment attempts of the two responders and thus the endogenous expected delay coincides in the bilateral model and the three-player version of the multilateral model. Naturally, what is left, once the endogenous delay is accounted for, will be split evenly, ex ante, between the three parties who have identical recognition and commitment success probabilities and who adopt identical strategies.

In this equilibrium, at the proposal-phase of each round, the proposer proposes sharing $(1 - 2\delta v^*, \delta v^*, \delta v^*)$ to two uncommitted responders (proposer's own share indicated first) and sharing $(\delta v^*, 1 - 2\delta v^*, \delta v^*)$ to a committed reponder and an uncommitted proposer (where the committed responder's share is indicated in the middle). If both responders are committed, the proposer makes any offer where at least one of the responders receives less than the committed share and thus the game proceeds to round $t + 1$. Thus the equilibrium is inefficient and exhibits costly delay. Equilibrium payoffs increase and commitments become less aggressive if players discount less (δ increases) or they are less likely to succeed with their commitments (q decreases).

Let us discuss why (1) constitutes part of an equilibrium. The discounted present value of utility in equilibrium equals v^*. In a stationary equilibrium all flexible players will renew their attempt to commit to x^* in the follow-up round. Thus the continuation value of a flexible responder at the bargaining stage equals δv^*. Notice first that (1) can be written as $\delta v^* = 1 - \delta v^* - c^*$. Thus the proposer is indifferent between inducing disagreement, on the one hand, and proposing $(\delta v^*, c^*, \delta v^*)$, on the other hand, and thus conceding to the aggressive demand of a single committed responder. On the same grounds, a responder who is uncommitted is indifferent between accepting and rejecting the proposal.

Consider then deviations at the commitment stage. A commitment matters only if the player is drawn to respond (recall that the commitment is automatically relaxed if one is drawn to propose). Since that all other players attempt commitment, deviating and not committing would automatically imply that the highest share this player can receive conditional on a deal (and a responder role) equals δv^* which is lower than the share one receives in equilibrium conditional being the responder and the other responder having a loophole, and equal in every other contingengy. Thus the deviation not to commit does not pay off.

Deviating by choosing a more aggressive commitment does not pay off since conceding to such a commitment will result in a payoff lower than δv^*. A deviation down to a less aggressive commitment strategy can only increase the payoff if it increases the probability of the deal at any given round (and thus reduces the expected conflict duration). In equilibrium, there is no deal at any round with probability q^2, i.e. in case both responders are committed to x^*. To induce a deal in this case, a deviating responder would have to have chosen a less aggressive commitment position y which the proposer is indifferent between accepting and rejecting when the other responder succeeds with his commitment, i.e. $\delta v^* = 1 - x^* - y$. Thus $y = \delta v^* = 1 - \delta v^* - x^*$ and the deviation does not pay off.

Proposition 1 illustrates that aggressive commitment tactics potentially lead to delay, inefficiency and asymmetric deals in multilateral bargaining much the same way as in bilateral bargaining (Ellingsen and Miettinen, 2008, 2014). In fact, in Section 4.2 below will show that the equilibrium with delay is the only symmetric stationary stationary equilibrium if $q < 1/2$ and thus inefficiencies necessarily arise in that case. Yet, the result hinges upon a key condition – that of unanimity. Only when mutual consent is needed from all parties, can the players force concessions from others in equilibrium. Proposition 2 below shows that any supermajority which does not require mutual consent is enough to remove any inefficiencies and all equilibria are efficient in this case, as in the original Baron and Ferejohn (1989) model.

Proposition 2. *In the three player game under majority rule, there is no inefficient symmetric stationary equilibrium.*

The intuition for this result is simple. Under majority, the responders compete for being included into the winning coalition. If no party is committed, deviating up and requiring concenssions from others will only imply that one is left out of the coalition. On the other hand, if everyone is aggressively committed to a symmetric position, any responder will be included to the winnning coalition with a probablity smaller than one. Yet, by deviating down and asking even very slightly less implies that one will be included to the winning coalition always when the number of committed players is greater than the required majority. A typical Bertrand competition argument then shows that commitments are competed down to a level where they have no effect payoffs and thus remaining flexible is in fact preferred. Thus the standard Baron & Ferejohn equilibrium outcome is the unique equilibrium outome in this case.

4 General analysis

In this section we first present the general structure of the aggressive symmetric commitment equilibria and after that present the main results of the paper. In subsection 4.1 we analyse the most aggressive and inefficient equilibrium which is shown to always exist. We first consider the unanimity case in four subsections. In subsection 4.2 we consider an efficient commiment equilibrium which exists only if q is sufficiently high with respect to the number of players. In subsection 4.3 we consider all other equilibria which are more efficient than the aggressive equilibrium but still not efficient. Finally we consider asymmetric equilibria in subsection 4.4 before turning to the analysis under the majority rule in subsection 4.5.

In a standard Baron-Ferejohn model, the proposer allocates a share equal to the continuation value to $\underline{n}-1$ responders, where $\underline{n}$ is the decision rule or the number of parties who need to agree in order for a deal to arise, and extracts the residual share of the pie herself. Without commitments, all equilibria are efficient and thus with symmetric recognition the continuation value equals δ/n. In the unanimity case, all parties must agree and thus each responders is proposed δ/n and the proposer receives the residual $1-(n-1)\delta/n$. When players are patient, the pie is thus shared equally and efficiently under unanimity.

When commitments are feasible, however, equilibria may be inefficient. As shown in the three-player case above, the aggressive commitment is targeted to extract (some of) the residual surplus from the proposer. The commitment is crafted to make the proposer indifferent when there is a given number of players who fail (have a loophole) with their commitments. If there are less loopholes that the given number, there will be delay; if there are more loopholes, the proposer receives rents above her continuation value.

In a symmetric commitment equilibrium, all players have an incentive to attempt commitment to extract rents. In this case the commitments are typically not mutually consistent. In the general case, each symmetric profile of commitment attempts can be characterized in terms of how large a chunk of the proposer rent each responder attempts to extract. More aggressive commitments demand a larger share of the surplus, and thus require a larger number of responders to fail with their commitment attempt in order for the commitments to be compatible and the agreement to arise. The more loopholes are needed, the longer it takes to reach an agreement. Therefore, greater number of required loopholes is associated with a greater commitment attempts in equilibrium, not only because there are fewer other committed players to share the extracted rents with, but also because the continuation value decreases with longer expected delay. Therefore, efficiency of the equilibrium is inversely releated to the number of loopholes required, on the one hand, and to how large a share of the pie each player commits, on the other hand.

Let us formalize these ideas. Denote the number of players by n and the number of loopholes required for an agreement to arise by h. Denote the symmetric equilibrium value by $v^*_{n,h}$ and the aggressive commitment attempted by all flexible players at the commitment stage of each period by $x^*_{n,h}$. A succesful commitment thus equals $s = x^*_{n,h}$. In equilibrium, if the number of flexible responders is at least h at the bargaining stage, all agents with a loophole will be offered $\delta v^*_{n,h}$, while those without will be offered $x^*_{n,h}$. As in the case of three players above, the commitment $x^*_{n,h}$ is chosen so as to make the proposer and the uncommitted responders indifferent between accepting and rejecting a deal when there are exactly $n-h-1$ committed responders. Each of the succesful responders receives $x^*_{n,h}$ in this case, and the proposer and the uncommitted responders each receive

$$\delta v^*_{n,h} = 1 - (n-h-1)x^*_{n,h} - h\delta v^*_{n,h}. \tag{2}$$

This expression reveals that the residual share for the proposer equals the continuation value (left hand side) exactly when h responders with a loophole receive their continuation value (last term on the right) and $n-h-1$ responders succeed with their commitment attempt and receive their commitment share. Solving for $x^*_{n,h}$ yields

$$x^*_{n,h} = \frac{1-(h+1)\delta v^*_{n,h}}{n-h-1},$$

showing that, if exactly h reponders have a loophole and $n-h-1$ responders succeed with their commitments, then the responders who succed share the residual pie in equal shares when those who have a loophole and the proposer (who is also assumed to have a loophole) are paid the continuation value. It is easy to see that when $n=3$ and $h=1$ the formula coincides with that in the three player case above.

In order to learn more about $x^*_{n,h}$ and $v^*_{n,h}$, we need to write explicit expressions of $v^*_{n,h}$. Let us first look at the proposer's expected payoff. Denote the probability that at least h *of* k *responders* will have a loophole by $\eta(k,h)$. Notice that $\eta(n-1,h)$ is the binomial cumulative distribution function measuring the number of responders (out of $n-1$) who *succeed* with their commitment, i.e. $\eta(n-1,h) = F(k-h; n-1, q)$ where $F(.,.,.)$ is the binomial cdf. Then we can think of the proposer paying all $(n-1)$ responders $x^*_{n,h}$and getting back $(x^* - \delta v^*)$ from all those who have a loophole as long as their number is equal to or exceeds h. Her expected payoff is therefore

$$\pi_P = (1-\eta(n-1,h))\delta v^* + \eta(n-1,h)\left[1-(n-1)x^* + E(l|h \le l \le n-1)(x^* - \delta v^*)\right]$$

In fact we know that $x^*_{n,h}$ is the optimal commitment targeted to induce proposer indifference when excatly h of the $(n-1)$ responders have a loophole, i.e. that $(n-h-1)x^*_{n,h} = 1-(h+1)\delta v^*_{n,h}$. Thus the above expression can be written as

$$\begin{aligned}\pi_P &= (1-\eta(n-1,h))\delta v^*_{n,h} + \eta(n-1,h)\left[1-(n-1)x^*_{n,h} + hx^*_{n,h} - h\delta v^*_{n,h} - hx^*_{n,h} + h\delta v^*_{n,h} + E(l|h \le l \le n-1)(x^*_{n,h} - \delta v^*_{n,h})\right] \\ &= \delta v^*_{n,h} + \eta(n-1,h)[E(l|h \le l \le n-1) - h](x^*_{n,h} - \delta v^*_{n,h})\end{aligned}$$

Next think about the expected payoff conditional being drawn to respond (which happens with probability $(n-1)/n$) when all agents make the candidate commitment x^*at the commitment stage. The value is given by

$$\pi_R = (1-q)\delta v^* + q\left(\eta(n-2,h)x^* + (1-\eta(n-2,h))\delta v^*\right)$$

where $\eta(n-2,h)$ denotes the probability that at least h *of the other* $(n-2)$ *responders* will have a loophole. In other words, if the responder has a loophole, she will receive her continuation value no matter whether an agreement will be reached or not. If the responder does not have a loophole, (probability q), then she will receive her commitment if at least h *of the other* $(n-2)$ *responders* have a loophole. Notice that $\eta(n-2,h)$ follows the binomial distribution, so for instance if we have 4 players and $h=2$, then conditional on i being a responder $\eta(2,2)$ equals $(1-q)^2$, and if $h=1$ then conditional on i being a responder $\eta(2,1)$ equals $2(1-q)q + (1-q)^2$.

To sum up, players expected payoff when all players attempt a commitment to $x^*_{n,h}$ in the commitment stage can be written as

$$v^*_{n,h} = \frac{1}{n}\pi_P + \frac{n-1}{n}\pi_R \tag{3}$$

We can now solve equations (2) and (3) to yield an explicit expressions for $v^*_{n,h}$and $x^*_{n,h}$(proof in Lemma 2 in the appendix)

$$v^*_{n,h} = \frac{\eta(n-1,h)}{n(1-\delta(1-\eta(n-1,h))},$$

which reflects the fact that the expected total payoff in the game comes from 1 dollar being paid out randomly at some point, with probability $\eta(n-1,h)$ in each period, i.e. the probability that it will be paid at a given period t is $\eta(n-1,h)(1-\eta(n-1,h))^{(t-1)}$and so the expected total payoff is $\sum_{t=1}^{\infty}\eta(n-1,h)[(1-\eta(n-1,h))\delta]^{(t-1)}$, which gives $v^*_{n,h}$ above.

Clearly, generally $\eta(n-1,h) < 1$ and thus $v^*_{n,h} < 1/n$ and therefore, if these strategies consisute an equilibrium, the equilibrium is inefficient. Moreover, the higher is h, the smaller is $\eta(n-1,h)$ and so is also $v^*_{n,h}$. Thus symmetric commitment profiles which require more loopholes for a deal to arise are also more inefficient. Yet, $x^*_{n,h}$ increases with h and thus, conditional on succeeding with one's own commitment, the earned share when the deal arises is larger the higher is h. In this sense commitment profiles with higher h generate longer conflict duration, greater inefficiency and greater asymmetries in the shares that the parties receives conditional on reaching an agreement. In this sense the commitment profile candidates can be very naturally ordered from the most aggressive one $h = n-2$ to the least aggressive one $h = 1$.

4.1 Aggressive equilibrium

Let us begin with considering the most aggressive symmetric commitment profile where each player aims to extract the entire surplus (beyond the continuation value) from the proposer. This requires that there are at least $h = n-2$ loopholes among the $n-1$ responders. Hence, agreement occurs only in two cases: (i) only one of the commitment attempts succeeds or (ii) none of the commitment attempts succeeds. In both cases, $n-1$ agents will get exactly the continuation value, and the residual, $1-(n-1)\delta v^*_{n,n-2}$, is secured either by the committed responder (note that x^*_{n-2} is exactly the residual) or by the proposer. Deviating to any $y \neq x^*_{n,n-2}$ affects the deviator's payoff only if she is drawn to respond and her commitment sticks. If $y > x^*_{n,n-2}$, the proposer will not want to pay her *even in the most favorable instance*, where everyone else has a loophole; so a more agressive commitment is out of the question. Consider then deviations to less agressive commitments, $y < x^*_{n,n-2}$, which have the property that the proposer will concede to them in cases where player's own commitment attempt as well as $k \geq 1$ others succeeds. The largest commitment that will be met when (at most) *one* additional person's commitment sticks is such that

$$y = 1 - x^*_{n,n-2} - (n-2)\delta v^*_{n,n-2}$$

But substituting $x^*_{n,n-2} = 1-(n-1)\delta v^*_{n,n-2}$ we see that this boils down to

$$y = \delta v^*_{n,n-2}$$

So this deviation is not profitable. It only makes a difference for the deviator's payoff in the instances where the deviator is a responder and her commitment sticks. Conditional on this, the deviator's payoff drops to the continuation value $\delta v^*_{n,n-2}$, whereas if she stays with the equilibrium demand, she will get $x^*_{n,n-2}$ in case all of the other responders have a loophole. It's clear that the argument can be extended to say that for all $k > 1$, there is also no profitable commitment that would be met if my own and k additional commitments stick, i.e. deviations of the type

$$y = 1 - kx^*_{n,n-2} - (n-1-k)\delta v^*_{n,n-2},$$

since *a fortiori* these commitments would be strictly *smaller* than the continuation value. This is enough to establish

Theorem 1. *Under unanimity, there always exists an equilibrium in which at least $n-2$ responder loopholes are required for agreement to be reached.*

Let us then analyze the characteristics of this most aggressive equilibrium. In particular, we are interested in understanding how the duration of the conflict and the fraction of the surplus demanded are affected by the number of players in the game.

Recall that the commitments are independently and identically distributed. A loophole arrives randomly with probability $(1-q)$ to each player at each round. The number of loopholes at a given round among the $n-1$ responders follows the binomial distribution.[9] The number of required loopholes $n-2$ increases with the number of players n. It is thus straightforward that the duration of the conflict increases with the number of players.

[9]This approaches the normal distribution with mean $(n-1)(1-q)$ and standard deviation $\sqrt{(n-1)(1-q)q}$ as n tends to infinty.

Corollary 1. *Consider the most aggressive commitment equilibrium with profile $x^*_{n,n-2}$. The duration of conflict increases as n increases.*

Moreover, since the duration of conflict increases with n, the expected arrival date of the deal also increases with n and thus the the equilibrium payoff and the continuation value, which is proposed to each flexible responder when the deal is done, decreases with n. Therefore the fraction of the pie that the deal allocates to the unique successful player, $x^*_{n,n-2}$, increases as the number of players increases.

Corollary 2. *The commitment share at the most aggressive commitment equilibrium with profile, $x^*_{n,n-2}$, increases as n increases .*

4.2 Efficient equilibrium

Now consider the opposite extreme, a commitment profile requiring no loopholes, i.e. $h = 0$. There are potentially uncountably many efficient equilibria where the symmetric commitment attempt $x^*_{n,0}$ belongs to the interval $[0, \frac{1}{n-1}(1-\frac{\delta}{n})]$ and where $x^*_{n,0} = 0$ indicates that the players stay flexible. In all these cases, the discounted expected equilibrium payoff equals

$$v^*_{n,0} = \frac{1}{n}$$

and when the commitment attempt is at the upper bound of the interval,

$$x^*_{n,0} = \frac{1}{n-1}\left[1 - \frac{\delta}{n}\right], \tag{4}$$

then, if *none of the responders has a loophole*, the $n-1$ responders are sharing what's left after the proposer is permitted to keep $\frac{\delta}{n}$. But when there are loopholes or when $x^*_{n,0} < \frac{1}{n-1}\left[1 - \frac{\delta}{n}\right]$, the proposer will secure a surplus. That is what creates a potential for more aggressive commitments. In fact, it is straightforward to notice that for any efficient equilibrium candidate with $x^*_{n,0} < \frac{1}{n-1}\left[1 - \frac{\delta}{n}\right]$ there is a profitable upward deviation which extracts rents from the proposer without risking the agreement and thus the only effcient equilibrium candidate satisfies (4).

Consider now a deviation to a more agressive commitment y with the property that it will be met if at least $k \geq 1$ of the *other* responders have a loophole. (I.e. thinking again conditional on the deviator's commitment sticking.) Then the most aggressive commitment that will be met with k loopholes is

$$y = 1 - (n-2-k)x^*_{n,0} - (1+k)\delta/n.$$

(To see this, note that $(n-2-k)$ committed responders will get x^*_0, the deviator will get y, and the proposer and k flexible players get $\delta v^*_{n,0}$.)

Substituting yields

$$\begin{aligned} y &= 1 - \frac{n-2-k}{n-1}\left[1 - \frac{\delta}{n}\right] - (1+k)\frac{\delta}{n} \\ &= \left(1 - \frac{n-2-k}{n-1}\right)\left[1 - \frac{\delta}{n}\right] - k\frac{\delta}{n} \\ &= \frac{1+k}{n-1}\left[1 - \frac{\delta}{n}\right] - k\frac{\delta}{n} \\ &= x^*_{n,0} + k[x^*_{n,0} - \frac{\delta}{n}]. \end{aligned}$$

The equation states that the deviating player is increasing his demand by $k(x^*_{n,0} - \frac{\delta}{n})$, which is the extra residual surplus that can be captured from another responder if that other responder has a loophole. Does this deviation pay off? Since it makes a difference only when the deviating player is a responder and her commitment sticks, all calculations can be done conditional on that event. Then the tradeoff is as follows. In all cases where *fewer* than k of the other $n-2$ responders have a loophole, the deviating player will *lose* $(x^*_{n,0} - \delta v^*_{n,0})$. (I.e. instead of getting an agreement in these cases, the deviator gets the continuation value.) This occurs with probability $1 - \eta(n-2, k)$. In all cases where at least k

of the other responders get a loophole, the deviating player will *gain* $k\left(x_0^* - \frac{\delta}{n}\right)$. This occurs with probability $\eta(n-2,k)$. So a deviation aiming at $k \geq 1$ loopholes pays off if

$$\eta(n-2,k)k\left(x_{n,0}^* - \frac{\delta}{n}\right) > (1-\eta(n-2,k))\left(x_{n,0}^* - \delta v_{n,0}^*\right).$$

This boils down to the following:

$$\eta(n-2,k)k > (1-\eta(n-2,k))$$

or simply $\eta(n-2,k) \leq \frac{1}{k+1}$.

Theorem 2. *An efficient equilibrium requiring no loopholes for agreement to be reached exists iff for all $k \in \{1,...,n-2\}$, we have $\eta(n-2,k) \leq \frac{1}{k+1}$.*

Notice that that the profitability of the deviation does not depend on δ. Since we are dealing with independently and identically distributed loopholes and $\eta(n-2,k)$ reflects the cumulative distribution of the binomial, the effect of increasing n while holding k constant is intuitive: when the number of responders increases, the probability that k of them will have a loophole will also increase. In particular, the effcient equilibrium will eventually be destabilized. For instace a small upward deviation with $k = 1$ pays off if

$$\eta(n-2,1) > \frac{1}{2},$$

which will be satisfied for n large enough

Corollary 3. *There exists an $\hat{n}$ such that if $n > \hat{n}$, then the symmetric efficient commitment profile does not consitute part of an equilibrium strategy profile.*

4.3 Intermediate equilibria

Consider a profile with symmetric commitments targeting to at least $h \in \{1, n-3\}$ responders with loopholes. We have

$$v_{n,h}^* = \frac{1}{n} \cdot \frac{\eta(n-1,h)}{1-\delta(1-\eta(n-1,h))}.$$

and

$$x_{n,h}^* = \frac{1}{n-1-h}\left[1-(h+1)\delta v_h^*\right].$$

That is, *when exactly h responders have a loophole*, the $n-1-h$ responders are sharing what's left after the proposer as well all those who have a loophole are permitted to keep $\delta v_{n,h}^*$.

Let us begin by considering downward deviations, to less aggressive commitments. Deviating to any $y < x_{n,h}^*$ affects the deviating player's payoff only if she becomes a responder and her commitment attempt succeeds. So like above we can do the analysis conditional on that event. All downward deviations have the property that they will be met if at most $k \geq (n-2-(h-1)) = n-1-h$ *other* commitment attempts succeed (among the $n-2$ responders), as opposed to $x_{n,h}^*$ which is met if at most $n-2-h$ other commitments stick. The largest commitment that will be met when at most $n-1-h$ other commitments attempts succeed is given by $y = 1-(n-1-h)x_{n,h}^* - h\delta v_{n,h}^*$. Like above, if we substitute $x_{n,h}^* = \frac{1}{n-1-h}\left[1-(h+1)\delta v_{n,h}^*\right]$, we obtain $y = \delta v_{n,h}^*$. Therefore, just as in the most aggressive equilibrium, a deviation designed to make agreement possible with one fewer loopholes is not profitable. It only makes a difference for the deviating player's payoff in the instances where she is a drawn to respond and her commitment sticks. Conditional on this, her payoff is reduced to the continuation value. It is clear that the argument can be extended to say that for all $k > n-1-h$, there is also no profitable commitment that would be met when there are even fewer than $h-1$ loopholes, since *a fortiori* these commitments would be strictly *smaller* than the continuation value. This is enough to establish the following lemma.

Lemma 1. *Define $v_{n,h} = \frac{1}{n} \cdot \frac{\eta(n-1,h)}{1-\delta(1-\eta(n-1,h))}$ and $x_{n,h} = \frac{1}{n-1-h}\left[1-(h+1)\delta v_{n,h}\right]$. Let x be a symmetric commitment profile where all players commit to $x_{n,h}$ for some $h \in \{1,...,n-1\}$. Then a unilateral downward deviation to any $y < x_{n,h}$ is not profitable.*

Next consider a deviation to a more agressive commitment y with the property that it will be met if at least $h+k \geq h$ of the *other* responders have a loophole. I.e. the deviation is targeted to succeed when at least k loopholes occur *in addition* to the h required in equilibrium. Once again, such a deviation makes a difference only in case the deviating player ends up being a responder and (a) her own commitment attempt succeeds and (b) at least h of the other $(n-2)$ commitments have a loophole. In this contingency, the outcome will depend on how many of the remaining $(n-2-h)$ responders have loopholes. Then the most aggressive commitment that will be met with *one additional* loophole (i.e. $h+1$ in total) is

$$y = 1 - (n-h-3)x^*_{n,h} - (h+2)\delta v^*_{n,h}.$$

where $(n-h-3)$ committed responders will get $x^*_{n,h}$, the deviator will get y, and the proposer and flexible players get $\delta v^*_{n,h}$. Substituting yields

$$y = \left(\frac{2}{n-1-h}\right)\left[1-(h+1)\delta v^*_{n,h}\right] - \delta v^*_{n,h} = x^*_{n,h} + (x^*_{n,h} - \delta v^*_{n,h}).$$

So this exactly replicates what we've already seen for the $h=0$ case.

This logic can be extended to yield that the best deviation targeting at least k loopholes is of the form $y = x^*_{n,h} + k\left(x^*_{n,h} - \delta v^*_{n,h}\right)$. To see whether a deviation targeting at $h+k$ loopholes pay off, note that it makes a difference only when (a) the deviating responder's commitment sticks and (b) at least h of the other $(n-2)$ commitments attempts fail. Thus we can do all calculations conditional on that event. Then the tradeoff is as follows. In all cases where *fewer* than k additional loopholes appear (out of $n-2-h$ chances), the deviating player will *lose* $(x^*_{n,h} - \delta v^*_{n,h})$. This occurs with probability $1-\eta(n-2-h,k)$. In all cases where at least k additional loopholes occur, the deviating player will *gain* $k\left(x^*_{n,h} - \delta v^*_{n,h}\right)$. This occurs with probability $\eta(n-2-h,k)$. So a deviation aiming at $k \geq 1$ additional loopholes strictly pays off if and only if

$$\eta(n-2-h,k)k\left(x^*_{n,h} - \delta v^*_{n,h}\right) > \left(1-\eta(n-2-h,k)\right)\left(x^*_{n,h} - \delta v^*_{n,h}\right)$$

or

$$\eta(n-2-h,k) > \frac{1}{k+1}. \tag{5}$$

This establishes the following. An equilibrium requiring $h \in \{1, n-3\}$ loopholes exists iff $\eta(n-2-h,k) \leq \frac{1}{k+1}$ for $k = \{1,...,n-h-2\}$. Defining $h = n-g$ we can restate this as follows.

Theorem 3. *For any $g \in \{3,...,n-1\}$, an equilibrium requiring at least $(n-g)$ loopholes exists iff $\eta(g-2,k) \leq \frac{1}{k+1}$. for all $k \in \{1,...,g-2\}$.*

Substantively, this condition says that no upward deviation pays off. The intuition is as follows. Since the equilibrium requires at least $n-g$ loopholes, there are up to $(n-2)-(n-g) = g-2$ pieces of the pie of size $x^*_{n,h} - \delta v^*_{n,h}$ that accrue to the proposer in case more than $n-g$ loopholes appear. The player who deviates upwards is aiming to extract some number of these pieces from the proposer, at the cost of higher probability of provisional impasse. There are up to $g-2$ variants of upward deviations which are ordered in terms of aggressivity, k, i.e. the higher number of pieces the deviator attempts to extract. Each of them generates a benefit of k times the size of the piece times the probability of at least k additional loopholes $\eta(g-2,k)$. Each deviation labelled by k is also associated with a provisional loss of the probability of less than k additional loopholes $(1-\eta(g-2,k))$ times one piece of size $x^*_{n,h} - \delta v^*_{n,h}$. The condition checks for each of these deviations, that the deviation does not pay off. It is a necessary and a sufficient condition since we verified above in Lemma 1 that a downward deviation is never profitable. The bargaining stage strategies are optimal in an obvious manner that we are familiar with from the existing literature.

In fact, we are confident that an even simpler characterization of the symmetric equilibria can be provided. Although we have not been able to prove it, extensive numerical verifications have increased our confidence that it is necessary and sufficient to check whether a deviation targeting one additional piece of size $x^*_{n,h} - \delta v^*_{n,h}$ and one additional loophole pays off. That $k=1$ is nesessary and sufficient is implied by the following conjecture.

Conjecture 1. *Let $g-2 \geq 1$. Suppose $\eta(g-2,k) \leq \frac{1}{k+1}$* for $k < g-2$. Then $\eta(g-2,k+1) \leq \frac{1}{k+2}$.

We are confident that this conjecture is true. Substantively, this would imply that an equilibrium targeting $n-g$ loopholes exists iff $\eta(g-2,1) \leq \frac{1}{2}$, i.e. if a deviation targeting *one* additional loophole is not profitable. This condition can be written in the following alternative forms: $g \leq 2 - \frac{\ln(2)}{\ln(q)}$or $q > 2^{-\frac{1}{g-2}}$.

Theorem 4. *For any $g \in \{3, ..., n-1\}$, an equilibrium requiring at least $(n-g)$ loopholes exists iff $\eta(g-2,1) \leq \frac{1}{2}$, or equivalently iff $g \leq 2 - \frac{\ln(2)}{\ln(q)}$or iff $q > 2^{-\frac{1}{g-2}}$.*

Theorem 4 will also have important comparative statics implications when studying the efficiency of equlibria when the number of players increases. Namely when one holds constant the number of loopholes, h, required and increases the number of players n, one must also increase g so that $n-g=h$ holds. Therefore the probability mass in the right-tale of the binomial cdf on the left-hand side of

$$\eta(g-2,k) \leq \frac{1}{k+1}$$

increases and eventually becomes greater than the right-hand side thus ruling out the equilibrium with h loopholes. Thus as the number of players in the game increases, the least delay of agreement in any equilibrium increases. Thus our theory gives a setwise prediction that delay times tend to increase as the number of players increases. This result is not contingent on the characterization above; an independent but related proof is provided in the footnote.[10] In conclusion, Theorem 4 and equation (6) show that the higher is the number of players, the higher is also, in any equilibrium, the lowest number of loopholes required (among the responders) for an agreement to arise.

Corollary 4. For any h, there exists an $\bar{n}$ such that if $n > \bar{n}$ then the symmetric commitment profile

$$x^*_{n,h} = \frac{1-(h+1)\delta v^*_{n,h}}{n-h-1}$$

does not constitute part of an equilibrium strategy profile.

Note moreover that the most efficient equilibrium exists only if this condition holds for $g=n$, meaning that the upper bound $\hat{n} = 2 - \frac{\ln(2)}{\ln(q)}$. This is displayed in Figure 1. For $q=0$, $\hat{n}=2$, so the efficient equilibrium does not exist. However note that in this case the commitments are ineffective. More interesting is the fact that $\hat{n}$ is very small (less than 10) even for q closer to 1. As $q \to 1$, $\hat{n} \to \infty$.

4.4 Asymmetric stationary equilibria

By now, we have learned that the incentives to deviate from a symmetric stationary pre-commitment equilibrium are centered around losses and gains which are both measured in shares of unit size $(x^* - \delta v^*)$ of the pie. An immediate question is whether asymmetric equilibria could exist where commitments differ and these shares are divided unevenly between the players whose commitments are succesful. In that case players' equilibrium values and continuation values, v_i^* and δv_i^* respectively, are player-specific.

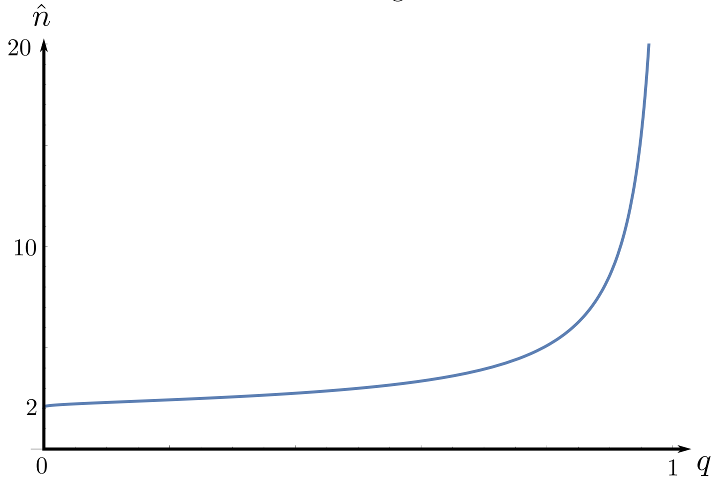
Studying whether such equilibria exist is complicated by the fact that, as in the symmetric case, the monotonicity properties are such that checking for any single deviating commitment pays off is not necessary and sufficient. To the contrary, one must verify the profitability of each potential devitation which makes any proposer indifferent. Formally, this is a challenging combinatoarial argument in the general case where commitments vary in their aggressivity and thus the agreement depends not only on how many players have a loophole but also who the players with the loopholes are.

[10] We can use (5) to establish an upper bound on the the number of players in the game for which an intermediate equilibrium exists. (If Conjecture 1 proves correct, the upper bound condition is in fact leads to a characterization of the equilibria.) Consider a deviation to a more aggressive commitment which requires one more loophole, i.e. $h+1$ loopholes among the other $n-2$ responders. Since $\eta(n-2-h,1) = 1-q^{n-2-h}$, the condition for a deviation to pay off $\eta(n-2-h,1) > 1/2$ can be written as $n > ln(1/2)/lnq + 2 + h$ and therefore

$$\bar{n} = \frac{ln(1/2)}{lnq} + 2 + h \tag{6}$$

provides an upper bound (in terms of the number of the players in the game) for the existence of an intermediate equilibrium with $h < (\bar{n}-2)$ loopholes. Notice however that there may be more a drastic upward deviation (counting in multiples of k, i.e. the number of additional loopholes required) which satisfies $\eta(n-2-h,k) > 1/(k+1)$ for a value of n lower than $\bar{n}$ and thus equation the above gives only a sufficient condition for non-existence.

Figure 1: Maximum n such that efficient equilibrium exists



Applications and anecdotal evidence suggest an interesting and relatively simple equilibrium candidate, however: one where there is a single negotiating party which attempts an aggressive commitment and thereby delays agreement holding all other parties hostage. We will consider such equilibria below and show that such asymmetric but stationary equilibria may exist.

4.4.1 Four players

Let us begin with an example. There are $n = 4$ players. Assume that player one attempts an aggressive commitment and all other players commit to a profile which results in an agreement immediately when player one has a loophole. This equilibrium is characterized by the following commitments: the first player commits to

$$x_H^* = 1 - 3\delta v_L^*$$

(players two, three and four receive their continuation value and are thus indifferent between the proposed deal and delaying) and all other players commit to

$$x_L^* = \frac{1 - \delta v_H^*}{3}, \tag{7}$$

where player one with the high commitment is recognized and thus has a loophole and proposes player two, three and four, who are succesful, their commitment values. The proposing player who had made an aggressive commitment then receives exactly her own continuation value (Notice that there are also other contingencies where an agreement arises, say, when the aggressive player is a responder and has a loophole. Yet, in this case the proposer receives more than her commitment value). Let us verify that this is an equilibrium for q sufficiently high. For the aggressive player one, the argument follows those we have seen earlier in the symmetric case: clearly no upward deviation of the aggressive player pays off since no-one will ever concede to so aggressive a demand. What about a downward deviation? Deviating down to δv_H^* would induce an immediate deal but it yields a lower payoff than v_H^* , i..e what player one receives in equilibrium when sticking to her commitment strategy.

What about players two to four? Again, a downward deviation must deviate all the way to δv_L^* to have any effect. This is lower than the equilibrium payoff. There are several ways to deviate upwards. One can attempt a very aggressive commiment extracting all residual rents when all other player have a loophole. This is as if one became player one, just that one must pay player one the higher continuation payoff δv_H^* and thus the share is a bit smaller than the one that goes to player one if this latter is alone succesful with her commitment . This deviation pays off if

$$(1 - 2\delta v_L^* - \delta v_H^*)(1-q)^2 + (1-(1-q)^2)\delta v_L^* > [\frac{1}{3} + \frac{2}{3}(1-q)](x_L^* - \delta v_L^*) + \delta v_L^*$$

or equivalently, using (7) and dividing through by $(x_L^* - \delta v_L^*)$,

$$3(1-q)^2 + \frac{2}{3}q > 1$$

implying that this deviation does not pay off if q is sufficiently large.

Consider then smaller upward deviations where the idea is that a reponder with a low commitment deviates to extract rents from the proposer in case one or two of the other low-committing reponders have a loophole (in addition to the loophole of the aggressive player one). Such deviation requiring one of the low committing responders to have a loophole pays off if

$$[\frac{1}{3}(2q(1-q)+(1-q)^2)+\frac{2}{3}(1-q)^2]2(x_L^*-\delta v_L^*) > [\frac{1}{3}+\frac{2}{3}(1-q)](x_L^*-\delta v_L^*),$$

capturing the idea that, given that the deviating player is a responder with successful commitment (the only event where commitment matters), if player one with the aggressive commitment is the proposer (conditional probability 1/3) then at least one of the other two responders must have a loophole (probability $2q(1-q)+(1-q)^2$) and if player one with the aggressive commitment is a responder (conditional probability 2/3 then both the aggressive responder and the other low-committing responder must have a loophole (probability $(1-q)^2$). In these two cases the deviating player gains one additional $(x_L^*-\delta v_L^*)$-share. Equivalently, this expression can be written as

$$(1-q)(1-\frac{1}{3}q) > \frac{1}{2}(1-\frac{2}{3}q),$$

which does not hold if q is sufficiently large.

The other deviation which targets both low-committing responders to have a loophole (in addittion to the aggressive player) pays off if

$$[\frac{1}{3}(1-q)^2+\frac{2}{3}(1-q)^2]3(x_L^*-\delta v_L^*) > [\frac{1}{3}+\frac{2}{3}(1-q)](x_L^*-\delta v_L^*).$$

If successful, the deviating player gains two additional $(x_L^*-\delta v_L^*)$-shares. The probability of success is smaller than if targeting only one other low-committing responder to have a loophole. The difference in probabilities arises conditional on the event that the aggressive player is the proposer. The condition is equivalent to

$$(1-q)^2\left(\frac{1}{3}+\frac{2}{3}\right) > \frac{1}{3}\left(\frac{1}{3}+\frac{2}{3}(1-q)\right)$$

which does not hold if q is sufficiently large. Thus for q sufficiently large, we have an equilibrium. This equilibrium is inefficient and asymmetric.

4.4.2 n players

Let us then consider the more general case of n players. The argument regarding the deviations of the aggressive player, and the downward deviation of the low-committing players are identical as in the four player case. Let us therefore consider the upward deviations of the low-committing players. There are two kinds of deviations: (i) targeting to make the proposer indifferent when all others, including the aggressive player, have a loophole, (ii) targeting to make the proposer indifferent and reach a deal when the aggressive player does not have a loophole but k low-committing responders do.

The deviation of type (i) pays strictly off if

$$(1-(n-2)\delta v_L^*-\delta v_H^*)(1-q)^{n-2}+(1-(1-q)^{n-2})\delta v_L^* > [\frac{1}{n-1}+\frac{n-2}{n-1}(1-q)](x_L^*-\delta v_L^*)+\delta v_L^*,$$

and the deviating player is a succesful responder. In the expression, $(1-q)^{n-2}$ indicates that all other responders but the deviating responder have a loophole. In this case, the deviating ensures that each of the other players receive their continuation payoffs and grabs the residual with her committed share. This expected payoff must be higher than that of a succesful player in the counterfactual case where she wouldn't deviate. This payoff on the right-hand side of the inequality reflects the fact that the aggressive player must have a loophole for the agreement to arise and this can happen either by the aggressive being recognized to propose, which happens with probability $1/(n-1)$ (conditional on the deviating player being a successful responder), or by the aggressive player having drawn to respond but having a loophole, which happens with probability $(n-2)/(n-1)$ (conditional on the deviating player being a successful responder).

Notice that $x_L^* = \frac{1-\delta v_H^*}{n-1}$ and thus $1-(n-1)\delta v_L^* - \delta v_H^* = (n-1)(x_L^* - \delta v_L^*)$ so that the condition can be written as

$$(n-1)(1-q)^{n-2} + q\frac{n-2}{n-1} > 1. \tag{8}$$

When n approaches infinity, the first term on the left in (8) approaches zero and the second term approaches q, so that the deviation does not pay off for a given q when n is sufficiently large.

Consider then smaller upward deviations which aim to extract rents from the proposer in case $0 < k < n-2$ of the other modestly committed responders have a loophole. These deviations are similar to those we studied in the symmetic case in subsections 4.2 and 4.3 above. Formally, such a deviation pays off if

$$[\frac{1}{n-1}\eta(n-2,k) + \frac{n-2}{n-1}(1-q)\eta(n-3,k)](x_L^* - \delta v_L^*)k > [\frac{1}{n-1} + \frac{n-2}{n-1}(1-q)](x_L^* - \delta v_L^*),$$

and the deviating player is a succesful responder. If one sticks to the initial modest commitment, the payoff is as before. Deviating up attempting to grab k additional $(x_L^* - \delta v_L^*)$ shares results in a deal if there are also k of the modestly committed with a loophole. This can happen either so that the aggressively committed player one is the proposer and k of the responders have a loophole. Or so that that one of the modestly committed is the proposer and $k-1$ of the modestly committed responders and player one have a loophole. Simplifying the condition yields

$$k > \frac{1+(n-2)(1-q)}{\eta(n-2,k) + (n-2)(1-q)\eta(n-3,k)}.$$

This condition is simlar to (5) and must hold for no k= $1, ...n-2$ for in an asymmetric equilibrium with one aggressive player.

4.5 Commitment and majority rules

We have now established a number of pathological properties regarding the usage of the unanimity rule in settings where the negotiating parties have access to aggressive commitment tactics. Let us finally show that a remedy is to adopt any supermajority rule (apart form unanimity).

Theorem 5. *There is no inefficient equilibrium under any supermajority rule.*

The intuition for this result is highlighted in Proposition 2of the three-player example of Section 3. When majority rule is used, there is competition between the responders to be included in the winning coalition. The responders with aggressive commitments will be left out. This triggers a Betrand-like competition in responder-demands. In equilibrium, the commitment cannot exceed the continuation value, which is what each responder receives without commitment anyway (Baron and Ferejohn 1989). Thus no aggressive commitments are made. The outcome is efficient and coincides with that of the Baron and Ferejohn (1989) model.

5 Conclusion

We extend the Baron and Ferejohn (1989) model of multilateral bargaining by allowing the players to attempt tying their hands prior to negotiating. A commitment binds a player not to accept any share of the pie below one's commitment. We compare the unanimity rule with majority rules. We find that precommitments lead to inefficiencies and delay under unanimity only. There are potentially many symmetric stationary subgame perfect equilibria that are inefficient under the unanimity rule. These can be ordered from the least to most inefficient/aggressive according to how how many uncommitted players are needed for an agreement to arise, or large a share of the pie each player commits to. With more aggressive commitments, the delay is longer and a greater number of uncommitted players are required. The most aggressive commitment profile exists independently of the number of players in the game, and the delay in this equilibrium is increasing in the number of players in the game. Yet, less aggressive commitment profiles cannot be sustained in equilibrium if the number of players in the game is sufficiently large. The delay and inefficiency at the least and at the most efficient equilibrium increase as the number of players increases. There is also an inefficient asymmetric equilibrium where a unique party holds others as hostage requiring lions share of the surplus, reflecting the stylized pattern in many multilateral negotiations in the field. Under any supermajority rule, however, there is no equilibrium with delay or inefficiency as competition to be included in the winning coalition ensures that no aggressive attempts to force concessions will be made. The predicted patterns are by and large consistent with observed inefficiencies in many international arenas including the European Union, WTO, and UNFCCC. The results suggest that the unanimity rule is particularly damaging if the number of legislators is large.

Appendix

Lemma

Lemma 2. *Consider a symmetric commitment equilibrium which requires that at least h responders are flexible for the agreement to arise. The expected equilibrium payoff equals* $v^*_{n,h} = \frac{\eta(n-1,h)}{n(1-\delta(1-\eta(n-1,h))}$.

Proof. We solve equations (2) and (3) to yield explicit expressions for $v^*_{n,h}$ and $x^*_{n,h}$ as functions of h.

$$\begin{cases} x^*_{n,h} = \frac{1-(1+h)\delta v^*_{n,h}}{(n-1-h)} \\ v^*_{n,h} = \delta v^* + [\frac{1}{n}\tilde{\eta}[E(l|h \leq l \leq n-1) - h] + \frac{n-1}{n}q\eta] \left(x^*_{n,h} - \delta v^*_{n,h}\right), \end{cases} \tag{9}$$

We can plug in the expression for $x^*_{n,h}$ into the second equation and solve for $v^*_{n,h}$. This yields

$$v^*_{n,h} = \frac{m(n-1,h)}{(n-1-h)(1-\delta(1-\frac{nm(n-1,h)}{(n-1-h)}))}$$

or

$$v^*_{n,h} = \frac{m(n-1,h)}{(1-\delta)(n-1-h) + \delta nm(n-1,h)}$$

where

$$m = \frac{1}{n}\eta(n-1,h)[E(l|h \leq l \leq n-1) - h] + \frac{n-1}{n}q\eta(n-2,h)$$

where

$$\eta(n-1,h)[E(l|h \leq l \leq n-1) - h] = \sum_{l=h}^{n-1}(l-h)\binom{n-1}{l}(1-q)^l q^{n-1-l}$$

and

$$\eta(n-2,h) = \sum_{l=h}^{n-2}\binom{n-2}{l}(1-q)^l q^{n-2-l}$$

so

$$m(n-1,h) = \frac{1}{n}\sum_{l=h}^{n-1}(l-h)\binom{n-1}{l}(1-q)^l q^{n-1-l} + \frac{n-1}{n}\sum_{l=h}^{n-2}\binom{n-2}{l}(1-q)^l q^{n-1-l}$$

$$m(n-1,h) = \sum_{l=h}^{n-2}\left[\frac{1}{n}(l-h)\binom{n-1}{l} + \frac{n-1}{n}\binom{n-2}{l}\right](1-q)^l q^{n-1-l} + \frac{1}{n}(n-1-h)(1-q)^{n-1}$$

$$m(n-1,h) = \frac{n-1-h}{n}\sum_{l=h}^{n-1}\binom{n-1}{l}(1-q)^l q^{n-1-l}$$

$$m(n-1,h) = \frac{1}{n}\eta(n-1,h)(n-1-h)$$

because

$$\eta(n-1,h) = \sum_{l=h}^{n-1}\binom{n-1}{l}(1-q)^l q^{n-1-l}$$

is the expected number of times that a player receives the "bonus" $(x^*_{n,h} - \delta v^*_{n,h})$ in a given period. □

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