Public Liquidity and Bank Lending: The Perils of Large Interventions

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August 27, 2020

Abstract

This paper studies optimal public liquidity injections by the Treasury and the central bank. I use a simple model in which banks can supply private liquidity and have better investment opportunities than the non-financial sector but are subject to moral hazard. Public liquidity injections crowd out private liquidity and reduce the supply of bank lending — an unintended consequence that decreases welfare. In some cases, the welfare function has multiple local maxima. Under some conditions, the optimal policy is a moderate injection that does not eliminate liquidity crises, even if taxation required to finance public liquidity entails no deadweight costs.

1 Introduction

What is the optimal size of public liquidity injections? Public liquidity in the form of Treasury securities and central bank reserves plays a crucial role in both normal and crisis times. In normal times, the supply of public liquidity is mostly affected by Treasury securities. In crisis times, both the Treasury and the central bank significantly affect public

*E-mail: robatto@wisc.edu. This paper includes a large revision of some material from a previous work circulated under the title "Flight to Liquidity and Systemic Bank Runs." For their comments and suggestions on an earlier draft, I am grateful to Douglas Gale, Guillermo Ordoñez, Vincenzo Quadrini, Jean-Charles Rochet, and seminar participants at Frankfurt University, the Macro-Finance Society Workshop at Boston College, the Summer Workshop on Money and Banking at the Bank of Canada, USC Marshall, and the Wharton Conference on Liquidity and Financial Fragility.

liquidity; Treasury debt typically increases in relation to fiscal policy responses, and the central bank intervenes in various ways. The economic consequences of the COVID-19 crisis have brought these interventions into the spotlight again and are now being implemented or discussed, highlighting the importance of understanding the effects and optimal design of liquidity policies.

Public liquidity injections have important effects on the financial system. While it is well understood that public liquidity affects the amount of liquid assets supplied by financial intermediaries (i.e., the supply of *private* liquidity), less is known about the impact of public liquidity policies on the provision of credit to the economy.[1] Yet, this is a crucial issue because if public liquidity policies affect banks' lending, they are likely to generate sizable effects on output, employment, and welfare.

The main argument of this paper is that public liquidity injections give rise to a trade-off which prevents the optimality of too-large injections. On the one hand, public liquidity injections increase the supply of high-quality liquid assets, which is good for the economy. On the other hand, however, my model shows that they crowd out banks' intermediation activities and, thus, increase borrowers' reliance on direct borrowing from savers (e.g., through bond and equity financing). This second effect produces an unintended consequence on welfare. Because banks play a crucial role in the process of screening and monitoring firms and other borrowers (Diamond, 1984; Freixas and Rochet, 2008), banks have a technological advantage which allows them to finance better and more productive projects. Hence, a public liquidity injection that shifts credit supply away from banks and toward direct financing generates a reduction in output and welfare.

The disintermediation triggered by liquidity injections reduces welfare because savers (i.e., agents outside the financial sector) are less productive than financial intermediaries at allocating credit. This is a common assumption used by several standard macro-finance frameworks (Bianchi, 2011; Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Lorenzoni, 2008) but typically overlooked in models that study public and private liquidity, such as Holmström and Tirole (1998) and the monetary

[1] In addition to theoretical studies, several empirical papers study the effects of public liquidity on the supply of private liquidity (see e.g. Greenwood, Hanson, and Stein 2015; Krishnamurthy and Vissing-Jorgensen 2015; Li 2019).

models with banks that build on Lagos and Wright (2005). Even if disintermediation might arise in those papers, it typically does not produce any negative welfare consequences. Hence, a large supply of liquidity that implements a Friedman-like rule is typically optimal in those models. In contrast, it is optimal to keep a positive premium on liquid assets in my model — equivalent to a positive interest rate in monetary models. Indeed, a high liquidity premium reduces banks' borrowing costs and prevents excessive disintermediation.

I distinguish between public liquidity injections made by the Treasury (in the form of an increase in Treasury securities) and those made by the central bank (in the form of purchases of financial assets or loans to financial intermediaries). Focusing on Treasury liquidity allows me to draw a direct comparison with Holmström and Tirole (1998) and present the main result in a simple baseline model. In that case, the optimal supply of public liquidity is always a moderate injection — as opposed to a large one. I then turn to liquidity injected by the central bank. This is relevant because, in practice, central banks lend to financial intermediaries and, thus, counteract the disintermediation effect. Nonetheless, under some (weak) conditions, the offsetting effect of central bank lending is not sufficient to undo the baseline result, and the unintended disintermediation arises even in this case. Thus, the resources intermediated by the financial sector remain positive and liquidity crises are not eliminated.

The model builds on Holmström and Tirole (1998)'s idea that financial intermediaries alone cannot meet the private sector's demand for liquidity if the economy is subject to aggregate shocks, and thus public liquidity improves welfare. However, the model I use is simpler and different in many respects.

In the model, financial intermediaries have access to better investment opportunities than non-financial agents but are plagued by moral hazard. Absent moral hazard, non-financial agents would hand all their endowments to intermediaries in exchange for debt securities, and intermediaries would make all the investments in the economy. However, moral hazard on the side of financial intermediaries prevents this from happening. In particular, financial intermediaries can misbehave and extract private benefits at the expense of debt holders and, thus, need to be compensated with positive rents. Crucially, the moral hazard friction is more severe than the technological advantage of intermediaries. As a result, absent liquidity considerations, financial intermediation would shut down because the rents extracted by intermediaries would reduce the return on intermediaries' debt too much. That is, non-financial agents would not buy intermediaries' debt and would make all investments directly, despite their technological disadvantage. Because intermediaries' debt provides liquidity, however, non-financial agents are willing to hold it even if such debt pays a lower return. Thus, the liquidity value of debt allows intermediaries to raise some resources and finance a fraction of the economy's investments.

In the baseline policy analysis, liquidity injected by the Treasury increase the amount of public liquidity available in the economy. As a result, the liquidity needs that must be met using intermediaries' debt are reduced by such an injection. Non-financial agents respond by holding less of intermediaries' debt — to economize on the cost that originates from the moral hazard friction — and by investing more directly. In other words, injections of public liquidity trigger a process of disintermediation that shifts investments from the financial sector to the non-financial sector. Overall, two main channels affect welfare. On the one hand, public liquidity injections provide more liquidity, which improves welfare through a better allocation of resources. On the other hand, the disintermediation reduces overall productivity and thus output because more investments are made by non-financial agents, which are less productive than financial intermediaries.

In the second part of the paper, I study liquidity provision in the form of central bank reserves. In particular, central bank interventions produce additional effects on the supply of credit, which in turn give rise to two local welfare maxima.

The disintermediation effect arises even if the interventions are made by the central bank, under some reasonable assumptions. In principle, the central bank can try to offset the disintermediation triggered by liquidity injections by purchasing securities issued by financial intermediaries or by lending to them — both approaches provide funding to the financial sector. However, disintermediation still occurs if private debt issued by intermediaries is subject to a repo-like haircut to a degree greater than that of public securities, as is the case in practice. To clarify this result, consider a simple example in which a private debt security issued by an intermediary has a 10% haircut, whereas a public security has a 0% haircut. Using private debt, you would need securities valued at about \$1.11 to obtain \$1 of liquidity, whereas you would need only \$1 of public securities to obtain \$1 of liquidity. Thus, a public liquidity injection of \$1 reduces the demand for intermediaries' debt by

$1.11>$1. Even if the central bank uses the newly issued $1 of liquidity to provide funds to intermediaries, disintermediation nonetheless occurs.

Welfare under central bank interventions has two local maxima. For small or moderate interventions, the same trade-off highlighted in the baseline model holds. Large injections, however, produce a richer outcome. First, they drive the amount of intermediaries' debt that is sold to non-financial private agents to zero (i.e., zero private liquidity). Second, the resources intermediated by the financial sector remain positive because of the funding that intermediaries receive from the central bank. With a very large injection — well above the minimum required to satiate the private sector's liquidity demand — a second local maximum arises, in which the economy is flooded with public liquidity and intermediaries use the resources provided by the central bank to finance the first-best level of investments.

The second local maximum — which implements the first best — is feasible only if there are no constraints on the central bank's loss-absorption capacity. Indeed, the large intervention required to implement the second local maximum exposes the central bank to large losses in the event of bad aggregate shocks that trigger intermediaries' default. In practice, there are often explicit or implicit constraints on the central bank's ability to absorb losses, for legal and institutional reasons. Therefore, it is useful to consider what the optimal policy is in the presence of constraints that limit central bank losses. If such constraints are sufficiently tight, the optimal policy is the first local maximum and, thus, qualitatively similar to the baseline model.

Crucially, the disintermediation effect plays a key role above and beyond any constraint faced by the central bank. To see this final point, it is useful to consolidates the Treasury with the central bank (Sargent and Wallace, 1981), so that constraints on central bank losses are equivalent to limits on fiscal capacity. Under a sufficiently tight limit, the optimal policy is the first local maximum, but any limit on fiscal capacity is never binding along the equilibrium path. In other words, the disintermediation effect reduces the optimal size of liquidity injections in comparison to a model that includes only distortionary taxation or limits on fiscal capacity.

1.1 Additional comparisons with the literature

The model I use builds on Benigno and Robatto (2019) — which in turn includes many elements of Lagos and Wright (2005) and the new monetarist literature (Lagos, Rocheteau, and Wright, 2017). However, my model has important differences compared with that paper. In Benigno and Robatto (2019), the optimal policy in the absence of tax distortions is similar to that of Holmström and Tirole (1998): an injection of public liquidity that drives private liquidity to zero and, possibly, financial intermediation. Indeed, the two elements that drive the result here — intermediaries' moral hazard and technological advantage — are missing from that paper.

This paper is closely related to the monetary literature and, in particular, the modern approach based on Lagos and Wright (2005). Similar to money, public liquidity is represented by assets that are traded above their intrinsic value because they relax liquidity constraints driven by frictions such as limited commitment and moral hazard. Private liquid debt is backed by investments in productive projects, similar to Bigio (2015), Geromichalos, Licari, and Suárez-Lledó (2007), Lagos and Rocheteau (2008), and Lagos (2010a). The main difference is that the Friedman rule is typically optimal in those models (Lagos, 2010b), whereas the disintermediation effect that arises here prevents its optimality. While some papers provide some justification for the non-optimality of the Friedman rule (e.g., Berentsen, Camera, and Waller, 2007), the disintermediation effect that I characterize here is, to my knowledge, novel. Another closely related paper is Gu, Mattesini, Monnet, and Wright (2013), in which banks are crucial both for investments and for the supply of liquid assets, and limited commitment frictions play a crucial role. That paper, however, abstracts from public liquidity altogether.

Several other papers study the effects of public liquidity and its interaction with the financial sector. In Greenwood, Hanson, and Stein (2015), public liquidity crowds out the financial sector too, but this effect is positive in their model because of an externality generated by private intermediaries and modeled along the lines of Stein (2012). Li (2017) also derives a risk associated with large public liquidity, which is complementary to mine. In Li (2017), public liquidity reduces liquidity premia and creates an incentive for intermediaries to take on more risk, which amplifies credit cycles and lengthens the duration of crises. Magill, Quinzii, and Rochet (2016) study a combination of central bank prudential, monetary, and balance sheet policies. Bolton and Huang (2017) study public liquidity in a framework in which domestic and foreign money provides liquidity. More generally, this paper is also related to a literature that highlights the perils of interventions during financial crises. Ennis and Keister (2009) focus on bank runs, and Malherbe (2014) highlights the risks associated with large interventions in markets plagued by adverse selection.

2 Model

The model includes two key features that have been extensively used in the literature: a moral hazard friction along the lines of Holmström and Tirole (1998) and Gertler and Kiyotaki (2010), and a technological advantage of the financial sector as in Bianchi (2011), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2010), He and Krishnamurthy (2013), and Lorenzoni (2008). The baseline structure, however, builds on Benigno and Robatto (2019), which in turn draws on Lagos and Wright (2005).[2]

There are two time periods ($t = 0, 1$), and time $t = 1$ is divided into two subperiods. There are three sets of players: households, financial intermediaries, and the government. At time $t = 1$, an aggregate shock realizes and affects the productivity of the investments made at $t = 0$. In particular, there are two possible states at $t = 1$: h (i.e., high) and l (i.e., low). State h realizes with probability $1 - \pi$ and state l with probability π. Depending on the conditions in the financial sector, a liquidity crisis can occur in state l.

The demand for liquid assets comes from households, which represent the non-financial sector, as in several other papers (Lagos and Wright, 2005; Nagel, 2016; Stein, 2012). The model could be reformulated by assuming that liquidity is used by entrepreneurs, as in the classic model of Holmström and Tirole (1998). However, the results would be unchanged but would require a richer structure, clouding the logic of the results.

At time $t = 0$, households have some endowment that can be invested in productive projects, either directly by households themselves or indirectly by financial intermediaries.

[2]The model I use has a representative agent, which makes the welfare analysis simpler in comparison to Holmström and Tirole (1998), which has two sets of agents. Indeed, I do not have to take a stand on the objective function of the government and the Pareto weights assigned to the various agents, whereas Holmström and Tirole (1998) note that altering the objective function can alter the optimal policy.

Financial intermediaries have no resources and finance their investments by issuing defaultable debt (i.e., a state-contingent security). Financial intermediaries do, however, have access to better investment opportunities, but they are plagued by a moral hazard problem. Absent moral hazard, welfare would be maximized if all investments were made by intermediaries. However, the moral hazard friction implies that intermediaries must be compensated with rents to make sure they do not misbehave. To earn sufficient rents, intermediaries pay a low return on the debt they issue, but households buy such debt anyway because of the liquidity it provides (i.e., intermediaries' debt represents the "private liquidity" in the economy).

In the first subperiod of $t = 1$, a friction motivated by limited enforcement implies that households' consumption must be financed using intermediaries' debt (i.e., private liquidity) or government debt (i.e., public liquidity). While government debt is risk-free and thus provides liquidity in all states, intermediaries' debt is fully defaulted on in the low state because intermediaries' projects do not produce any output in that contingency. As a result, in the low state, liquidity is effectively provided only by public debt.

2.1 Environment

I first describe the environment and then analyze households' and financial intermediaries' choices.

Preferences. Households consume in both subperiods at $t = 1$. Their utility is given by

$$(1 - \pi) [\log C_h + X_h] + \pi [\log C_l + X_l] , \tag{1}$$

where C_h and C_l denote consumption in the first subperiod of $t = 1$, and X_h and X_l denote consumption in the second subperiod at $t = 1$.

Endowments. At $t = 0$, households are endowed with an amount $\overline{B}$ of government debt and an amount $\overline{Y}$ of goods. The debt $\overline{B}$ is a liability of the government and I model it as a zero-coupon security with a unitary payoff at $t = 1$. As a result, the debt $\overline{B}$ can be interpreted as a Treasury security, although I extend the analysis to include the central bank in Section 5. The endowment of goods, $\overline{Y}$, will only be invested in productive projects, either directly by households or by intermediaries.

At $t = 1$, in the first subperiods, households have an additional endowment of goods whose amount is possibly state-contingent: $\overline{Y}_h$ in the high state and $\overline{Y}_l$ in the low state.[3] These time-1 endowments can be viewed as output of a non bank-financed sector, and thus are assumed to be exogenous. Endowments $\overline{Y}_h$ and $\overline{Y}_l$ can be consumed in the first subperiods of $t = 1$ and/or stored to be consumed in the second subperiod of $t = 1$.

All endowments of goods (i.e., $\overline{Y}$, $\overline{Y}_h$, and $\overline{Y}_l$) are assumed to be sufficiently large so that none of the results are affected by a shortage of such goods.

Technology. At $t = 0$, households and financial intermediaries can invest in productive projects, using the endowment of goods $\overline{Y}$. These projects may produce in the second subperiod of $t = 1$. More precisely, the productivity depends on whether the project is run by households or financial intermediaries and on the realization of the aggregate state. That is, (i) financial intermediaries are more productive than households, and (ii) productivity in the high state is higher than in the low state.

If a project is run by an intermediary, the productivity is $A_h > 0$ in the high state and $A_l = 0$ in the low state. The average productivity is normalized to one, that is, $(1 - \pi) A_h + \pi A_l = 1$. Thus the assumption $A_l = 0$ implies $A_h = 1/(1 - \pi)$.

If a project is run by a household, the productivity is only a fraction $1 - \phi$ of that of financial intermediaries. That is, the productivity is $A_h (1 - \phi)$ in the high state and $A_l (1 - \phi) = 0$ in the low state.[4]

In summary, letting K^I be the resources invested at $t = 0$ by financial intermediaries, the resource constraints are

$$C_h + X_h \leq \overline{Y}_h + A_h K^I + A_h (1 - \phi) \left(\overline{Y} - K^I\right), \tag{2}$$

$$C_l + X_l \leq \overline{Y}_l, \tag{3}$$

in the high and low state, respectively, in addition to non-negativity constraints C_h, C_l, X_h,

[3]The results do not depend on the state-contingent nature of the time-1 endowment, and thus one can assume that $\overline{Y}_h = \overline{Y}_l$.

[4]The zero-productivity in the low state follows from the assumption that $A_l = 0$.

$X_l \geq 0$, and the feasibility constraint $K^I \leq \overline{Y}$.

Financial intermediaries operate in a fashion similar to Gertler and Kiyotaki (2010, 2015). That is, a fraction of households become bankers at $t = 0$ and operate financial intermediaries in the households' interest. Because households have linear utility at $t = 2$, bankers simply maximize the expected profits. Any realized profit is then returned to households at $t = 2$.

Financial frictions. Absent financial frictions, the first best could be achieved by turning all endowments to financial intermediaries, which are more productive than households, and by choosing consumption so that the marginal utilities across subperiods at $t = 1$ are equalized:

$$K^I = \overline{Y}, \quad \frac{1}{C_h} = 1, \quad \frac{1}{C_l} = 1.$$

With no financial frictions, this outcome could be achieved without government debt. However, the financial frictions introduced next may prevent this outcome and give rise to a role for government securities.

First, following Gertler and Kiyotaki (2010, 2015) and Holmström and Tirole (1998), intermediaries can extract private benefits from managing their projects at the expense of the resources that can be produced and are pledgeable to depositors. If intermediaries misbehave, they can extract, in the high state, an amount θA_h per unit invested, with $\theta < 1$.[5] The resources obtained by misbehaving are returned to the shareholders, and nothing is left for depositors.[6] As a result, to ensure that intermediaries do not misbehave, they need to be compensated in the high state with a rent θA_h per unit of capital.

Crucially, I assume that the private benefits that can be extracted by financial intermediaries exceed the technological advantage that such intermediaries have with respect to households. More precisely,

$$\theta > \phi \tag{4}$$

[5]Because the productivity is zero in the low state anyway, intermediaries cannot extract any benefits by misbehaving in that state.

[6]This approach is an extreme version of the moral-hazard problem introduced by Holmström and Tirole (1998). In that paper, shirking reduces the probability of success, and I assume here that shirking reduces that same probability to zero.

where θ measures the degree of the moral hazard problem and ϕ measures the gap between the technologies available to intermediaries and households, as described above. Under this assumption, the moral hazard friction is severe enough that households prefer to manage some of the time-zero investments directly, even if they their technology is worse than that of the financial intermediaries. If, instead, the technological advantage of intermediaries were greater than the moral hazard friction (i.e., $\theta \leq \phi$), it would be optimal for households to turn all their time-zero endowment $\overline{Y}$ over to intermediaries because the technological advantage of the financial sector would more than offset the rent extracted to prevent the moral hazard problem.

Second, following Lucas and Stokey (1987), households cannot consume their own endowment of goods in the first subperiod of $t = 1$. Instead, they must purchase C_h and C_l from other households in a centralized market. This assumption is made for convenience, but the model can be reformulated by distinguishing between buyers and sellers as in Lagos and Wright (2005).

Third, households' purchases of C_h and C_l must be paid immediately using financial instruments issued by the government or by the intermediaries. Alternatively, the amount due can be paid at $t = 2$, but households must hand in collateral in the form of government debt or intermediaries' debt. In other words, households cannot buy using unsecured credit (i.e., pay in the second subperiod for the purchases made in the first subperiod, without any collateral), and they cannot settle transactions by handing over the investments they made at $t = 0$ or handing over claims on the output produced by such investments.[7] While the main result of the paper can be understood by taking this assumption as exogenous, I describe next how it can be derived endogenously if one imposes two further assumptions. First, the inability to use unsecured credit can be derived endogenously by assuming that households lack the commitment to repay debt , as discussed by Lagos, Rocheteau, and Wright (2017). Under this assumption, any purchase made in the first subperiod of $t = 1$ using unsecured credit would give rise to a debt that would be always defaulted on by the household in the second subperiod. The inability to settle transactions by handing over the investments they made at $t = 0$, or claims on them, is ruled out by introducing a simple

[7]Because capital has positive productivity only in state h, handing over claims on output would be available only in such a state.

moral hazard problem on the side of households, similar to that of financial intermediaries. In particular, investments made at $t = 0$ by a certain household are productive at $t = 1$ only if that household exerts some effort. If instead the household shirks, it extracts in the high state private benefits γA_h per unit invested. Assuming that the non-pledgeable rents from misbehaving, γA_h, are equal to the pledgeable output per unit of capital if the household exerts effort, $(1 - \phi) A_h$, implies that the investments made by the household are not used as collateral in equilibrium. Indeed, if a household were able to convince a counterparty to take some of its investments as collateral, even with some haircut, the household would find it profitable to misbehave at $t = 2$ to extract rents γA_h, so that the collateral would be worthless. The assumption $\gamma A_h = (1 - \phi) A_h$ simplifies the analysis because it reduces the collateral value of households' investments to zero. However, the analysis can be relaxed to the case $0 < \gamma A_h < (1 - \phi) A_h$, in which households' investments would be partially collateralizable.

2.2 Households' choices

I now turn to the analysis of households' choices. I first describe households' constraints, which follow from the environment described in Section 2.1, and then present the households' optimality conditions.

At $t = 0$, households choose investments K, holdings of intermediaries debt D, and holdings of government bonds B subject to the budget constraint

$$K + Q^D D + Q^B B \leq \overline{Y} + Q^B \overline{B}, \tag{5}$$

where Q^D is the price at which intermediaries issue their debt, and Q^B is the price of government debt. Similar to government debt, intermediaries' debt is modeled as a zero-coupon security with unitary face value.[8]

In the first subperiod of $t = 1$, households choose consumption C_h and C_l. The frictions

[8]Formally, D is a state-contingent security with payoff equal to one in state h and zero in state l because D is fully defaulted on in the low state, as explained below.

described in Section 2.1 give rise to a liquidity constraint of the form

$$C_h \leq B + D, \tag{6}$$

$$C_l \leq B, \tag{7}$$

in the high and low state, respectively. That is, households can purchase C_h in the high state using government debt B and intermediaries debt D as means of payment, whereas they can use only government debt in the low state. This is because intermediaries' investments have zero productivity in the low state, and thus intermediaries' debt is fully defaulted on in that state.

In the second subperiod of $t = 1$, consumption X_h and X_l is residually determined by all the resources that are left to each household:

$$X_h \leq \overline{Y}_h + \Pi_h + (B + D - C_h) + A_h (1 - \phi) K - T_h \tag{8}$$

$$X_l \leq \overline{Y}_l + \Pi_l + (B - C_l) - T_l. \tag{9}$$

In the high state, a household's resources are represented by its endowment $\overline{Y}_h$, the profits Π_h received from intermediaries, the liquidity $B+D$ that was not used in the first subperiod to purchase C_h, and the output produced by the investments K, net of lump-sum taxes T_h that must be paid to the government. In the low state, the resources available are determined similarly, with the differences being that government bonds B are the only liquid assets, and investments K produce no output.

Households maximize utility (1) subject to (5)-(9). Let μ_h and μ_l be the Lagrange multiplier of the liquidity constraints (6) and (7).

The optimal choice of government debt at $t = 0$ is determined by the first-order condition:

$$1 + (1 - \pi) \mu_h + \pi \mu_l = Q^B (1 - \pi) A_h (1 - \phi). \tag{10}$$

Equation (10) equates the marginal benefits of holding an additional unit of government debt B to its marginal cost. The benefits are represented by the payoff, which is one, plus the liquidity value of relaxing the liquidity constraint in the high state, μ_h, and in the low state, μ_l. The marginal cost is the payoff that can be obtained by investing in projects directly. Government debt costs Q^B and, if these resources were instead invested in projects, they would produce $A_h(1-\phi)$ in the high state, that is, with probability $1-\pi$.

The optimal choice of intermediaries' debt at $t = 0$ is determined by the first-order condition

$$(1-\pi) + (1-\pi)\mu_h \leq Q^D (1-\pi) A_h (1-\phi) . \tag{11}$$

Equation (11) is expressed as an inequality to account for the possibility that the households choose $D = 0$; this case is indeed possible in equilibrium, as discussed in Section 3. The expression is similar to that in Equation (10), but the marginal benefits differ because intermediaries' debt D is repaid and provides liquidity only when it is not defaulted on — in the high state — and thus with probability $1-\pi$.

To clarify the household's choice of intermediaries' debt D, consider the spread between the return that households earn by investing directly in projects and the return earned on D. It is useful to define such a spread as:

$$S^D \equiv \frac{(1-\pi) A_h (1-\phi)}{(1-\pi)\frac{1}{Q^D}} - 1. \tag{12}$$

The spread S^D is the ratio of the gross return earned by investing directly (i.e., $(1-\pi) A_h (1-\phi)$) over the gross return on intermediaries' debt (i.e., $(1-\pi)/Q^D$) minus one. When the two returns are the same, the spread S^D is thus zero; when the return earned on intermediaries' debt is lower, the spread is positive.

Using the definition of the spread S^D in (12), the first-order condition (11) can be expressed as

$$\mu_h \leq S^D.$$

When this equation holds with equality, households choose D to equalize the benefit and cost of holding securities D. The benefit is given by the fact that an additional unit of D relaxes the liquidity constraint (6) in state h, whose value is captured by the Lagrange multiplier μ_h of (6). The cost is given by the spread S^D, that is, the additional return earned by investing directly in projects (which are illiquid) rather than holding D.

At $t = 1$, consumption choices are determined subject to the liquidity constraints (6) and (7). Such constraints might introduce a wedge between the marginal utility in the first subperiod (i.e., $1/C_h$ in state h and $1/C_l$ in state l) and that in the second subperiod (i.e., one in both states). As a result, the optimality conditions are

$$\frac{1}{C_h} = 1 + \mu_h, \qquad \frac{1}{C_l} = 1 + \mu_l, \tag{13}$$

where μ_h and μ_l are the Lagrange multipliers of the liquidity constraints (6) and (7), respectively, as described above. When the liquidity constraints (6) and (7) are not binding, their Lagrange multipliers are $\mu_h = \mu_l = 0$, and the marginal utilities of C_h and C_l are equalized to those of the second subperiod.

2.3 Financial intermediaries' choices

At time $t = 0$, an intermediary issues zero-coupon debt D at price Q^D to finance investments K^I. Thus, the budget constraint is

$$K^I \leq Q^D D. \tag{14}$$

At $t = 1$, in the high state, the intermediary earns the output A_h per unit of the project and must reimburse its debt D. Thus, profits are given by

$$\Pi_h = A_h K^I - D. \tag{15}$$

Profits in the low state are zero, $\Pi_l = 0$, because the productivity of the investments is $A_l = 0$ in that state and the intermediary fully defaults on its debt.

The moral hazard friction described in Section 2.1 implies that the intermediary's profits must be at least as large as the private benefits that the intermediary can extract by misbehaving:

$$\Pi_h \geq \theta A_h K^I. \tag{16}$$

Using $A_h = 1/(1 - \pi)$ (see Section 2.1), equations (14), (15), and (16) imply that intermediaries are willing to supply a positive amount of securities, $D > 0$ as long as

$$Q^D \geq \frac{1-\pi}{1-\theta}, \tag{17}$$

whereas they supply $D = 0$ if the price Q^D does not satisfy (17).

Without the moral hazard friction (i.e., if $\theta = 0$), the condition (17) would simplify to $Q^D \geq 1 - \pi$. This is because issuing a debt security at price $Q^D = 1 - \pi$ allows intermediaries to invest an additional $1-\pi$ units of resources, which produces $(1-\pi) A_h = 1$ unit of output in the high state at $t = 1$. This unit of output suffices to repay the debt security but leaves the intermediary with no profits. If instead there is moral hazard (i.e., $\theta > 0$), the intermediary issues a security at a price $Q^D = \frac{1-\pi}{1-\theta} > 1 - \pi$. As a result, the intermediary produces more than one unit of output in the high state for each unit of debt issued, and earns positive profits that provide an incentive to exert effort.

2.4 Government

The government collects lump-sum taxes T_h and T_l from households in the high and low state, respectively, to repay the initial debt $\overline{B}$. This section abstracts from policy interventions, which are instead analyzed in Sections 4 and 5.

Because the government must repay the debt $\overline{B}$ in both states, taxes T_h and T_l are the same:

$$T_h = T_l = \overline{B}. \tag{18}$$

In this paper, I focus on lump-sum non-distortionary taxation. Abstracting from distortionary taxation allows me to highlight a new channel that prevents the optimality of a large injection of public liquidity. Several papers in which the government supplies public liquidity and can use lump-sum taxes assume that the government does not flood the market with its debt, even if such policy would be optimal. Others, such as Greenwood, Hanson, and Stein (2015), introduce distortionary taxation, which generates a cost of a too-large supply of public liquidity. The results derived in this paper are thus complementary to those of papers that use distortionary taxation and highlight a new, different channel that prevents the optimality of a large supply of public liquidity.

3 Equilibrium with no policy intervention

I now present the equilibrium of the model with no policy intervention. I first formalize the equilibrium concept, although it is standard, and then present and discuss the results.

3.1 Equilibrium definition

The equilibrium definition is standard. An equilibrium is a collection of

- Prices of government debt and intermediaries' debt, Q^B and Q^D;
- Intermediaries' debt, D, and households' holding of government debt, B;
- Time-zero investments by intermediaries and households, K^I and K;
- Intermediaries' profits in the high and low state: Π_h, Π_l;
- Time-one consumptions in the high and low state: C_h and X_h, and C_l and X_l;

such that households maximize their utility (1) subject to (5)-(9); intermediaries maximize profits (15) subject to their budget constraint (14) and the moral-hazard constraint (16); the government budget constraint (18) holds; and the time-zero bond market clears, $B = \overline{B}$.

3.2 Equilibrium

To simplify the exposition, the rest of the paper restricts attention to a special case in which the moral-hazard problem that plagues financial intermediaries is "mild." I define a "mild" moral hazard problem as the case in which the private benefits that intermediaries can extract are only slightly larger than their technological advantage, as formalized below. Appendix A presents the equilibrium in the full model that does not include this restriction.

Formally, I consider the limiting economy in which the assumption $\theta > \phi$ introduced in (4) holds, but in which θ is arbitrary close to ϕ (i.e., the gap between θ and ϕ is arbitrarily small). As I explain below, I do not focus on the case $\theta = \phi$ because of a discontinuity in the limit.

In general, given arbitrary θ and ϕ with $\theta > \phi$, households' choice of intermediaries' debt D trades off the benefits due to the liquidity of this security with the cost due to the rents extracted by intermediaries. The latter takes the form of a lower return on D — namely, the spread S^D defined in (12) between the return on D and the return that households earn by investing directly in projects. The implication of focusing on the case $\theta \to \phi$ is that intermediaries' rents due to the moral hazard are (almost) exactly offset by the technological advantage of the financial sector. As a result, the spread on intermediaries' debt becomes arbitrarily close to zero.

The reason I focus on the limiting economy in which $\theta \to \phi$ but $\theta > \phi$, rather than the limit case $\theta = \phi$, is the presence of a discontinuity in the limit. As long as $\theta > \phi$, the moral hazard premium is slightly more severe than the intermediaries' technological advantage. Thus, intermediaries must earn rents to avoid misbehavior, and the spread S^D must be positive — albeit very small. If, instead, $\theta = \phi$, the spread S^D is exactly zero because the intermediaries' rents are fully covered by the intermediaries' technological advantage. In this case, the amount of resources intermediated by the financial sector is not uniquely defined because households are indifferent between investing directly in projects or buying intermediaries' debt.

With a close-to-zero spread, households essentially have access to a liquid security (i.e., D) that pays almost the same return as direct investments (which are illiquid). As a result, they find it optimal to hold as much D as needed to satiate their liquidity needs in the high state. That is, the high-state level of consumption C_h is arbitrary close to one, equating the marginal utility of C_h with that of X_h.

The low state is characterized by a financial liquidity crisis. Liquidity is scarce in the low state because D is defaulted on in that state. Thus, consumption C_l is financed only by government debt: $C_l = \overline{B}$. If $\overline{B} < 1$, the liquidity constraint is binding and, thus, the allocation of consumption is inefficient in comparison to the first best. Section 4 studies this issue in more details.

Even though the spread is zero in equilibrium, intermediaries earn positive profits: $\Pi_h > 0$.[9] This is because the moral hazard parameter θ and the technological parameter ϕ are positive, even if the gap between the two is arbitrarily close to zero. As a result, intermediaries earn positive profits thanks to the better technology they have access too. In turn, these profits provide the incentive to intermediaries to behave properly and not extract private benefits.

The next proposition characterizes the equilibrium under mild moral hazard and focuses

[9]Formally, profits are positive and bounded away from zero.

on the case in which public liquidity is $\overline{B} \leq 1$. A supply of public liquidity $\overline{B} = 1$ is sufficient to satiate the liquidity needs of the economy; hence, for this baseline model, focusing on the case $\overline{B} \leq 1$ is without loss of generality. All the proofs are provided in the Appendix.

Proposition 1. *Assume $\overline{B} \leq 1$. In the limiting economy in which $\theta \to \phi$ but $\theta > \phi$, the equilibrium is characterized by:*

- *Investments by intermediaries, K^I, and households, K, at $t = 0$:*

$$K^I = \frac{\left(1 - \overline{B}\right)(1 - \pi)}{1 - \phi}, \quad K = \overline{Y} - \frac{\left(1 - \overline{B}\right)(1 - \pi)}{1 - \phi};$$

- *Intermediaries' debt:*

$$D = 1 - \overline{B};$$

- *Price of government debt, Q^B, and of intermediaries' debt, Q^D:*

$$\begin{aligned} Q^B &= \frac{1}{1 - \phi}\left[(1 - \pi) + \pi \frac{1}{\overline{B}}\right], \\ Q^D &= \frac{1 - \pi}{1 - \phi}, \end{aligned}$$

- *Spread: $S^D = 0$;*
- *Consumption at $t = 1$, first subperiod:*

$$C_h = 1, \quad C_l = \overline{B},$$

  and thus $C_l < C_h < 1$;
- *Consumption at $t = 1$, second subperiod:*

$$\begin{aligned} X_h &= \overline{Y}_h + \overline{Y}\frac{1 - \phi}{1 - \pi} - 1 + \frac{\phi\left(1 - \overline{B}\right)}{1 - \phi}, \\ X_l &= \overline{Y}_l - \overline{B}. \end{aligned}$$

- *Intermediaries' profits in the high state: $\Pi_h = \phi\frac{1-\overline{B}}{1-\phi}$.*

4 Policy analysis: liquidity injections

This section analyzes the optimal supply of public liquidity by the government. Without policy interventions, Section 3.2 has shown that the equilibrium is characterized by (i) positive rents for intermediaries, which provide them with incentives to avoid misbehavior, and (ii) default, in the low state, on the private liquidity D issued by financial intermediaries, which results in a liquidity crisis in that state.

The optimal policy is a moderate supply of public liquidity. Under the optimal policy, financial intermediaries are active in equilibrium and liquidity crises occur in the low state.

Public liquidity injections are modeled in this section as an increase in the endowment of government debt $\overline{B}$ at the beginning of $t = 0$. As a result, the policy analysis follows in a straightforward way from the results of the previous section. On the one hand, increasing the supply of public liquidity at $t = 0$ allows households to better cope with the liquidity crisis in the low state. Absent any other effect, the optimal policy would thus be to flood the economy with as much liquidity as needed to satiate the demand from households, as in Holmström and Tirole (1998), thereby shutting down the financial sector. On the other hand, however, the same increase in public liquidity crowds out financial intermediation. This second effect is detrimental for welfare because financial intermediaries have access to better investment opportunities than do households, reducing the optimal size of public liquidity. Hence, a large supply of public liquidity that eliminates liquidity crises is not optimal.

Section 5 studies the alternative case in which liquidity injections are implemented by the central bank, which contemporaneously increases public liquidity and buys intermediaries' debt. This intervention is thus akin to an asset purchase program — in which the assets that are purchased are issued by the financial sector — or to a provision of liquidity to financial intermediaries. An additional effect of this approach is that the central bank provides funds to intermediaries, offsetting the crowding out of financial intermediation. However, under a small extension of the model, the policy intervention nonetheless produces the same disintermediation that arises here.

4.1 Liquidity injections: higher supply of government bonds

This section considers the supply of public liquidity by assuming that the government chooses the initial amount of government debt $\overline{B}$ that households are endowed with.[10] In practice, even in non-crisis times, the supply of government debt shows significant variability (Krishnamurthy and Vissing-Jorgensen, 2012). Thus, the analysis presented here could be interpreted as the choice of the optimal supply of government debt $\overline{B}$ in normal times, which is then taken as given once the economy enters a period of financial distress. Analyzing the optimal policy in this way is particularly straightforward and thus allows for the main result and its logic to be conveyed very simply. Nonetheless, Section 5 extends the analysis to the case in which liquidity injections are implemented by the central bank, and newly created liquidity is used to provide funding to financial intermediaries.

Because the government implements its policy by choosing households' endowment of bonds $\overline{B}$ at $t = 0$, the equilibrium for any given $\overline{B}$ is just given by Proposition 1. The only difference is that $\overline{B}$ must now be interpreted as a policy variable rather than as an exogenous endowment.

The optimal policy maximizes the households' utility, (1). Liquidity injections provide a direct benefit because they allow households to finance consumption C_h and C_l at $t = 1$ (in particular, consumption C_l in the low state, in which government debt is the only liquid asset) but crowd out debt D issued by intermediaries and intermediaries' investments K^I. This second effect shrinks the financial sector and increases direct investments by households. Thus, higher $\overline{B}$ reduces total output available at $t = 1$ because households' investment opportunities are worse than the intermediaries' opportunities. The next proposition characterizes the optimal policy.

Proposition 2. *Consider the limiting economy in which $\theta \to \phi$ but $\theta > \phi$. When government chooses the initial level $\overline{B}$ of government debt held by households, the optimal policy is*

$$\left(\overline{B}\right)^* = \frac{\pi\left(1-\phi\right)}{\pi\left(1-\phi\right)+\left(1-\pi\right)\phi} < 1. \tag{19}$$

[10] Alternatively, one can keep $\overline{B}$ as exogenous and assume that the government "prints" and transfers L units of bonds to the households at $t = 0$, so that households start time $t = 0$ with $\overline{B} + L$ units of government bonds. The results are unchanged.

Under the optimal policy, financial intermediaries are active (i.e., $D > 0$ and $K^I > 0$), and the low state is characterized by a liquidity crisis (i.e., $C_l < 1$).

The key result of Proposition 2 is that optimality is reached under a supply of government liquidity that is less than one and, thus, does not satiate households' liquidity needs. That is, under the optimal policy, the liquidity constraint in the low state, (7), is binding. It is useful to contrast this result with that of a model in which intermediaries have no technological advantage with respect to households (i.e., $\phi = 0$); the results in this case follow as a corollary of Propositions 1 and 2.

Corollary 3. *Assume $\phi = 0$ and consider the limiting economy in which $\theta > \phi$ but $\theta \to \phi$. When government chooses the initial level $\overline{B}$ of government debt held by households, the optimal policy is $\left(\overline{B}\right)^* = 1$. Under the optimal policy, intermediaries do not operate: $D = 0$ and $K^I = 0$.*

The case $\phi = 0$ produces a result similar to that of Holmström and Tirole (1998). It is useful to look in detail at this case to clarify the relevant one studied in Proposition 2.

When $\phi = 0$, and thus when intermediaries have no technological advantage with respect to households, the optimal policy is to set public liquidity to $\left(\overline{B}\right)^* = 1$. This result corresponds to the Friedman rule. With this amount of public debt, households can finance consumption $C_h = C_l = 1$, thereby equating the marginal utilities C_h and C_l with the marginal utilities of X_h and X_l. Effectively, the government floods the economy with liquidity. Financial intermediation is shut down, but this outcome does not affect welfare because intermediaries are no better than households at making investments under the assumption $\phi = 0$.

When instead $\phi > 0$, Proposition 2 shows that the optimal supply of government debt is less than one and thus away from the Friedman rule. That is, the optimal supply of liquidity is not too high; indeed, any injection of liquidity, while helping households to finance consumption C_l, crowds out financial intermediation:

$$\frac{\partial K^I}{\partial \overline{B}} = -\frac{1-\pi}{1-\phi} < 0, \quad \frac{\partial D}{\partial \overline{B}} = -1 < 0, \tag{20}$$

where the results follow from Proposition 9. As shown in Proposition 9, financial intermediaries are active under the optimal policy (i.e., they issue debt $D > 0$ and make investments $K^I > 0$), in contrast to the case highlighted in Corollary 3.

The result in (20) resembles that in Geromichalos, Licari, and Suárez-Lledó (2007). In that model, reducing liquidity premia through monetary injections (i.e., liquidity injections) is optimal because it reduces an excessive accumulation of capital. Here, in contrast, reducing liquidity premia rises intermediaries' borrowing costs and thus reduces the size of the banking sector, which is detrimental for welfare.

4.2 Optimal supply of public liquidity: crisis versus non-crisis times

What should be the optimal supply of public liquidity in non-crisis times? The model can be used to answer this question by analyzing the equilibrium under the assumption that $\pi = 0$ (i.e., the probability of the low state is zero).

With no crises, Proposition 2 implies that the optimal supply of public liquidity should be zero: $\left(\overline{B}\right)^* = 0$. That is, absent financial crises, liquidity should be provided only by financial intermediaries through their debt D. Indeed, under $\left(\overline{B}\right)^* = 0$, the government avoids crowding out resources from financial intermediaries, and the amount of resources intermediated by the financial sector is maximal.

Finally, it is worth noting that the optimal supply of public liquidity derived in Proposition 2 is increasing in the probability π of a crisis state:

$$\frac{\partial \left(\overline{B}\right)^*}{\partial \pi} = \frac{\phi\left(1-\theta\right)}{\left[\phi + \pi\left(1-\theta-\phi\right)\right]^2} > 0.$$

As a crisis becomes more likely, government liquidity is increasingly beneficial because of its role in financing consumption in the low state, despite the crowding out of resources intermediated by the financial sector. This result justifies the need to intervene rapidly during crises — a task undertaken by central banks in practice. The next section turns to the analysis of liquidity injections implemented by central banks.

5 Liquidity injections as central bank interventions

The analysis of the baseline model has established that government liquidity injections crowd out the private liquidity issued by financial intermediaries (i.e., their debt D) and the resources intermediated by the financial sector. The result is derived under the assumption that the government changes the supply of Treasury bonds that are then held by the private non-financial sector. While such an approach delivers very simple results, central banks can also inject liquidity and provide funding to the financial sector, especially during crises. Hence, the objective of this section is to study the robustness of the results when the central bank simultaneously injects liquidity and provides funding to the financial sector.

Under a simple extension of the model, the disintermediation created by large liquidity injections arises even when the central bank simultaneously injects liquidity and provides funding to intermediaries.[11] This result is not obvious, as funds provided by the central bank to intermediaries can be used to finance investments, offsetting the disintermediation that arises in the baseline model. That is, even if central bank interventions do provide resources that can be invested by the financial sector, they do not fully counteract the drop in financial intermediation in the extended model.

The welfare analysis is now more complicated because the central bank interventions give rise to two local maxima. The first one is qualitatively similar to the optimal policy of the baseline model. The second one is achieved by a very large liquidity injection — much larger than what is necessary to satiate households' liquidity needs. In this second case, (i) intermediaries' debt D sold to households is zero, and thus liquidity is only provided by government securities; (ii) funding to intermediaries is provided by the central bank; and (iii) in the low state, the central bank incurs in large losses because of intermediaries' default. This second local maxima can be ruled out if there are limits on the central bank's loss-absorption capacity, which is the case in practice because of institutional and legal constraints. This last result is equivalent to a limit on fiscal capacity, if one consolidates the central bank with the fiscal authority (Sargent and Wallace, 1981). Nonetheless, even in this case, the disintermediation effect amplifies the limit on fiscal capacity, so that the optimal supply of public liquidity is smaller in comparison to a model that includes only a

[11] The extension studied here, if applied to the baseline model, would only reinforce the results of Section 4.

limit on fiscal capacity.

5.1 Model with central bank liquidity injections: overview

To study central bank liquidity injections, I make two extensions to the model.

a. Liquidity injections are made by a central bank that faces a constraint on the maximum amount of losses it can make.
b. Households' payment made at $t = 1$ using intermediaries' debt are subject to a haircut χ.

Without Item (a), welfare as a function of public liquidity displays two local maxima. One of such maxima, however, is achieved only if the central bank can sustain very large losses in the low state, thereby requiring to be recapitalized with large transfers from the Treasury.[12] In practice, though, most central banks face institutional constraints that restrict their loss-taking abilities. Therefore, I will study the policy analysis both with and without Item (a).[13]

Item (b) is crucial to obtain the result that liquidity injections reduce the resources intermediated by the financial sector. With no haircut, a central bank policy that swaps intermediaries' debt with public debt does not affect the resources intermediate by the financial sector. The easiest way to understand the implications of Item (b) is to think of consumption purchases in the first subperiods of $t = 1$ as repo-like transactions. That is, households buy consumption goods in the first subperiod of $t = 1$ and use government securities and intermediaries' debt as a collateral for a payment that will be settled in the second subperiod of $t = 1$. While government securities are accepted as collateral at face value, intermediaries' debt is subject to a haircut $\chi > 0$, so a one-dollar security only purchases $1 - \chi$ dollars of goods.

[12]Because the only assets available for purchases are risky securities D that are fully defaulted on in the low state, central bank interventions will always result in *some* losses in the low state. The constraint introduced by Item (a) limits the amount of such losses. More generally, it is possible to extend the model and introduce a technology that allows intermediaries to issue safe securities, so that one could force the central bank to buy only such securities and earn non-negative profits. This approach, however, delivers the same qualitative results but is less transparent because it would require solving the model numerically.

[13]A similar limit on central bank losses is used by Benigno and Nisticò (2019) to study the non-neutrality of open market operations and their possible effects on inflation.

While the results are derived here by taking the haircut χ as exogenous, it is possible to derive it endogenously. A possible approach to do so is based on Li, Rocheteau, and Weill (2012). In their model, a household can fabricate fraudulent securities — either by literally counterfeiting means of payment, or more generally by constructing securities backed by dubious collateral, as in the origination of asset-backed securities based on fraudulent mortgages and deficient lending practices that took place during the mortgage crisis. The fabrication of such securities happens at a cost, however. As in standard models of asymmetric information, households that have non-fraudulent securities can signal the quality of such assets by using only a fraction of their high-quality assets as means of payment — in this case, a fraction $1 - \chi$.[14]

5.2 Model with central bank liquidity injections: details

Government. It is useful to begin the description of this extended model by highlighting how the government injects liquidity in the economy.

At $t = 0$, the central bank purchases intermediaries' debt and finances this operation by creating new reserves.[15] Let D^{cb} be the intermediaries' debt purchased by the central bank, and L be the new reserves. Reserves L are modeled as zero-coupon debt with unitary face value. As a result, households' endowment $\overline{B}$ can be understood as Treasury securities, and the total amount of zero-coupon government securities is $L + \overline{B}$. It is useful to clarify that both L and $\overline{B}$ provide liquidity at $t = 1$ to households. More generally, one could extend the model to introduce other financial institutions — such as small commercial banks — that hold L and $\overline{B}$ and issue deposits that are used by households at $t = 1$. For simplicity, however, I assume that households can make transactions at $t = 1$ directly using L and $\overline{B}$.

The purchase of securities D^{cb} can be interpreted as a central bank asset purchase or, more generally, as loans made by the central bank to financial intermediaries. Under the second interpretation, the amount D^{cb} denotes the face value of the loan.

[14] In the baseline model of Li, Rocheteau, and Weill (2012), the possibility of fraudulent assets remain a threat, in the sense that such assets are not produced in equilibrium. However, they also present an extension in which fraudulent assets are produced in equilibrium.

[15] The approach is similar to that of a monetary model in which a central bank could purchase securities using newly printed money.

At $t = 0$, the budget constraint of the central bank is

$$Q^D D^{cb} \leq Q^B L,$$

where D^{cb} represents the face value of the risky securities purchased by the central bank, Q^D denotes their price, and liquidity L is issued at price Q^B because it has the same payoff and liquidity as the Treasury debt $\overline{B}$.

At the end of $t = 1$, the central bank earns the payoff on the securities acquired at $t = 0$ and withdraws the liquidity L. As a result, the central bank's profit are

$$\Pi_h^{cb} = D^{cb} - L \tag{21}$$

$$\Pi_l^{cb} = -L \tag{22}$$

in the high and low state, respectively. In particular, securities D^{cb} do not appear in (22) because they are fully defaulted on in the low state

The central bank's profits are rebated to the Treasury, so that the budget constraint of the Treasury is

$$T_h + \Pi_h^{cb} = \overline{B}$$

$$T_l + \Pi_l^{cb} = \overline{B}$$

in the high and low state, respectively. That is, the Treasury uses taxes, T_h and T_l, and central bank profits, Π_h^{cb} and Π_h^{cl}, to repay the initial stock of debt $\overline{B}$.

As discussed in Section 5.1, I assume that the central bank must conduct its policy subject to a bound on the maximum losses it can incur. Denoting $\underline{\Pi} \in (-\infty, 0]$ to be such bound, the constraint takes the form:

$$\Pi_h^{cb} \geq \underline{\Pi}, \quad \Pi_l^{cb} \geq \underline{\Pi}. \tag{23}$$

That is, the central bank can incur in some losses — at most, $\underline{\Pi}$ — but such losses are limited. This constraint may arise if the Treasury faces a limit on its ability to recapitalize the central bank in the event of large losses. In practice, this is indeed the case, and many central banks have statutes that de-facto limit the extent of their loss-taking abilities. For instance, after the 2008 crisis, article 13(3) of the Federal Reserve statute was modified to prevent the Fed from taking too much risk in the event of another financial meltdown. In Europe, various policies of the European Central Bank have been scrutinized because of their possible fiscal impact and, typically, are designed to limit such an impact. In any case, I will analyze the optimal policy both with and without (23).

In equilibrium, the constraint (23) is relevant only in the low state because the central bank will earn positive profits in the high state. In particular, (22) and (23) imply a bound on the maximum size of the liquidity injection: $L \geq -\underline{\Pi}$. (Recall that $\underline{\Pi} < 0$, and thus the central bank essentially faces an upper bound on L.)

Households. In this extended model, households' payments at $t = 1$ are subject to a haircut if made using intermediaries' debt D. As described in Section 5.1, it is thus useful to think of transactions in the first subperiod of $t = 1$ as repos. In particular, households buy consumption goods in the first subperiod of $t = 1$ and use government securities B and intermediaries' debt D as collateral for a payment that will be settled in the second subperiod of $t = 1$. Securities B include both the initial Treasury debt $\overline{B}$ as well as the liquidity injected by the central bank L. While securities B are accepted as a collateral at face value, securities D are subject to a haircut $\chi > 0$, so a one-dollar security allows to purchase only $1 - \chi$ dollars of goods. The collateral is returned to the household in the second subperiod of $t = 1$, when the transaction is settled. Because the household owes only $1 - \chi$ at the payment stage, the remaining value χ of the collateral can be used to finance consumption in the second subperiod.

Under the assumption that only a fraction $1 - \chi$ of securities D can be used for transactions at $t = 1$, the liquidity constraint in the high state, (6), is replaced by

$$C_h \leq B + (1 - \chi) D, \tag{24}$$

whereas the liquidity constraint in the low state, (7), is unchanged because D is fully defaulted on in that state. Consumption X_h, which is determined by (8) in the baseline model, is now given by

$$X_h \leq \overline{Y}_h + \Pi_h + [B + (1-\chi) D - C_h] + \chi D + A_h (1-\phi) K - T_h. \qquad (25)$$

Note that, even though a fraction χ of securities D cannot be used to purchase C_h in the first subperiod of $t = 1$, such resources are available in the second subperiod to finance X_h. Equation (9), which refers to X_l, is unchanged.

The first-order condition that governs the optimal choice of intermediaries' debt is now given by

$$(1-\pi) + (1-\pi)(1-\chi)\mu_h \leq Q^D (1-\pi) A_h (1-\phi) \qquad (26)$$

because only a fraction $1-\chi$ of such securities can be used to relax the liquidity constraint. Alternatively, using the definition of the spread S^D in Equation (12), the first-order condition (26) can be rearranged as

$$(1-\chi)\mu_h \leq S^D.$$

That is, the benefits of holding an extra unit of intermediaries' debt D — rather than investing directly in projects — is the availability of an additional $1-\chi$ units of liquidity, which relaxes the liquidity constraint (24); the term μ_h denotes the Lagrange multiplier of (24), similar to the baseline model.

The other equations and first-order conditions that describe households' behavior are unchanged.

Financial intermediaries. At $t = 0$, intermediaries finance their activity by issuing securities D to households and D^{cb} to the central bank. Securities D^{cb} are modeled the same way as D, that is, as zero-coupon bonds with a unitary face value that is fully repaid in state h and fully defaulted on in state l. Thus, both D and D^{cb} are issued at the same price, namely, Q^D. The budget constraint of an intermediary is thus

$$K^I \leq Q^D D + Q^D D^{cb}. \qquad (27)$$

If one interprets policy interventions as loans to intermediaries, $Q^D D^{cb}$ represents the loans extended by the central bank to the financial intermediary.

At $t = 1$, profits in the high state are given by

$$\Pi_h = A_h K^I - D - D^{cb} \tag{28}$$

and profits in the low state are $\Pi_l = 0$, similar to the baseline model.

The moral hazard constraint is the same as in the baseline model, (16). Combining (27), (28), and (16), we obtain that intermediaries are willing to supply of positive amount of securities, $D + D^{cb} > 0$, provided that (17) holds.

5.3 Equilibrium under central bank liquidity injections

The equilibrium concept is the same described in Section 3.1, with the addition that the policy L is announced at the beginning of $t = 0$, before the time-zero market opens. The liquidity injection implies that the market clearing condition for government bonds is now given by

$$B = \overline{B} + L. \tag{29}$$

That is, at $t = 0$, the securities issued by the central bank end up in the hands of households. Indeed, intermediaries hold no government securities in equilibrium. Thus, they trade the securities they receive from the central bank in exchange for households' endowments of goods, which are then invested in the projects K^I.[16]

While it is still possible to solve for the equilibrium for any set of parameters, I maintain a focus on the limiting economy with a mild moral hazard, similar to Section 3.2. In addition, I normalize the endowment of government debt $\overline{B}$ to zero: $\overline{B} = 0$. In this section, I abstract from any restriction on central bank losses — formally, I consider the case in

[16] In this sense, the intermediaries that receive loans from the central bank can be interpreted as large banks that borrow from the central bank and, in turn, use the borrowed resources to make investments. In equilibrium, somebody must hold the central bank's lending, and in the model, the only remaining agents that can do so are households. In practice, private non-financial agents typically hold deposits at smaller banks, so that a liquidity injection would increase such deposits and the reserves held by these banks. In this sense, the households in the model can be interpreted as a sector that consolidates non-financial private agents and smaller banks.

which $\underline{\Pi} = -\infty$, so that the central bank can incur in any loss. I will highlight the role of this constraint in the next section.

The equilibrium differs depending on whether the liquidity injection is $L < 1$ or $L > 1$. As I highlight in the discussion below, the properties of the equilibrium to a marginal change in L are also very different depending on whether $L < 1$ or $L > 1$. I begin the analysis with the case $L < 1$.

Proposition 4. *Assume $\overline{B} = 0$, $\underline{\Pi} = -\infty$, and consider the limiting economy in which $\theta \to \phi$ but $\theta > \phi$. Under a liquidity injection of size $L \in (0,1)$, the equilibrium is characterized by:*

- *Investments by intermediaries, K^I, and households, K, at $t = 0$:*

$$K^I = \frac{1 - L(1-\pi)\chi - \pi\chi}{(1-\phi)(1-\chi)}, \quad K = \overline{Y} - \frac{1 - L(1-\pi)\chi - \pi\chi}{(1-\phi)(1-\chi)};$$

- *Intermediaries' debt:*

$$D = \frac{1-L}{1-\chi};$$

- *Price of government debt, Q^B, and of intermediaries' debt, Q^D:*

$$\begin{aligned} Q^B &= \frac{1}{1-\phi}\left[(1-\pi) + \pi\frac{1}{L}\right], \\ Q^D &= \frac{1-\pi}{1-\phi}, \end{aligned}$$

- *Spread: $S^D = 0$;*
- *Consumption at $t = 1$, first subperiod:*

$$C_h = 1, \quad C_l = L < 1,$$

- *Consumption at $t = 2$, second subperiod:*

$$\begin{aligned} X_h &= \overline{Y}_h + A_h\phi\frac{1 - L(1-\pi)\chi - \pi\chi}{(1-\phi)(1-\chi)} + A_h(1-\phi)\overline{Y} - 1, \\ X_l &= \overline{Y}_l - L. \end{aligned}$$

- *Intermediaries' profits in the high state:* $\Pi_h = \phi \frac{1-L(1-\pi)\chi-\pi\chi}{(1-\phi)(1-\pi)(1-\chi)}$.

The equilibrium under $L < 1$ is broadly similar to that of Proposition 1. An important difference is that, even as $L \to 1$, the amount of investments K^I made by intermediaries does not drop all the way to zero. Indeed, here we have

$$K^I \to \frac{1}{1-\phi} > 1 \quad \text{as } L \to 1.$$

This result arises despite private liquidity D goes to zero as $L \to 1$, as in the equilibrium Proposition 1. The key difference with the baseline model is that loans to intermediaries provide funds that are used for investments K^I. Thus, even if $D \to 0$ as $L \to 1$, intermediaries can invest a positive amount thanks to the funding provided by the central bank.

The sensitivity of intermediaries' investments K^I to a marginal public liquidity injections (i.e., to a marginal change in L) is negative:

$$\frac{\partial K^I}{\partial L} = -\frac{(1-\pi)\,\chi}{(1-\phi)\,(1-\chi)} < 0 \quad \text{for } L \in (0,1)\,. \tag{30}$$

This result, which follows directly from Proposition 4, is also similar to that of the baseline model. Here, however, it is crucial that the haircut χ on private liquidity D is positive. Indeed, with $\chi = 0$, (30) implies $\partial K^I/\partial L = 0$.

To understand the effect played by the haircut χ on liquidity injection, first consider the case $\chi = 0$. In this case, any public liquidity injection crowds out private debt D one-for-one. As a result, the lower funds received by households are exactly offset by the higher central bank lending. In contrast, when $\chi > 0$, a public liquidity injection crowds out "net" private liquidity $(1-\chi)\,D$ one-for-one. However, $\chi > 0$ implies that D drops *more* than one-for-one with a liquidity injection. The overall effect is thus a contraction of the resources K^I invested by financial intermediaries.[17]

As a second step, I solve for the equilibrium under a large injection — more precisely, $L \in \left[1, \overline{Y}\,(1-\phi)\right]$. Absent limit on central bank losses, the injection $L = \overline{Y}\,(1-\phi)$ implements the first best, as shown in the next section. Thus, I do not consider injections above this threshold. When $L \in \left[1, \overline{Y}\,(1-\phi)\right]$, the central bank provides enough liquidity

[17] Adding the haircut χ to the baseline model would only reinforce the results of Section 4.

to satiate households' demand. In addition, because debt D reaches zero once the liquidity injection reaches $L = 1$, no more crowding out takes place if more liquidity is injected. This is due to the fact that intermediaries' debt must be non-negative, and thus cannot drop below zero.[18] As a result, as L increases above one, intermediaries get more resources and their investments increase in L:

$$\frac{\partial K^I}{\partial L} = \frac{1}{1-\phi} > 0 \quad \text{for } L \in \left(1, \overline{Y}\left(1-\phi\right)\right). \tag{31}$$

This result is the opposite of what happens when $L < 1$ — compare (31) with (30). This stark difference gives rise to the non-monotonic welfare effects of liquidity injections, as explained in details in Section 5.4.

The next proposition provides the full characterization of the equilibrium when $L \in \left[1, \overline{Y}\left(1-\phi\right)\right]$.

Proposition 5. *Assume $\overline{B} = 0$, $\underline{\Pi} = -\infty$, and consider the limiting economy in which $\theta \to \phi$ but $\theta > \phi$. Under a liquidity injection of size $L \in \left[1, \overline{Y}\left(1-\phi\right)\right]$, the equilibrium is characterized by:*

- *Investments by intermediaries and households, at $t = 0$:*

$$K^I = \frac{L}{1-\phi}, \quad K = \overline{Y} - \frac{L}{1-\phi};$$

- *Intermediaries' debt: $D = 0$;*
- *Price of government debt and of intermediaries' debt:*

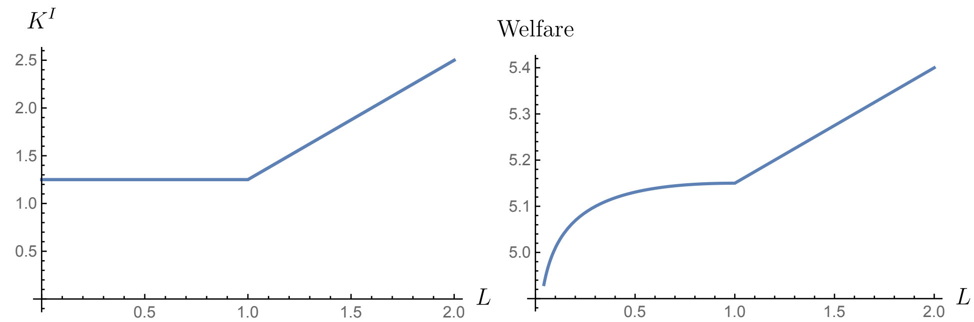
$$Q^B = \frac{1}{1-\phi}, \quad Q^D = \frac{1-\pi}{1-\phi};$$

- *Consumption at $t = 1$, first subperiod: $C_h = C_l = 1$;*
- *Consumption at $t = 2$, second subperiod:*

$$X_h = \overline{Y}_h + A_h\left(1-\phi\right)\overline{Y} + A_h\phi\frac{L}{1-\phi} - 1,$$

[18] If households could short debt D (i.e., borrow from intermediaries), the crowding out would still take place. However, as discussed in Section 2.1, households lack the commitment to repay debt and thus cannot borrow in equilibrium. This assumption endogenously generates the constraint $D \geq 0$.

Figure 1: Policy effects with $\underline{\Pi} = -\infty$ and $\chi = 0$



Left panel: effects of injections L on K^I; Right panel: effects of injections L on welfare. Parameter values: $\overline{B} = 0$; $\phi = 0.1$; $\overline{Y} = 2.5$; $\pi = 0.1$; $\chi = 0$; $\overline{Y}_h = 4$; $\overline{Y}_l = 3$; $\theta \to \phi$.

$$X_l \quad = \quad \overline{Y}_l - 1.$$

- *Intermediaries' profits in the high state:* $\Pi_h = 0$.

5.4 Optimal central bank policy

I now turn to the determination of the optimal central bank lending L to financial intermediaries. To clarify the role of the limit on central bank losses and the haircut on intermediaries' debt (i.e., Items (a) and (b) introduced in Section 5.1), I first solve for the optimal policy without these assumptions. Then, I add the assumptions one at the time, and finally present the optimal policy in the full model. Similar to the previous sections, all the results are provided under the normalization $\overline{B} = 0$ and for the case of a mild moral hazard (i.e., $\theta \to \phi$).

Optimal policy with $\underline{\Pi} = -\infty$ and $\chi = 0$. I begin by deriving the optimal policy when I shut down Items (a) and (b) introduced in Section 5.1. Formally, I set the limit on central bank losses $\underline{\Pi}$ to $\underline{\Pi} = -\infty$, so that the central bank can incur in arbitrarily large losses in the low state, and I set the haircut on private intermediaries' debt χ to $\chi = 0$. The next proposition characterizes the results in this case.

Proposition 6. *Assume* $\underline{\Pi} = -\infty$ *and* $\chi = 0$. *Normalize* $\overline{B} = 0$ *and consider the limiting* *economy in which $\theta > \phi$ but $\theta \to \phi$. The optimal liquidity policy is*

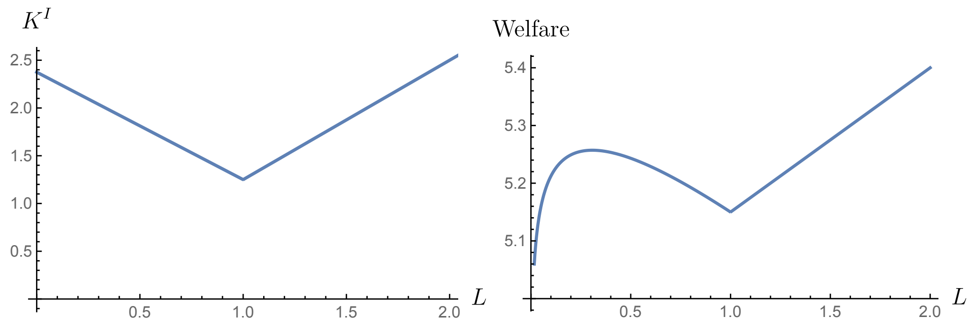
$$L^* = \overline{Y}\,(1-\phi) > 1$$

and implements the first best: $C_h = C_l = 1$, $K^I = \overline{Y}$, $K = 0$.

To understand the result of Proposition 6, consider the plots in Figure 1. As L ranges from 0 to 1, changing L does not affect the amount of investments K^I made by intermediaries, as shown in the left panel. This result can be understood by looking at the liquidity constraint (24) and the time-zero budget constraint of intermediaries, (27). Recall that, because of the moral hazard of financial intermediaries, households want to limit the amount of securities D they buy. As discussed in the previous section, any additional unit of government liquidity crowds out one-for-one the amount of private debt D purchased by households, without affecting households' ability to finance their consumption in the high state of $t = 1$. On the intermediaries' side, however, the lower amount of debt D issued at $t = 0$ is exactly offset by a higher issuance of securities D^{cb} that are purchased by the central bank. As a result, the amount of investments made by intermediaries is unchanged. Turning to the welfare as L ranges from 0 to 1, the effect of increasing L is to increase liquidity, which is beneficial to households. As a result, welfare is strictly increasing in this range. When $L = 1$, there is enough liquidity to satiate households' demand using public debt only. As a result, households choose $D = 0$. At that point, intermediaries' private debt D cannot decrease any further. Thus, any additional unit of liquidity provided by the central bank is used by intermediaries to finance more investments, explaining the increase in K^I for $L > 1$. Welfare increases in this range too because more capital is intermediated by intermediaries, which are more productive than households. For a sufficiently high L, all the time-zero endowment $\overline{Y}$ is invested by intermediaries.

To sum up, with $\chi = 0$ and no restrictions on central bank losses, a sufficiently large liquidity injection can both (i) satiate households' liquidity needs and (ii) provide enough resources to intermediaries so that they invest all the endowment $\overline{Y}$. Thus, this injection achieves the first best. In the numerical example of Figure 1, the injection that achieves the first best is $L = 2$.

Figure 2: Policy effects with $\underline{\Pi} = -\infty$ and $\chi > 0$



Left panel: effects of injections L on K^I; Right panel: effects of injections L on welfare. Parameter values: $\overline{B} = 0$; $\phi = 0.2$; $\overline{Y} = 2.5$; $\pi = 0.1$; $\chi = 0.5$; $\overline{Y}_h = 4$; $\overline{Y}_l = 3$; $\theta \to \phi$.

Optimal policy with $\underline{\Pi} = -\infty$ and $\chi > 0$. As a second step, I consider the case in which the haircut χ on private debt is positive, but there are still no limits on central bank losses.

Proposition 7. *Assume $\underline{\Pi} = -\infty$ and $\chi > 0$. Normalize $\overline{B} = 0$ and consider the limiting economy in which $\theta > \phi$ but $\theta \to \phi$. Liquidity injections give rise to two local welfare maxima:*

$$L^* = \frac{\pi(1-\phi)(1-\chi)}{\phi\chi + \pi(1-\phi-\chi)} < 1 \qquad \textit{and} \qquad L^{**} = \overline{Y}(1-\phi) > 1. \tag{32}$$

The global maximum is achieved by L^{**}, which implements the first best: $C_h = C_l = 1$, $K^I = \overline{Y}$, $K = 0$.

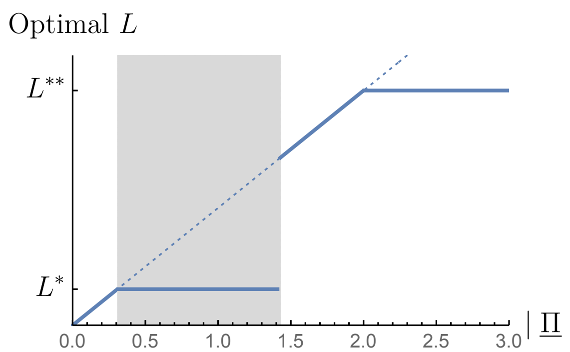
In this case, a non-monotonicity arises, giving rise to two local maxima (see Figure 2). As L lies between 0 and 1, a liquidity injection now reduces K^I. As before, an injection of central bank, public liquidity crowds out private liquidity D. However, now that the haircut is positive, every additional unit of L crowds out *net* liquidity $D(1-\chi)$, and thus D drops more than one-for-one because $\chi > 0$. That is, the haircut on D amplifies the crowding out effect. As a result, intermediaries' resources drop and their time-zero investments K^I decrease. Turning to welfare as L ranges from 0 to 1, a local maximum is reached at some intermediate value of L. That's because higher liquidity injections have two effects: they provide liquidity to the economy — which is good for improving the allocation of resources at $t = 1$ — but they crowd out intermediaries' investments — which is bad for welfare. As $L = 1$, private liquidity D reaches $D = 0$, as in the case with no haircut, and thus any additional liquidity injected by the central bank increases K^I. That is, for $L > 1$, the effects of liquidity injections on K^I and welfare are independent of χ.

In summary, absent any limit on the central bank's interventions, the first best can still be achieved by a sufficiently large injection ($L = 2$ in the example of Figure 2). Contrary to the case with $\chi = 0$, though, an injection that is barely sufficient to satisfy liquidity needs (i.e., $L = 1$) is highly suboptimal, in the sense that a slightly higher or slightly lower injection is preferred. That is, $L = 1$ is a local minimum.

Optimal policy with $\underline{\Pi} > -\infty$ and $\chi > 0$. The optimal policy in the full model with $\chi > 0$ and a constraint on the central bank's losses follows from the previous analysis. Absent the constraint on the central bank's losses, the welfare function is the same as the one depicted in Figure 2. Thus, to choose the optimal (and feasible) policy subject to the constraint on losses, the central bank could first compute the unconstrained welfare, then exclude the injections that violate the constraint on profits (23), and finally choose the optimal injection among those that are feasible. The outcome is represented in Figure 3. The horizontal axis denotes the maximum loss that the central bank can take, that is, the absolute value of $\underline{\Pi}$. Thus, moving to the right along the horizontal axis means that the central bank can take larger losses.

It is useful to start from the extreme case in which $\underline{\Pi} = 0$ (i.e., the central bank cannot sustain any loss). In this case, the central bank cannot implement any policy because any purchase $D^{cb} > 0$ would produce losses in the low state. As the constraint on losses (23) is relaxed (i.e., as we move to the right on the horizontal axis of Figure 3), the central bank can do some intervention, and the policy L chosen by the central bank moves one-for-one with the loosening of the constraint (i.e., along the 45-degree line in Figure 3). As (23) gets looser and looser, the central bank can implement the first local maximum of the welfare function, that is, L^* defined in (32). Because L^* is a local maximum, a further loosening of the constraint (23) — corresponding to the shaded area of Figure 3 — does not change the optimal stance, that remains L^*. This is the central result of this section. That is, the central bank's optimal policy is such that the constraint on losses (23) is not binding in

Figure 3: Optimal policy with $\underline{\Pi} > -\infty$ and $\chi > 0$



Horizontal axis: Maximum loss that can be incurred by the central bank (in absolute value); Solid line: Optimal liquidity injection L. Dotted line: 45-degree line. The shaded area represents the region in which the limit on central bank losses does not bind in equilibrium because of the disintermediation generated by liquidity injections. Parameter values: $\overline{B} = 0$; $\phi = 0.2$; $\overline{Y} = 2.5$; $\pi = 0.1$; $\chi = 0.5$; $\overline{Y}_h = 4$; $\overline{Y}_l = 3$; $\theta \to \phi$.

equilibrium, not even in the low state.

As the constraint gets very loose — to the right of the shaded area in Figure 3 — it becomes optimal to make a much bigger injection, and the optimal policy jumps from $L^* < 1$ to some $L > 1$. Further loosening of (23) increase the optimal L even more along the 45-degree line, up to the point at which the central bank can inject L^{**} defined in (32). With $L = L^{**}$, the central bank implements the first best, and thus the ability to take further losses does not change the policy stance.

Note that, for values of $\underline{\Pi}$ corresponding to the shaded area in Figure 3, the constraint (23) is slack along the equilibrium path in which the central bank implements the optimal policy. Yet, this constraint does matter for the result in the sense that, absent (23), the central bank would have chosen a different policy. This result is the byproduct of the disintermediation that gives rise to the non-monotonic link between L and welfare.

The next proposition summarizes the results, formalizing the optimal policy as the constraint (23) is loosened.

Proposition 8. *Normalize $\overline{B} = 0$ and consider the limiting economy in which $\theta \to \phi$ but*

$\theta > \phi$. Let L^* and L^{**} be the local maxima defined in Proposition 7, and let

$$\hat{L} \equiv -\frac{\phi\left(1-\pi\chi\right)+\pi\left(1-\chi+\chi\phi-\phi\right)\log\left[\frac{\pi(1-\chi)(1-\phi)}{\pi(1-\chi-\phi)+\chi\phi}\right]}{\phi\left(1-\chi\right)}.$$

Then:

a. *If $\underline{\Pi} \in (-L^*, 0]$, the optimal policy is $L = -\underline{\Pi}$ and (23) binds in equilibrium;*
b. *If $\underline{\Pi} \in (-\hat{L}, L^*]$, the optimal policy is $L = L^*$ and (23) does not bind in equilibrium;*
c. *If $\underline{\Pi} \in (-L^{**}, \hat{L}]$, the optimal policy is $L = -\underline{\Pi}$ and (23) binds in equilibrium;*
d. *If $\underline{\Pi} \in (-\infty, L^{**}]$, the optimal policy is $L = L^{**}$ and (23) does not bind in equilibrium.*

6 Conclusions

This paper studies public liquidity injections in a model in which financial intermediaries have access to better investment opportunities than other agents but are plagued by a moral hazard problem. Liquidity injections crowd out private liquidity and give rise to a process of disintermediation wherein more investments are made by non-financial agents, and fewer are made by intermediaries. This effect decreases welfare and, under some conditions, prevents the optimality of a large supply of public liquidity. When compared to monetary models, the disintermediation effect prevents the optimality of the Friedman rule, in contrast to the result that arises in a very large number of settings.

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Appendix

A Equilibrium with more severe moral hazard

This appendix analyzes the equilibrium in the general case with an arbitrary severe moral hazard problem, that is Equation (4) holds and there are no further restrictions on θ and ϕ. The main trade-off described in Section 3.2 still holds. That is, households trade off the liquidity of D (i.e., its ability to relax the constraint (6)) with the opportunity cost of holding D, namely, the spread S^D defined in (12), except now S^D is now possibly large. The financial liquidity crisis in the low state is the same as described in Section 3.2. That is, the default on intermediaries' debt D is independent of moral hazard considerations, and thus consumption in state l is financed only with government debt, too.

When no restriction is imposed on the moral-hazard parameter θ other than (4), the spread S^D is positive in equilibrium and possibly large. As a result, households find it optimal to make choices so that their high-state consumption, C_h, is below the first-best level $C_h = 1$.

The positive spread on D is equivalent to a higher price Q^D at which intermediaries issue D, in comparison to the economy with a mild moral hazard problem of Section 3.2. This higher price generates additional profits, in comparison to those earned by intermediaries of the economy of Section 3.2. Such higher profits are necessary to provide incentives to intermediaries to behave well because the moral hazard problem is in general more severe here.

While the above paragraphs implicitly assume that intermediaries are active in equilibrium, the equilibrium is featured by a lack of financial intermediation for some levels of $\overline{B}$ sufficiently close to one — and even if $\overline{B} < 1$. While this extreme case is not central to the analysis, it is helpful to discuss it to clarify the mechanics of the model. When $\overline{B}$ is sufficiently close to one, there is enough liquidity in circulation so that households choose to finance a large amount of consumption C_h using only public debt, and hold $D = 0$. In this case, the value associated with relaxing the liquidity constraint in the high state, (6), is small, and so is its Lagrange multiplier μ_h. As a result, households' benefits from holdings intermediaries' debt D are lower than the cost, represented by the spread defined in (12), confirming that households do prefer to chose $D = 0$. Alternatively, one can think that households would be willing to buy D only if the spread is small enough. However, with a small spread, intermediaries' profits are not high enough to discipline their behavior. Financial intermediation, thus, shuts down, as any attempt to intermediate resources would result in intermediaries' misbehavior and thus a default on their debt D even in the high state. The equilibrium with no financial intermediation arises even for values of government debt $\overline{B} > (1-\theta)\,/\,(1-\phi)$, where $(1-\theta)\,/\,(1-\phi) < 1$ because of (4). In addition, if $(1-\theta)\,/\,(1-\phi) < \overline{B} < 1$, not only will the financial sector shut down (i.e., $D = K^I = 0$) but there also will not be enough liquidity to satiate households' demand, and thus $C_h, C_l < 1$. If instead $\overline{B} = 1$, there is plenty of liquidity to finance $C_h = C_l = 1$, as discussed in Section 4.

Proposition 9. *The equilibrium is:*

- *Investments by intermediaries at $t = 0$,*

$$K^I = \begin{cases} \frac{(1-\pi)\left[1-\theta-\overline{B}(1-\phi)\right]}{(1-\theta)(1-\phi)} & \text{if } \overline{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

  and households, $K = \overline{Y} - K^I$;

- *Intermediaries' debt :*

$$D = \begin{cases} \frac{1-\theta-\overline{B}(1-\phi)}{1-\phi} & \text{if } \overline{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}; \end{cases}$$

- *Price of government debt, Q^B, and of intermediaries' debt, Q^D:*

$$\begin{aligned} Q^B &= \begin{cases} (1-\pi)\,\frac{1}{1-\theta} + \pi\frac{1}{\overline{B}(1-\phi)} & \text{if } \overline{B} < \frac{1-\theta}{1-\phi} \\ \frac{1}{\overline{B}(1-\phi)} & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}, \end{cases} \\ Q^D &= \frac{1-\pi}{1-\phi}; \end{aligned}$$

- $Spread = (\theta - \phi) / (1 - \theta)$;
- *Consumption at $t = 1$, first subperiod:*

$$C_h = \begin{cases} \frac{1-\theta}{1-\phi} & \text{if } \overline{B} < \frac{1-\theta}{1-\phi} \\ \min\left\{1, \overline{B}\right\} & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

and $C_l = \min\left\{1, \overline{B}\right\}$*;*

- *Consumption at $t = 2$, second subperiod:*

$$X_h = \begin{cases} \overline{Y}_h + \overline{Y}\frac{1-\phi}{1-\pi} - \frac{1-\theta-\phi}{1-\phi} - \overline{B}\frac{\phi}{1-\theta} & \text{if } \overline{B} < \frac{1-\theta}{1-\phi} \\ \overline{Y}_h + \overline{Y}\frac{1-\phi}{1-\pi} - \overline{B} & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}, \end{cases}$$

$$X_l = \overline{Y}_l - \overline{B};$$

- *Intermediaries' profits in the high state:*

$$\Pi_h = \begin{cases} \theta\frac{1+\theta-\overline{B}(1-\phi)}{(1-\theta)(1-\phi)} & \overline{B} < \frac{1-\theta}{1-\phi} \\ 0 & \text{if } \overline{B} \geq \frac{1-\theta}{1-\phi}. \end{cases}$$

B Proofs

Proof of Proposition 1. The result follows directly from the definition of the equilibrium in Section 3.1. That is, given $\overline{B}$, I solve for Q^B, Q^D, D, B, K^I, K, Π_h, C_h, C_l, X_h, X_l using households' first-order conditions (10) and (11), with the Lagrange multipliers given by (13), the households' budget constraint at $t = 0$, (5), the households' liquidity constraints (6) and (7), and the time-1 resource constraints (2) and (3); the intermediaries' budget constraint (14), its profits (15), and the moral-hazard constraint (16); and the market-clearing condition for government bonds $B = \overline{B}$. The spread can then be computed using (12). The results are then evaluated at $\theta \to \phi$. □

Proof of Proposition 2. The optimal policy is computed by maximizing the utility function of households, (1), evaluated at the equilibrium of Proposition 1, with respect to $\overline{B}$. That is, after plugging in the values of C_h, C_l, X_h, X_l from Proposition 1 into (1), I take the first-order condition with respect to $\overline{B}$ and solve for $\overline{B}$, which yields (19). The results $D > 0$ and $K^I > 0$ follow from plugging in (19) into the equilibrium values of D and K^I derived in Proposition 1. □

Proof of Proposition 4. The results are derived similarly to that of Proposition 1. That is, given $\overline{B} = 0$ and $L \in (0, 1)$, I solve for Q^B, Q^D, D, B, K^I, K, Π_h, C_h, C_l, X_h, X_l using households' first-order conditions (10) and (26), with the Lagrange multipliers given by (13), the households' budget constraint at $t = 0$, (5), the households' liquidity constraints (24) and (7), and the time-1 resource constraints (2) and (3); the intermediaries' budget constraint (27), its profits (28), and the moral-hazard constraint (16); and the market-clearing condition for government liquidity (29). The spread can then be computed using (12). The results are then evaluated at $\theta \to \phi$. □

Proof of Proposition 5. Given $\overline{B} = 0$, $L > 1$ and the market-clearing condition for government liquidity (29), the time-1 households' liquidity constraints (24) and (7) are not binding because the households can finance $C_h = C_l = 1$, so that (13) holds with $\mu_h = \mu_l = 0$. As a result, the first-order condition with respect to intermediaries' debt D, (26), implies $D = 0$ because the lack of commitment described in Section 2.1 implies that households cannot borrow in equilibrium: $D \geq 0$. I can then solve for the remaining variables Q^B, Q^D, K^I, K, Π_h, X_h, X_l using the households' budget constraint at $t = 0$, (5), households' first-order condition (10), the time-1 resource constraints (2) and (3), the intermediaries' budget constraint (27), its profits (28), and the moral-hazard constraint (16). The results are then evaluated at $\theta \to \phi$. □

Proof of Proposition 6. The proof is conceptually similar to that of Proposition 2. That is, given $\overline{B} = 0$ and $\chi = 0$, I compute welfare by evaluating the utility function of households, (1), at the equilibrium described by Propositions 4 or 5, depending on the value of L. Then, I obtain

$$\left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in(0,1)} = \frac{1}{L} - 1 > 0, \qquad \left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in\left(1,\overline{Y}(1-\phi)\right)} = \frac{\phi}{1-\phi} > 0.$$

Thus, welfare is monotonically increasing in L in the interval for $L \in \left(0, \overline{Y}\,(1-\phi)\right)$. Then, evaluating the equilibrium of Proposition 5 at $L = \overline{Y}\,(1-\phi)$, I obtain $K^I = \overline{Y}$ and $K = 0$, that is, all the time-0 investments are made by banks. This result, in conjunction with $C_h = C_l = 1$ from Proposition 5, implies that $L = \overline{Y}\,(1-\phi)$ implements the first best. Thus, $L = \overline{Y}\,(1-\phi)$ is the optimal policy. □

Proof of Proposition 7. The proof is similar to that of Proposition 6, but now $\chi > 0$. I compute welfare by evaluating the utility function of households, (1), at the equilibrium described by Propositions 4 or 5, depending on the value of L. Then, I obtain

$$\begin{aligned} \left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in(0,1)} &= \frac{\pi\,(1-\phi-\chi)\,(1-L) - \phi\chi\,(L-\pi)}{L\,(1-\phi)\,(1-\chi)}, \\ \left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in\left(1,\overline{Y}(1-\phi)\right)} &= \frac{\phi}{1-\phi} > 0. \end{aligned}$$

Given $\chi > 0$, welfare is not monotonic in L, different from the proof of Proposition 6. In particular:

$$\left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in(0,1)} \to +\infty \quad \text{as } L \downarrow 0, \qquad \left.\frac{\partial\,(\text{Welfare})}{\partial L}\right|_{L\in(0,1)} \to -\frac{(1-\pi)\,\phi\chi}{(1-\phi)\,(1-\chi)} \quad \text{as } L \uparrow 1.$$

Thus, welfare is non-monotone in L and there exists a local maximum in $(0,1)$. Solving the first-order condition $\left.\frac{\partial(\text{Welfare})}{\partial L}\right|_{L\in(0,1)} = 0$ and solving for L, I obtain the local maximum L^* stated in the proposition. The other local maximum, L^{**}, is the one that implemented the first best, and thus is the same as the one derived in the proof of Proposition 6. Because L^* implies $C_l < 1$ and $K^I < \overline{Y}$, it does not implement the first best. Hence, the optimal policy is the global maximum L^{**}. □

Proof of Proposition 8. The value $\hat{L}$ is derived by equating welfare under the policy L^* and welfare for some $L > 1$, and solving for L. Thus, $\hat{L}$ represent the lowest value of $\underline{\Pi}$ such that L^* is optimal. For lower values of $\underline{\Pi}$ (i.e., when (23) loosens), $L = -\underline{\Pi}$ is optimal, up to $L = \overline{Y}(1 - \phi)$, which implement the first best, so that any further injections cannot improve the equilibrium further. Finally, in the region $\underline{\Pi} \in (-L^*, 0)$, neither L^* nor L^{**} can be implemented and thus the optimal policy is to choose an injection L that makes (23) binding in equilibrium: $L = -\underline{\Pi}$. □